SIGNALS AND SYSTEMS

For
ELECTRICAL ENGINEERING
INSTRUMENTATION ENGINEERING
ELECTRONICS & COMMUNICATION ENGINEERING

SIGNALS AND SYSTEMS

SYLLABUS

ELECTRONICS AND COMMUNICATION ENGINEERING

Definitions and properties of Laplace transform, continuous-time and discrete-time Fourier series, continuous-time and discrete-time Fourier Transform, DFT and FFT, z-transform. Sampling theorem. Linear Time-Invariant (LTI) Systems: definitions and properties; causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay. Signal transmission through LTI systems.

ELECTRICAL ENGINEERING

Representation of continuous and discrete-time signals; shifting and scaling operations; linear, time-invariant and causal systems; Fourier series representation of continuous periodic signals; sampling theorem; Fourier, Laplace and Z transforms.

INSTRUMENTATION ENGINEERING

Periodic and aperiodic signals. Impulse response, transfer function and frequency response of first- and second order systems. Convolution, correlation and characteristics of linear time invariant systems. Discrete time system, impulse and frequency response. Pulse transfer function. IIR and FIR filters.

ANALYSIS OF GATE PAPERS

	ELECTRONICS			ELECTRICAL			INSTRUMENTATION		
Exam Year	1 Mark Ques.	2 Mark Ques.	Total	1 Mark Ques.	2 Mark Ques.	Total	1 Mark Ques.	2 Mark Ques.	Total
2003	4	3	10	-	-	-	3	9	21
2004	3	6	15	1	2	5	3	8	19
2005	6	6	18	-	4	8	3	3	9
2006	3	3	9	3	4	11	-	8	16
2007	1	4	9	2	5	12	2	3	8
2008	2	8	18	2	7	16	3	7	17
2009	3	5	13	1	3	7	2	4	10
2010	2	3	8	2	4	10	6	3	12
2011	3	4	11	2	2	6	7	3	13
2012	2	3	8	2	4	10	4	3	10
2013	7	3	13	6	1	8	7	3	13
2014 Set-1	4	4	12	2	3	8	4	2	8
2014 Set-2	3	3	9	2	2	6	-	-	-
2014 Set-3	4	4	12	3	3	9	-	-	-
2014 Set-4	4	4	12	-	-	-	-	-	-
2015 Set-1	2	3	8	2	2	6	2	6	14
2015 Set-2	5	4	13	1	4	9	-	-	-
2015 Set-3	4	6	16	-	-	-	-	-	-
2016 Set-1	4	4	12	4	3	10	4	4	12
2016 Set-2	1	2	5	5	2	9	-	-	-
2016 Set-3	3	4	11	-	-	-	-	-	-
2017 Set-1	3	4	11	2	2	6	2	3	8
2017 Set-2	2	3	8	2	2	6	-	-	-
2018	3	2	7	2	2	6	2	3	8

CONTENTS

Topics **Page No**

1 INTRODUCTION TO SIGNALS & SYSTEMS

1.1 INTRODUCTION TO SIGNAL

Signals play a major role in our life. In general, a signal can be a function of time, distance, position, temperature, pressure, etc., and it represents some variable of interest associated with a system. For example, in an electrical system the associated signals are electric current and voltage. In a mechanical system, the associated signals may be force, speed, torque etc. In addition to these, some examples of signals that we encounter in our daily life are speech, music, picture and video signals.

"A signal is a function representing a physical quantity which conveys some amount of information."

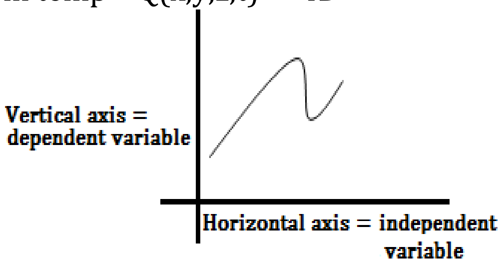
Signals can vary by more than one independent variables.

Example Speech (passing through a telephone line), one variable → A cosωt

Image – I(x,y) (2D)

T.V. Picture – I(x,y,t) → 3D

Room temp – Q(x,y,z,t) → 4D



1.2 CLASSIFICATION OF SIGNALS

1.2.1 CONTINUOUS-TIME AND DISCRETE - TIME SIGNALS

Signals can be classified based on their nature and characteristics in the time domain. They are broadly classified as (i) continuous–time signals and (ii) discrete time signals. A continuous-time signal is a mathematically continuous function and the function is defined continuously in the time domain. On the other hand, a discrete-time signal is specified only at certain time instants. The amplitude of the discrete-time signal between two time instants is not defined.

A. Continuous time and Discrete time Signals:- (Change in Horizontal Axis)

A signal x(t) is a continuous –time signal if it is a continuous variable. If it is a discrete variable than it is defined as discrete time signal.

Notations:

x(t) → continuous time signal

x[n] → discrete time signal

B. Discrete-time Signals – Sequences

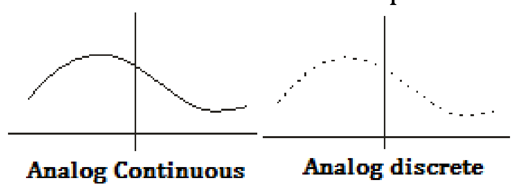
A discrete–time signal has a value defined only at discrete points in time and a discrete-time system operates on and produces discrete-time signals. A discrete-time signal is a sequence is a sequence which is a function defined on the positive and negative integers, that is, x(n) = {x(n)} = {.....x(-1), x(0), x(1), ...} where the up-arrow ↑ represents the sample at n = 0.

If a continuous-time signal x(t) is sampled every T seconds, a sequence x(nT)results. In the sample interval, T for convenience, the sample interval T is taken as 1 second and hence x(n) represents the sequence.

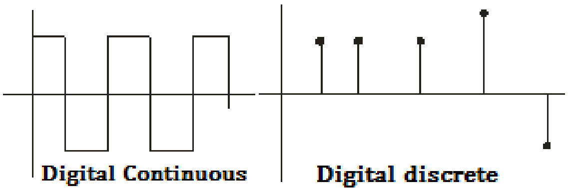
1.2.2 ANALOG AND DIGITAL SIGNALS: (Change in Vertical Axis)

1) Analog:- A Signal can take on infinite number of distinct values in amplitude.

2) Digital:- A Signal can take on finite number of distinct values in amplitude.

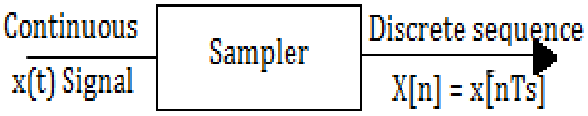


3) Digital Continuous



1. Analog Continuous Signal
2. Analog Discrete Signal
3. Digital Continuous Signal
4. Digital Discrete Signal

Sampler: It is a device (switch) which convert continuous time signal to discrete time sequence.



$T_S \rightarrow$ Sampling time

$t = nT_S \quad n = 0, \pm 1, \pm 2, \pm 3$

$T_S \rightarrow$ sampling time

Example: $x(t) = e^{-2t}\, u(t)$ in discrete form

Solution: $x(nT_S) = e^{-2nTS} \quad [nT_S]$, for T_S=1 sec.

$x[n] = e^{-2n}\, u[n]$

→ Sampling – discretizing – X – axis

→ Quantization–discretizing–Y – axis

Example:-For given the continuous time signal

$$x(t) = \begin{cases} 1-|t| & -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

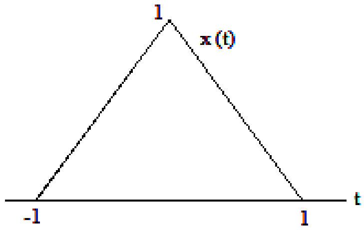
Determine the discrete time sequence obtained by uniform sampling of x(t) with a sampling interval of

a) 0.25 sec.

b) 0.5 sec.

c) 1.0 sec.

Solution: - Draw x(t)



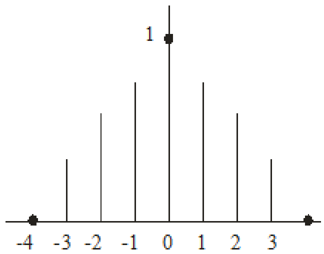
$$x(t) = \begin{cases} 1-t & 0 \le t \le 1 \\ 1+t & 0 \ge t \ge -1 \end{cases}$$

a) T_S = .25sec., $t = n.T_S$

n = 0,1,2,3,4 …………………

$x[n] = x[n/4]$

$x[n]$ = {0,.25, 05, .75, 1, .75, .5, .25, 0 ………………} ↑



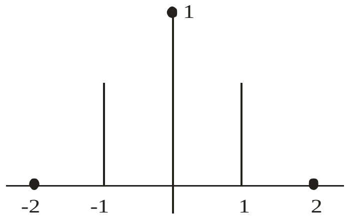
b) T_S = .5sec. $t = nT_S$

n = 0,1,2,3,4 …………………

$x_2[n] = x[n/2]$

$x[n]$ = [.0, 0.5, 1, .5, 0]

↑



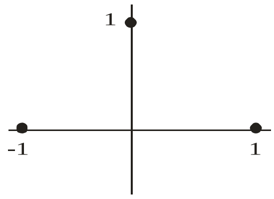
c) T_S =1sec., $t = nT_S$

n = 0,1,2,3,4

$x_3[n] = x[n]$

$x[n]$ = {…………0, 1, 0…………} = s[n]

↑



T_S ↑ no. of samples in given interval deceases f_S ↓, T_S ↑, or f_S ↑, T_S↓

1.2.3 REAL AND COMPLEX SIGNAL

A signal x(t) is a real signal if its value is a real number, if x(t) complex number, then it complex Signal.

$x(t) = x_1(t) + jx_2(t)$ → imaginary part

Real part

$j = i = \sqrt{-1}$

$x_1(t)$ → real part = $x_r(t)$

$x_2(t)$ → imaginary part = $x_i(t)$

$$\angle x(t) = \tan^{-1}\left(\frac{x_2(t)}{x_1(t)}\right) \qquad x = r\cos\theta \qquad y = r\sin\theta$$

Two form to representational

1) Cartesian form
2) Polar form

(1) Cartesian form :- x + jy,

Example :

a) $e^{-j\pi/2}$

b) $e^{-j5\pi/2}$

c) $\sqrt{2}e^{-j\pi/4}$,

d) $\sqrt{2}e^{-j9\pi/4}$

Solution :

a) –j b) +j

c) $\sqrt{2}\left[\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}}\right]=1+j$ d) 1 –j

(2) Polar Form :- $re^{j\theta}$

Example :

a) 5

b) -2 -3j

c) j(1 –j)

d) $\frac{1+j}{1-j}$

e) $\left[\frac{(\sqrt{2}+j\sqrt{2})}{(1+j\sqrt{3})}\right]$, $-\pi \le \theta \le \pi$

Solution :

a) $5e^{j\theta}$ b) $\sqrt{13}e^{-j\tan^{-1}\left(\frac{3}{2}\right)}$

c) $e^{-j\pi/4}.\sqrt{2}$ d) $e^{j\pi/2}$

e) $e^{-j\pi/12}$

1.2.4 DETERMINISTIC AND RANDOM SIGNALS

Deterministic and Non-deterministic Signals

Deterministic signals are functions that are completely specified in time. The nature and amplitude of such a signal at any time can be predicted. The pattern of the signal is regular and can be characterized mathematically examples of deterministic signals are

(i) x(t) = α t This is a ramp whose amplitude increases linearly with time and slope is α.

(ii) x(t) = A sin ωt. The amplitude of this signal varies sinusoidally with time and its maximum amplitude is A.

(iii) $x(n) = \begin{cases} n \ge 0 \\ 0 \quad \text{otherwise} \end{cases}$ is a discrete-time signal whose amplitude is 1 for the sampling instants n ≥ 0 and for all other samples, the amplitude is zero.

A non-deterministic signal is one whose occurrence is random in nature and its pattern is quite irregular. A typical example of a non deterministic signal is thermal noise in an electrical circuit.

Deterministic: It's values are completely specific for any given time. (Equation or formula).

Random Signal: - It takes random values at any given time.

1.3 OPERATION ON A SIGNAL

(A) Shifting
(B) Scaling
(C) Inversion / Folding or reflection

A) Time Shifting: - $x(t - t_0)$

x(t) shifted in time by t_0

$t_0 > 0$ Right shift (delay)

$t_0 < 0$ Left shift (Advance)

(B) Time Scaling x(at)

x(t) is time scaled by a

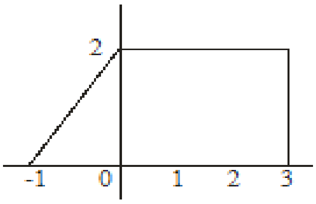
$|a| > 1$, It is multiplication or compression

$|a| < 1$, It is division or expression

(C) Time inversion x(-t)

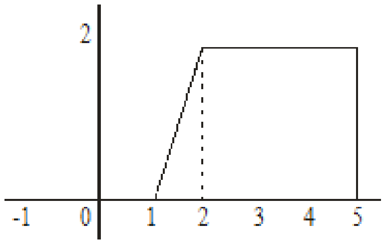
Folded about y – axis

Example : Let signal x(t) $-1 \le t \le 3$

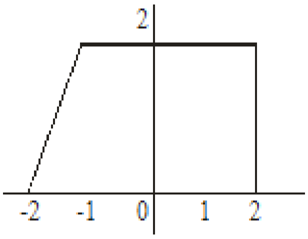


Solution :

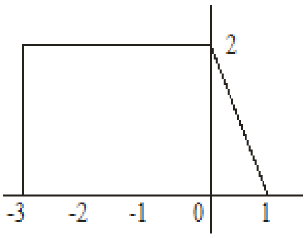
a) x(t) $+1 \le t \le 5$, delay



b) x(t+1) $-2 \le t \le 2$, Advance

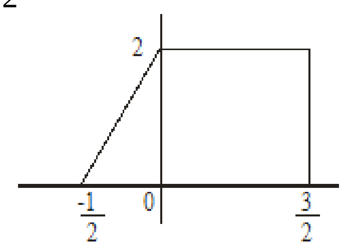


c) x(-t) $-3 \le t \le 1$, Inversion

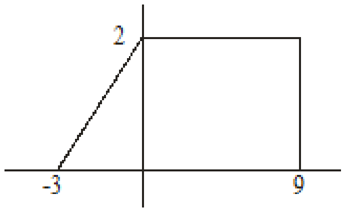


d) x(2t) $-1 \le t \le 3$

$$-\frac{1}{2} \le t \le \frac{3}{2}$$

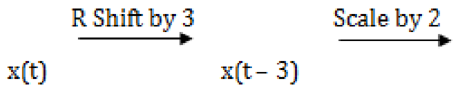


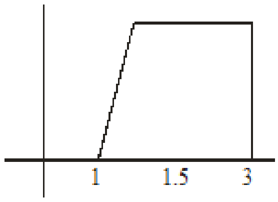
e) x(t/3) $-3 \le t \le 9$



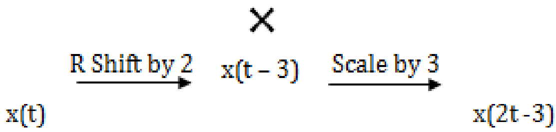
f) x(2t – 3)

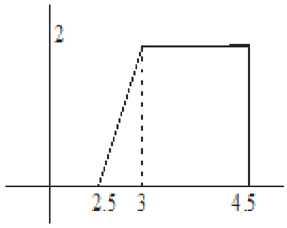
Case – 1





Case - 2





Scaling can't be done before shifting

$-1 \le 2t - 3 < 3$

$2 \le 2t < 6$

$1 \le t \le 3$

Combined Operations

x(at – b) This can be realized in two possible sequences of operation:

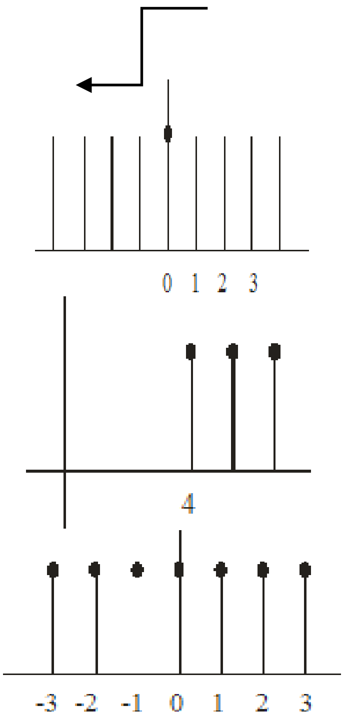
1. **Time** – shift x(t) by b to obtain x(t – b). Now time – scale the shifted signal x(t – b) by a (i.e., replace t with at) to obtain x(at – b).

2. **Time** – scale x(t) by a to obtain x(at). Now time-shift x(at) by b/a (i.e., replace t with t – (b/a) to obtain x[a(t – b/a)] = x(at – b). In either case, if a is negative, time scaling involves time reversal.

For example, the signal x(2t -6) can be obtained in two ways. We can delay x(t) by 6 to obtain x(t-t), and then time – compress this signal by factor 2(replace t with 2t) to obtain x(2t – 6). Alternately, we can first time-compress x(t) by factor 2 to obtain x(2t), then delay this signal by 3(replace t with t – 3) to obtain x(2t – 6).

Example: Given x[n] = 1- ∑ δ[n–1–k] k=3 Such that x[n] – u [mn – n_0] find m & n_0.

Solution: $x[n] = 1 - \sum_{k=3}^{\infty} \delta(n-1-k)$



u[-n] to get x[n] shift it by 3 unit
x[n] = u[-(n -3)]
x [n] = u[-n + 3]
Company with x[n] = u[mn – n_0]
m = -1, n_0 = -3

Example:-Given a sequence x[n], to generate y[n]= x[3 - 4n], which of following procedures would be correct?

Solution: → Let x(n) → $N_1 \le x(n) \le N_2$

x[3 - 4n] ↔ $N_1 \le x(n) \le N_2$

↔ $N_1 \le x(n) \le N_2$

↔ $N_1 \le 3 - 4n \le N_2$

↔ $N_1 - 3 \le -4n \le N_2 - 3$

↔ $\frac{3 - N_1}{4} \ge n \ge \frac{3 - N_2}{4}$

A) First delay x(n) by 3 Samples to generate $Z_1(n)$

$N_1 \le x_1(n) \le N_1 \rightarrow Z_1(n) - N_1 \le n - 3 \le N_2$

$\rightarrow Z_1(n) - N_1 + 3 \le n \le N_2 + 3$

$Z_2(n)$ ↔ pick every 4th sample of $Z_1(n)$

m = $1/4$, down sampling

$\frac{N_1+3}{4} \le n \le \frac{N_2+3}{4} \quad Z_2(n)$

$\rightarrow N_1 + 3 \le 4n \le N_2 + 3$

$x(3-4) \ne y(n) \qquad y(n) = Z_2(-n)$

Time inverse

$y(n) \rightarrow -\frac{N_1+3}{4} \ge n \ge -\frac{N_2+3}{4}$

B) First advance x(n) by 3 Sample to generate Z_1 (n)

$N_1 \le x(n) \le N_2 \quad - \quad N_1 \le n + 3 \le N_2$

$- \quad N_1 - 3 \le n \le N_2 - 3$

$Z_2(n)$ ↔ pick every 4th sample of Z_1 (n)

m = 1/4 down sampling

$Z_2 \rightarrow \frac{N_1-3}{4} \le n \le \frac{N_2-3}{4}$

$y(n) \leftrightarrow Z_2(-n)$

$y(n) = \frac{-N_1+3}{4} \ge n \ge \frac{-N_2+3}{4}$

$x(3-4n) = y(n)$

C) x(n) ↔ $N_1 \le x(n) \le N_2$

(Pick every 4th sample of sequence)

$V_1(n) \rightarrow N_1 \le 4n \le N_2$

$V_1(n) \rightarrow \frac{N_1}{4} \le n \le \frac{N_2}{4}$

$V_2(n) = V_1(-n)$ → time reverse

$V_2(n) \rightarrow \frac{N_1}{4} \le -n \le \frac{N_2}{4} \rightarrow -\frac{N_1}{4} \ge n \ge -\frac{N_2}{4}$

y(n) = V_2 (N + 3), V_2 time advance 3 unit to obtain y (n)

$V_2(n) \rightarrow \frac{N_1}{4} \ge n + 3 \ge \frac{-N_2}{4}$

$\rightarrow -\frac{N_1}{4} - 3 \ge n \ge -\frac{-N_2}{4} - 3$

y(n) ≠ x(3 -4n)

D) First pick every forth sample of x(n) sequence

$V_1(n) \rightarrow N_1 \le 4n \le N_2,$

$V_1(n) \rightarrow \frac{N_1}{4} \le n \le \frac{N_2}{4}$

$V_2(n) = V_1(-n) - \text{time reverse}$

$V_2(n) \rightarrow \frac{N_1}{4} \le -n \le \frac{N_2}{4}$

$\rightarrow V_2(n) \rightarrow \frac{-N_1}{4} \ge n \ge \frac{-N_2}{4}$

y(n) = V2(n-3), V2 time delay 3 unit to obtain y(n)

$y(n) = -\frac{N_1}{4} \ge n - 3 \ge -\frac{N_2}{4}$

$\rightarrow y(n) = -\frac{N_1}{4} + 3 \ge n \ge -\frac{N_2}{4} + 3$

y(n) ≠ x [3 – 4n]

Example: x[n] = [-3, 5, 4, -2, 3] then write x[3n], x[2n]

Solution: Decimation or down sampled discard (n – 1) samples w.r.t. to origin

$x[3n] = [5, 3], \text{ discard 2 sample}$

$\uparrow$

$x[2n] = [5, -2], \text{ discard 1 sample}$

$\uparrow$

$x(n) \xrightarrow{x[n/2]} Z(n) \xrightarrow{Z[2n]} x(n)$

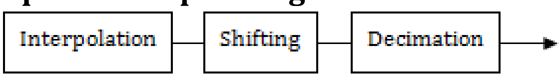
Decimation is the inverse of interpolation but interpolation is not inverse of decimation.

$x(n) \xrightarrow{x[2n]} Z(n) \xrightarrow{Z[n/2]} x(n)$ not equal

if x(n -2) → Signal is R shift by 2

if $x(n\text{-}1, 3) \leftrightarrow x\left(\frac{10n - 13}{10}\right)$

Sequence of operating



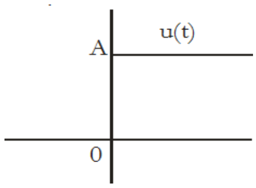
Interpolation should be proceeding by decimation.

1.4 IMPORTANT SIGNALS

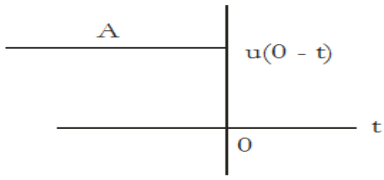
1.4.1

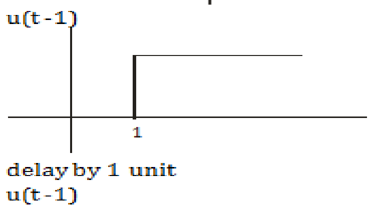
(1) Step Signal :-

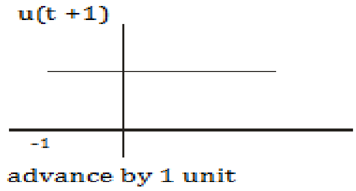
$$Au(t) = \begin{cases} A & t > 0 \\ 0 & t > 0 \end{cases}$$

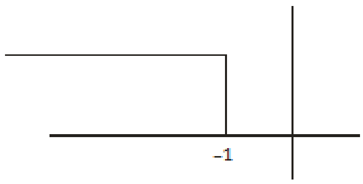


$$Au(t) = \begin{cases} A & t < 0 \\ 0 & t > 0 \end{cases}$$







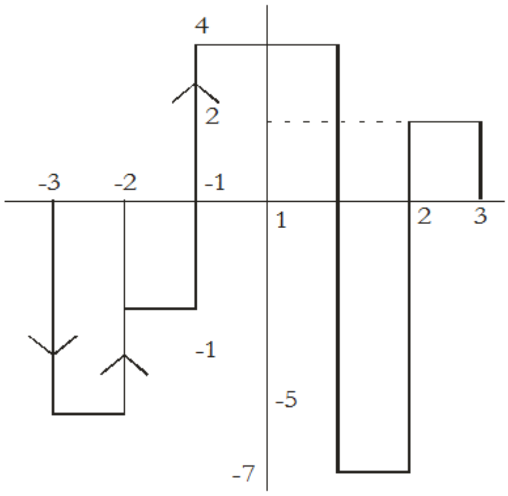


0 < - t – 1 <∞

1 < - t <∞

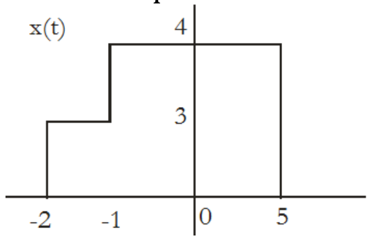
-1 > t > 1- ∞

Example: Write expression



$x(t) = -5u(t+3) + 4u(t+2)$
$+5u(t+1) - 11u(t-1)$
$+9u(t-2) - 2u(t-3)$

Example: Write expression

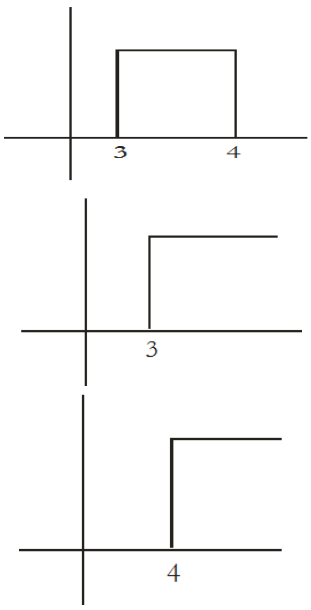


$x(t) = 3u(t+2) + u(t+1) - 4u(t-5)$

Example: Possible expression in terms of steps:

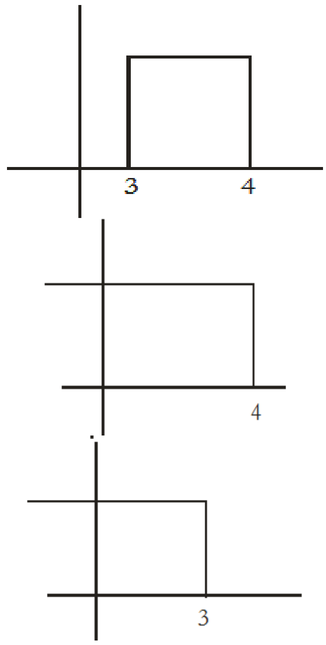
Solution:

a)



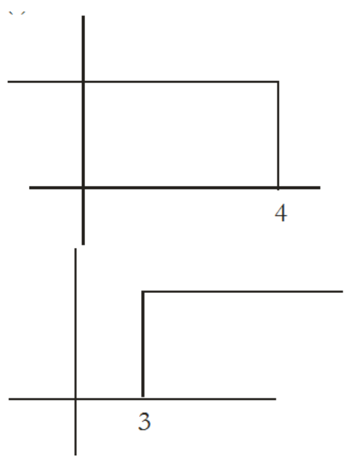
a) $u(t-3) - u(t-4)$

b)



b) $u(-t+4) - u(-t+3)$

c)



c) $u(t-3) \times u(t+4)$

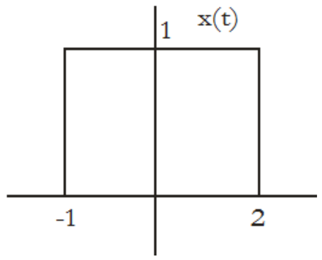
Relation between unit step signal and impulse signal

$$\delta(t) = \frac{d}{dt}u(t)$$

$$u(t) = \int_{-\infty}^{\infty} \delta(t)dt$$

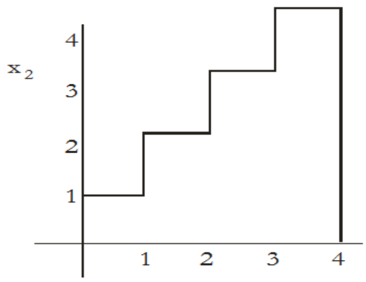
$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

Example:



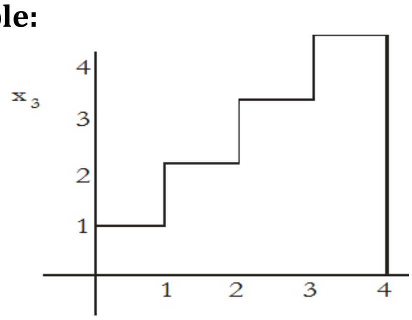
Solution: $[u(t+1)-u(t-2)]$

Example:



Solution:
$[u(t-1)+u(t)+u(t-2)+u(t-3)-4u(t-4)]$

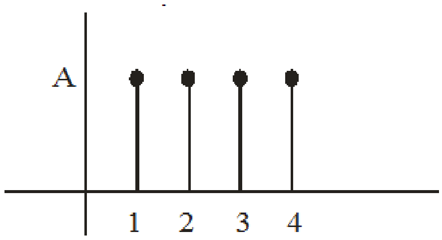
Example:



Solution:
$4u(t-1)-u(t-1)-u(t-2)-u(t-3)-u(t-4)$

1.4.2 Discrete Step Signal

$$Au[n]=\begin{cases} A & \text{at } n \geq 0 \\ 0 & \text{at } n < 0 \end{cases}$$

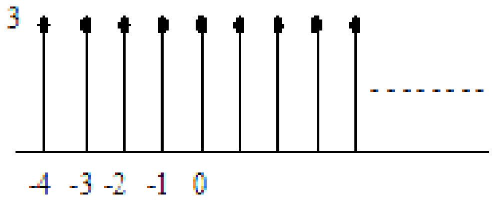


Example: Sketch the following
(1) 3u(n + 4)
(2) u(n – 6)
(3) 3u (6 – 2n)
(4) u(n) – u(n – 1) = δ(n)

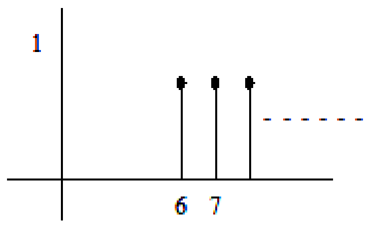
First difference of step function known as impulse response
= S(n) – S(n – 1)

$$u(n) = \sum_{k=-\infty}^{\infty} \delta(k)$$

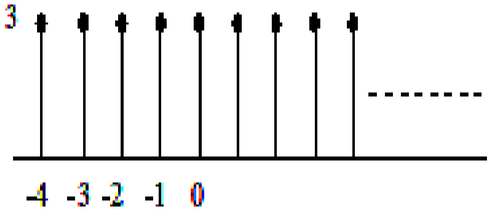
$$x[n] = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$



$0 \leq n + 4 < \infty$
$-4 \leq n \leq \infty$



$0 \leq n - 6 < \infty$
$6 \leq n \leq \infty$



$0 \leq 8 - 2n < \infty$
$-8 \leq -2n \leq \infty$
$-4 \leq -n \leq \infty$

1.4.3 Impulse Function

Unit-impulse Function
Indicate that the area of the impulse function is unity and this area is confined to an infinitesimal interval on the t-axis and concentrated at t = 0. The unit impulse function is very useful in continuous-time system analysis. It is system characteristics. In discrete-time domain, the unit-impulse signal is called a unit-sample signal.
Unit impulse function or direct delta function.

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \text{ or } \int_{-\epsilon}^{\epsilon} \delta(t)dt = 1$$

Properties of impulse signal

1) Scaling: $\delta(at) = \frac{1}{|a|}\delta(t)$

or δ (a(t – t_0) = $\frac{1}{|a|}\delta$ (t – t_0)

if a = -1

$\delta(-t) = \delta(t)$ Even function

2) $\int_{-\infty}^{\infty} \delta(t - t_0)dt = \int_{-\infty}^{\infty} \delta(t)dt = 1$

3) Sampling: Multiplication of signal x(t) with impulse with be impulse f^n but strength of impulse is governed by x(t) at location of impulse.

x(t) δ(t –t_0) = x(t_0) δ(t – t_0)

x(t) δ(t) = x(0) δ(t)

4) Shifting:

1) $\int_{-\infty}^{\infty} x(t)\delta(1)\,(t - t_0)dt = x(t_0)$

2) $\int_{-\infty}^{\infty} x(t)\delta(t)dt = x(0)$

3) $\int_{-\infty}^{\infty} x(t)\delta(1)(t - t_0)dt = x(t_0)$

if a< t_0< b

0t_0< a

0 t_0> b

Undefined a = b

4) $\delta(t) = \frac{d}{dt}u(t)$

5) Impulse response =

$\frac{d}{dt}$ (step response)

Step response =

$\int_{-\infty}^{t} (\text{impulse response})dt$

6) $u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = \int_{0}^{\infty} \delta(t - \tau)d\tau$

Simplify the following Questions

1) $t\delta 1.(t) = 0$

2) $\text{Sin } t\delta 1.(t) = 0$

3) $\text{Cos } t\delta(t - \pi) = -\delta 1.(t - \pi)$

4) $\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi}\delta(f)$

5) $\delta(3 - 2t) = \delta - 2(t - 3/2) = +\frac{1}{2}\delta(t - 3/2)$

6) $e^{-2t}\delta(-3t + 2) = e^{-2t}\delta\left[-3\left(t - \frac{3}{2}\right)\right]$

$= \frac{e^{-2+2/3}}{3}\delta(t - 2/3)$

7) $(1 + 2t + 3t^2)\delta(\omega + u)$

$= \left[1 + 2(-1) + 3(-1)^2\right]\delta(t + 1)$

$= 2\delta 1.(t + 1)$

8) $\frac{2 - 3\omega j}{3 + 2\omega j}\delta(\omega + 4)$

$= \frac{2 - 3(-4)j}{3 + 2(-4)j}\delta(\omega + 4)$

$= \frac{\sqrt{148}}{\sqrt{73}}\delta(\omega + 4)$

9) $\cos(3\tau)\delta\left(\tau - \frac{\pi}{3}\right) = \cos\left(\frac{3\pi}{3}\right)\delta(\tau - \pi/3)$

$= -\delta 1.(\tau - \pi/3)$

10) $\frac{\sin kt}{kt}\delta(t) \to$ L – Hospital rule

$\frac{K\cos kt}{K}\delta(t)\delta = (t)$

11) $\int_{-1}^{1} (3t^2 + 1)\delta(t)dt = 1$

12) $\int_{-1}^{1} (3t^2 + 1)\delta 1.(t)\ dt = 1$

13) $\int_{1}^{2} (3t^2 + 1)\delta(t)\ dt = 0$

14) $\int_{-\infty}^{\infty} (t^2 + \cos\ \pi t)\delta(t - 1)dt = 0$

15) $\int_{-\infty}^{\infty} e^{-t}\delta(2t - 2)dt = \frac{1}{2}\int_{-\infty}^{\infty} e^{-t}\delta(t - 1)dt$

$$= \frac{1}{2} e^{-t}{}_{t=1} = \frac{1}{2e}$$

16) $\int_{-\infty}^{\infty} e^{-t}\delta'(t)dt = -\frac{d}{dt}\left(e^{-t}\right)\Big|_{t=0} = e^{-t}\Big|_{t=0} = 1$

$$\int_{-\infty}^{\infty} \varnothing(t)u'(t)dt = -\int_{-\infty}^{\infty} \varnothing'(t)u(t)dt$$

17) $\int_{-\infty}^{\infty} \varnothing(t)\delta'(t)dt = -\varnothing'(0)$

18) $t\delta'(t) = -\delta(t)$

Generalized Derivates : -

$$\int_{-\infty}^{\infty} \varnothing(t)(t)dt = (-1^n)\int_{-\infty}^{\infty} \varphi^n(t)\ g(t)\ dt$$

$g(t) \rightarrow$ generalized f^n, $g^n(t) \rightarrow n^{th}$ derivation
$\varphi(t) \rightarrow$ testing f^n, $\varphi^n(t) \rightarrow n^{th}$ derivation

(1) $\int_{-\infty}^{\infty} e^{-ut}\delta(t+0.3)dt = e^{1.2} = \int_{-4}^{0} e^{-ut}\delta(t+0.3)dt$

(2) $\int_{0}^{10} e^{-ut}\delta(t+0.3)dt = 0$

Example : Step response of LTI System $e^{-3t}u(t)$ find impulse response.

Solution :

$= \frac{d}{dt}$[step response]

$= \frac{d}{dt}\left[e^{-3t}u(t)\right]$

$= \frac{d}{dt}\left[e^{-3t}u(t) + e^{-3t}u'(t)\right]$

$= -3e^{-3t}u(t) + e^{-3t}\delta(t)$

$= -3e^{-3t}u(t) + \delta(t)$

Discrete time impulse: Unlike δ (t), δ (n) does not represent area under the signal rather it represent the amplitude.
δ (n) = 0 for n ≠ 0
1 for n = 0

Properties of δ (n)

1. $\delta(an) = \delta(n)$ **
2. $x(n)\ \delta\ (n-k) = x(k)\ \delta\ (n-k)$
3. $\delta\ (n) = u(n) - u(n-1)$
4. $u(n) = \sum^{n} \delta(k)$
   $k = +\infty$
   $\sum_{k=0}^{k=+\infty} {}^{\infty}\delta(n-k) = \sum_{k=-\infty} {}^{n}\delta(k)$
   there fore step response =
   $\sum_{k=-\infty} {}^{\infty}$impulse response

Example: Step response of LTI system is given by $\left(-\frac{2}{3}\right)^n u(n)$, find the impulse response.
Impulse response
$= S(n) - S(n-1)$

$= \left(-\frac{2}{3}\right)^n u(n) - \left(-\frac{2}{3}\right)^{n-1} u(n-1)$

$= \left(-\frac{2}{3}\right)^n u(n) + \left(-\frac{2}{3}\right)^n \times \frac{3}{2} u(n-1)$

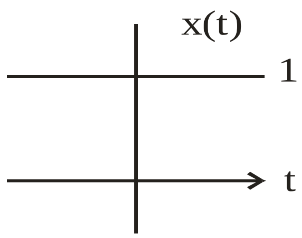
$= \left(-\frac{2}{3}\right)^n \left[u(n) + \frac{3n}{2} u(n-1)\right]$

1.4.4 EXPONENTIAL FUNCTION

The complex exponential signal is defined as $x(t) = e^{st}$ where $s = \sigma + j\omega$, a complex number. Then this signal x(t) is known as a general complex exponential signal whose real part $e^{\sigma t}$ cos ωt and imaginary part $e^{\sigma t}$ sin ωt are exponentially increasing (σ > 0) or decreasing σ , 0 respectively. If $_s$ = σ, then x(t) = eσt which is a real exponential signal. If (σ > 0), then x(t) is a growing exponential; and if (σ < 0), then x(t) is a decaying exponential.

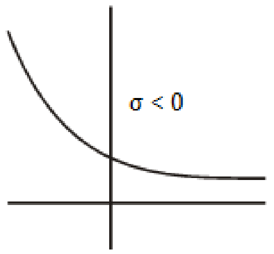
Continuous Time
General form $x(t) = e^{st}$
$S = \sigma + j\omega$

Case 1. σ = 0, ω = 0, s = 0
x(t) = 1 dc

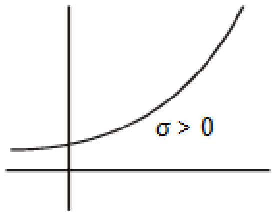


Case 2. $\sigma \neq 0, \omega = 0, \quad s = \sigma$

$x = e^{\sigma t}$



Decay exponential

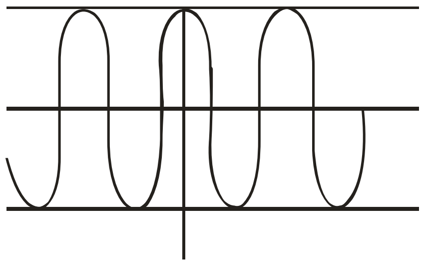


Growing exponential

Case 3. $\sigma = 0, \omega \neq 0, \quad s = j\omega t$

$x(t) = e^{+j\omega t} = \cos \omega t + j \sin \omega t$

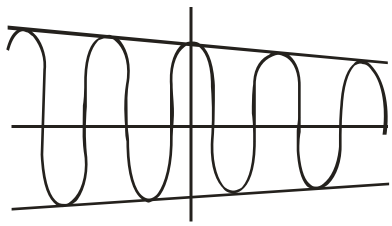
Will oscillate from +1 to -1



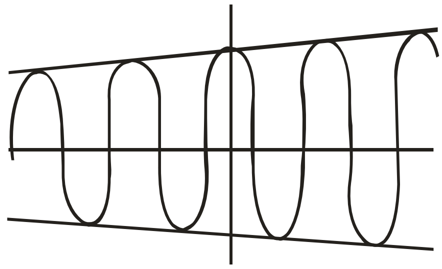
Case 4. $\sigma \neq 0, \omega \neq 0, \quad s = \sigma + j\omega$

$x(t) = e^{0t}e^{j\omega t}$

(i) $\sigma < 0$ $\quad x(t) = e^{0t}e^{j\omega t}$ Under damped oscillation



(ii) Over damped oscillation



$\sigma > 0$. $x(t) = e^{0t}e^{j\omega t}$

1.5 EVEN AND ODD SIGNAL

If a signal exhibits symmetry in the time domain about the origin, it is called an even signal. The signal must be identical to its reflection about the origin. Mathematically, and even signal satisfies the following relation.

For a continuous-time signal

For a discrete-time signal

An odd signal exhibits anti-symmetry. The signal is not identical to its reflection about the origin, but to its negative. An odd signal satisfies the following relation.

For a continuous-time signal

For a discrete-time signal

$x_1(t) = \sin \omega t$ and $x_2(t) = \cos \omega t$ are good examples of odd and even signal, respectively. An even signal which often occurs in the analysis of signals is the sinc function.

A signal can be expressed as a sum of two components, namely, the even component of the signal and the odd component of the signal. The even and odd components can be obtained from the signal itself.

$x(t) = x_{even}(t) + x_{odd}(t)$

$x_{even}(t) = \frac{1}{2}[x(t) + x(-t)]$ and

$x_{odd}(t) = \frac{1}{2}[x(t) + x(-t)]$

A signal x(t) or x[n] is an even signal if

$x(-t) = x(t)$

$x[-n] = x[n]$

an odd signal if $\quad x(-t) = -x(t)$

$x[-n] = -x[n]$

Any signal x(t) or x[n] can be expressed as

$x(t) = x_e(t) + x_o(t)$

$x[n] = x_e[n] + x_o[n]$

$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ even part

$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$ even part

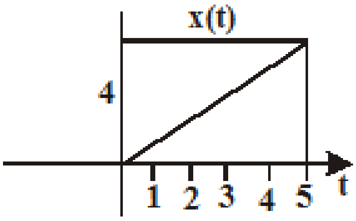
$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$ odd part

$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$ odd part

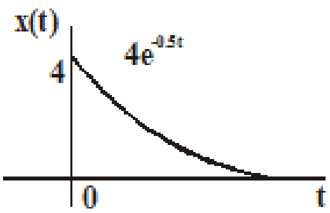
Note:- The product of
Two even or odd signal = an even signal
An even signal an odd signal = an odd signal

Example :Sketch even and odd components of signals

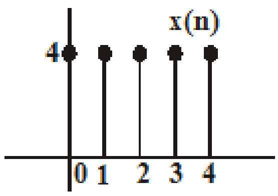
(a)



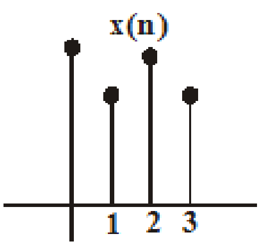
(b)



(c)

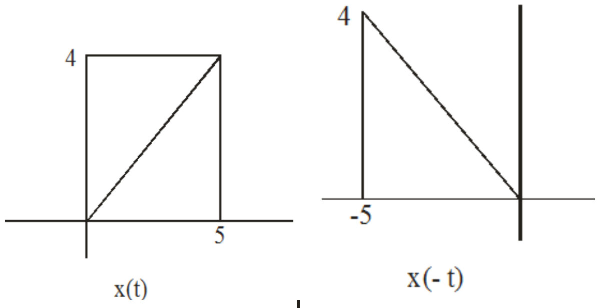


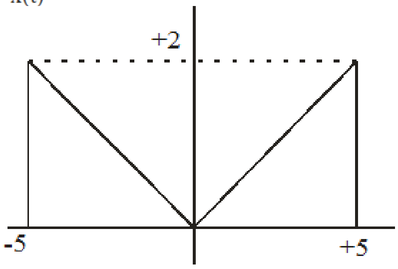
(d)



Solution :

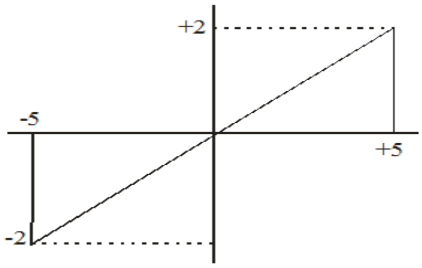
(a)



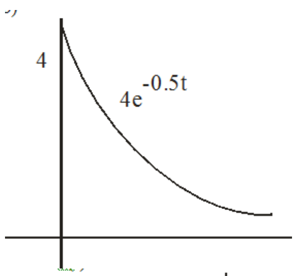


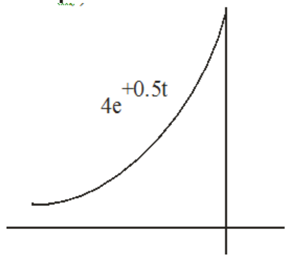
$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_0(t) = \frac{x(t) - x(-t)}{2} =$$

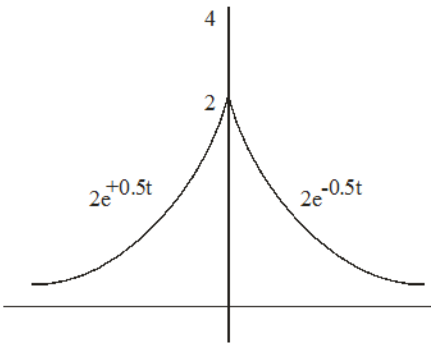


(b)



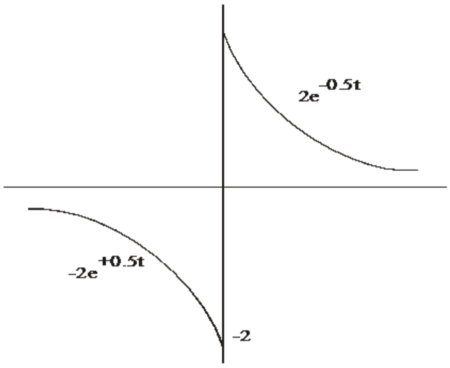


x(t)
x(-t)

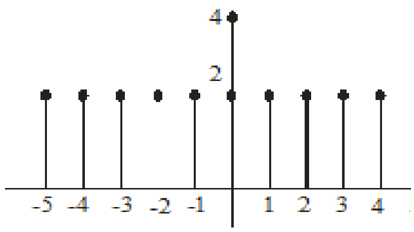


$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

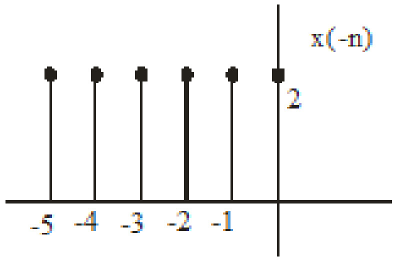
$$x_0(t) = \frac{x(t) - x(-t)}{2} =$$



(c)

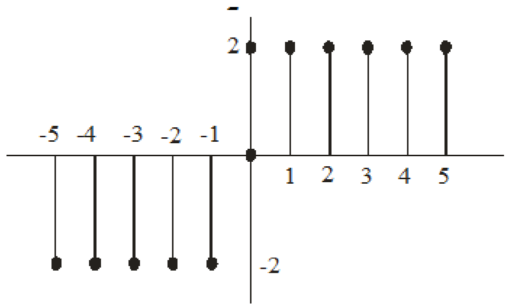


x(n)

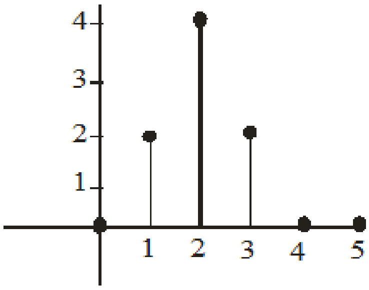


$$x_e(n) = \frac{x(n) - x(-n)}{2}$$

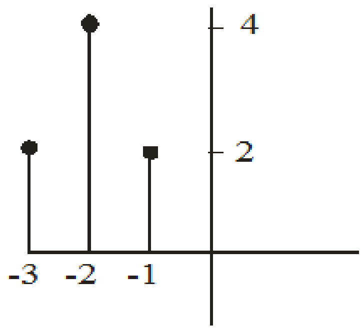
$$x_0(n) = \frac{x(n) - x(-n)}{2}$$



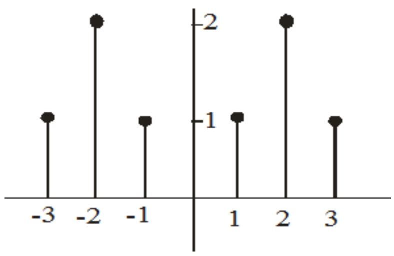
(d)



x(n)



x(-n)



$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$x_0(n) = \frac{x(n) - x(-n)}{2}$$

Properties of impulse signal

1) Scaling : $\delta(at) = \frac{1}{|a|}\delta(t)$

or $\delta\left(a(t - t_0)\right) = \frac{1}{|a|}\delta\left(t - t_0\right)$

if a = -1

$\delta(-t) = \delta(t)$ even function

2) $\int_{-\infty}^{\infty}\delta(t-t_0)dt=\int_{-\infty}^{\infty}\delta(t)dt=1$

3) Sampling: Multiplication of signal x(t) with impulse with be impulse f^n but strength of impulse is governed by x(t) at location of impulse.

$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

$x(t)\,\delta(t) = x(0)\,\delta(t)$

4) Shifting :

1) $\int_{-\infty}^{\infty}x(t)\delta(t-t_0)(1)\,dt = x(t_0)$

2) $\int_{-\infty}^{\infty}x(t)\delta(t)dt = x(0)$

3) $\int_{-\infty}^{\infty}x(t)\delta(t-t_0)dt = x(t_0)$

if $a< t_0< b$

0 $t_0< a$

0 $t_0> b$

Undefined $a = b$

5) $\delta(t)=\frac{d}{dt}u(t)$

6) Impulse response $=\frac{d}{dt}$(step response)

Step response $=\int_{-\infty}^{t}$(impulse response)dt

7) $u(t)=\int_{-\infty}^{t}\delta(\tau)d\tau=\int_{0}^{\infty}\delta(t-\tau)d\tau$

Simplify the following Questions

1) $t\,\delta(t) = 0$

2) $\sin t\,\delta(t) = 0$

3) $\cos t\,\delta(t-\pi) = -\delta(t-\pi)$

4) $\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi}\delta(f)$

5) $\delta(3-2t)=\delta(-2\,(t-3/2)=+\frac{1}{2}\delta(t-3/2)$

6) $e^{-2t}(-3t+2)=e^{-2t}\left[-3\left(t-\frac{3}{2}\right)\right]$

$=\frac{e^{-2+2/3}}{3}\delta(t-2/3)$

7)

$(1+2t+3t^2)\delta(\omega+u)=\left[1+2(-1)+3(-1)^2\right](t+1)$

$=2\delta(t+1)$

8) $\frac{2-3\omega j}{3+2\omega j}\delta(\omega+4)=\frac{2-3(-4)j}{3+2(-4)j}\delta(\omega+4)$

$=\frac{\sqrt{148}}{\sqrt{73}}\delta(\omega+4)$

9) $\cos(3\tau)\delta\left(\tau-\frac{\pi}{3}\right)=\cos\left(\frac{3\pi}{3}\right)\delta(\tau-\pi/3)$

$=-\delta(\tau-\pi/3)$

10) $\frac{\sin kt}{kt}\delta(t)\rightarrow$ L – Hospital rule

$\frac{K\cos kt}{K}\delta(t)=\delta(t)$

11) $\int_{-1}^{1}(3t^2+1)(t)dt=1$

12) $\int_{1}^{2}(3t^2+1)\delta(t)\,dt = 0$

13) $\int_{-\infty}^{\infty}(t^2+\cos\pi t)\delta(t-1)dt=0$

14) $\int_{-\infty}^{\infty}e^{-t}\delta(2t-2)dt=\frac{1}{2}\int_{-\infty}^{\infty}e^{-t}\delta(t-1)dt$

$=\frac{1}{2}e^{-t}\Big|_{t=1}=\frac{1}{2e}$

15) $\int_{-\infty}^{\infty}e^{-t}\delta(2t-2)dt=\frac{1}{2}\int_{-\infty}^{\infty}e^{-t}\delta(t-1)dt$

$=e^{-t}\,\Big|_{t=0}=1$

$\int_{-\infty}^{\infty}\varnothing(t)u'(t)dt=-\int_{-\infty}^{\infty}\varnothing'(t)u(t)dt$

16) $\int_{-\infty}^{\infty}\varnothing(t)\delta'(t)dt=-\varnothing'(0)$

17) $t\delta'(t)=-\delta(t)$

Generalized Derivatives : -

$\int_{-\infty}^{\infty}\varnothing(t)g^n(t)dt=(-1^n)\int_{-\infty}^{\infty}\varphi^n(t)\,g(t)dt$

$g(t)\rightarrow$ generalized f^n, $g^n(t)\rightarrow n^{th}$ derivation

$\varphi(t)\rightarrow$ testing f^n, $\varphi^n(t)\rightarrow n^{th}$ derivation

1)

$\int_{-\infty}^{\infty}e^{-ut}\delta(t+0.3)dt=e^{1.2}=\int_{-4}^{0}e^{-ut}\delta(t+0.3)dt$

2) $\int_{0}^{10} e^{-ut}\delta(t+0.3)dt = 0$

Example : Step response of LTI System $e^{-3t}u(t)$ find impulse response.

Solution :

$= \frac{d}{dt}[\text{step response}]$

$= \frac{d}{dt}\left[e^{-3t}u(t)\right]$

$= \frac{d}{dt}\left[e^{-3t}u(t) + e^{-3t}u'(t)\right]$

$= -3e^{-3t}u(t) + e^{-3t}\delta(t)$

$= -3e^{-3t}u(t) + \delta(t)$

Discrete time impulse : Unlike $\delta(t)$, $\delta(n)$ does not represent area under the signal rather it represent the amplitude.

$\delta(n) = 0$ for $n \neq 0$

1 for $n = 0$

Properties of δ (n)

1) $\delta(an) = \delta(n)$ **

2) $x(n)\,\delta(n-k) = x(k)\,\delta(n-k)$

3) $\delta(n) = u(n) - u(n-1)$

4) $u(n) = \sum^{n}\delta(k)$

$k = +\infty$

$\sum_{k=0}^{k=+\infty} {}^{\infty}\delta(n-k) = \sum_{k=-\infty} {}^{n}\delta(k)$

therefore step response

$= \sum_{k=-\infty} {}^{\infty}$impulse response

Example : Step response of LTI system is given by $\left(-\frac{2}{3}\right)^{n} u(n)$, find the impulse response. Impulse response

$= S(n) - S(n-1)$

$= \left(-\frac{2}{3}\right)^{n} u(n) - \left(-\frac{2}{3}\right)^{n-1} u(n-1)$

$= \left(-\frac{2}{3}\right)^{n} u(n) + \left(-\frac{2}{3}\right)^{n} \times \frac{3}{2} u(n-1)$

$= \left(-\frac{2}{3}\right)^{n} \left[u(n) + \frac{3n}{2} u(n-1)\right]$

1.4.4 EXPONENTIAL FUNCTION

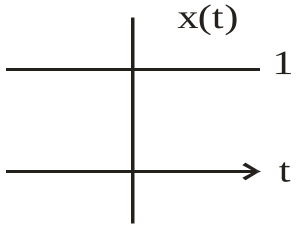
The complex exponential signal is defined as $x(t) = e^{st}$ where $s = \sigma + j\omega$, a complex number. Then this signal x(t) is known as a general complex exponential signal whose real part $e^{\sigma t} \cos \omega t$ and imaginary part $e^{\sigma t} \sin \omega t$ are exponentially increasing ($\sigma > 0$) or decreasing σ , 0 respectively. If $_s = \sigma$, then $x(t) = e\sigma t$ which is a real exponential signal. If ($\sigma > 0$), then x(t) is a growing exponential; and if ($\sigma < 0$), then x(t) is a decaying exponential.

Continuous Time

General form $x(t) = e^{st}$ $S = \sigma + j\omega$

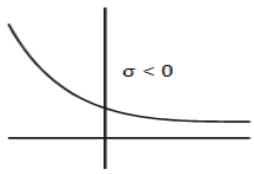
Case 1. $\sigma = 0, \omega = 0$, $s = 0$

$x(t) = 1$ dc

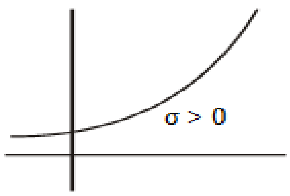


Case 2. $\sigma \neq 0, \omega = 0$, $s = \sigma$

$x = e^{\sigma t}$



Decay exponential

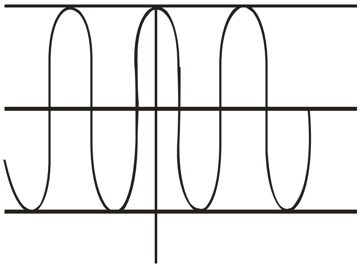


Growing exponential

Case 3. $\sigma = 0, \omega \neq 0$, $s = j\omega t$

$x(t) = e^{+j\omega t}$ $= \cos \omega t + j \sin \omega t$

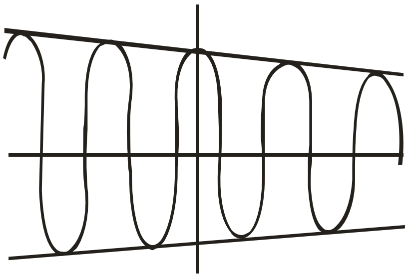
Will oscillate from +1 to -1



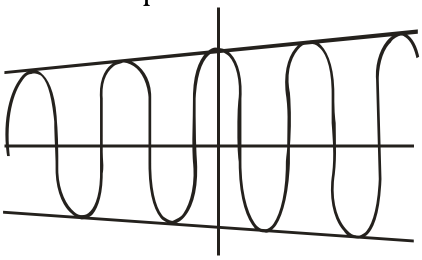
Case 4. $\sigma \neq 0, \omega \neq 0, \quad s = \sigma + j\omega$

$x(t) = e^{0t}e^{j\omega t}$

(i) $\sigma < 0 \quad x(t) = e^{0t}e^{j\omega t}$ Under damped oscillation



(ii) Over damped oscillation



$\sigma > 0.\ x(t) = e^{0t}e^{j\omega t}$

1.5 EVEN AND ODD SIGNAL

If a signal exhibits symmetry in the time domain about the origin, it is called an even signal. The signal must be identical to its reflection about the origin. Mathematically, and even signal satisfies the following relation.

For a continuous-time signal

For a discrete-time signal

An odd signal exhibits anti-symmetry. The signal is not identical to its reflection about the origin, but to its negative. An odd signal satisfies the following relation.

For a continuous-time signal

For a discrete-time signal

$x_1(t) = \sin \omega t$ and $x_2(t) = \cos \omega t$ are good examples of odd and even signal, respectively. An even signal which often occurs in the analysis of signals is the sinc function.

A signal can be expressed as a sum of two components, namely, the even component of the signal and the odd component of the signal. The even and odd components can be obtained from the signal itself.

$$x(t) = x_{even}(t) + x_{odd}(t)$$

$$x_{even}(t) = \frac{1}{2}[x(t) + x(-t)] \text{ and}$$

$$x_{odd}(t) = \frac{1}{2}[x(t) + x(-t)]$$

A signal x(t) or x[n] is an even signal if

$x(-t) = x(t)$

$x[-n] = x[n]$

an odd signal if $\quad x(-t) = -x(t)$

$x[-n] = -x[n]$

Any signal x(t) or x[n] can be expressed as

$x(t) = x_e(t) + x_o(t)$

$x[n] = x_e[n] + x_o[n]$

$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ even part

$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$ even part

$x_0(t) = \frac{1}{2}[x(t) - x(-t)]$ odd part

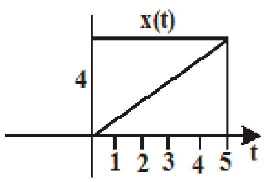
$x_0(n) = \frac{1}{2}[x(n) - x(-n)]$ odd part

Note:- The product of Two even or odd signal = an even signal

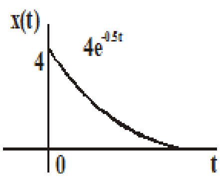
An even signal an odd signal = an odd signal

Example: Sketch even and odd components of signals

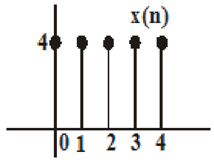
a)



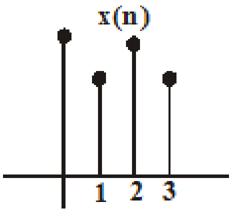
b)



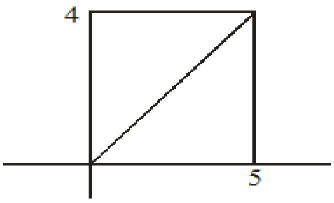
c)



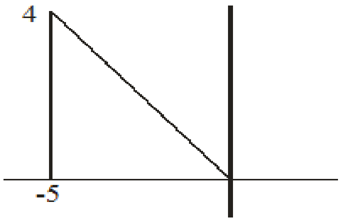
d)



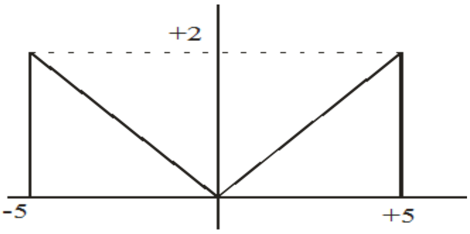
Solution :



x(t)

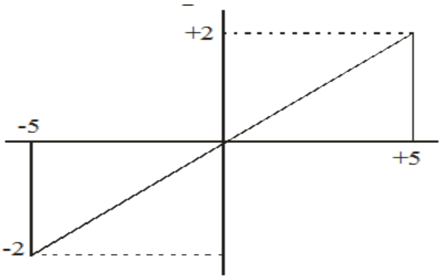


x(-t)

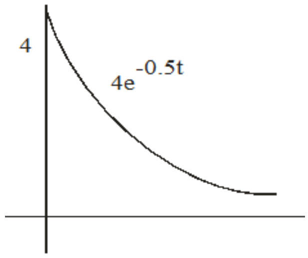


$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

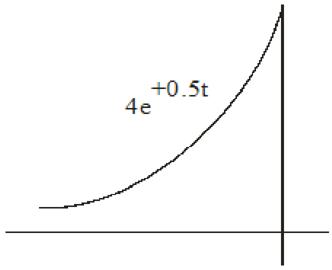
(a) $$x_0(t) = \frac{x(t) - x(-t)}{2} =$$



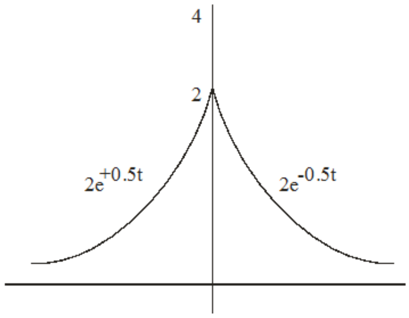
(b)



x(t)

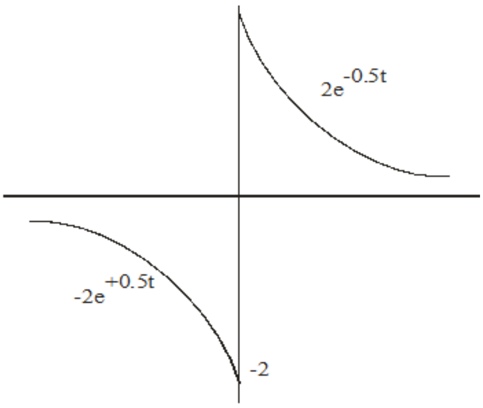


x(-t)

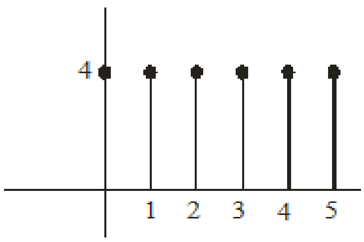


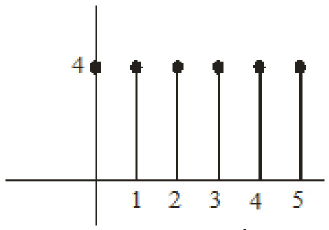
$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

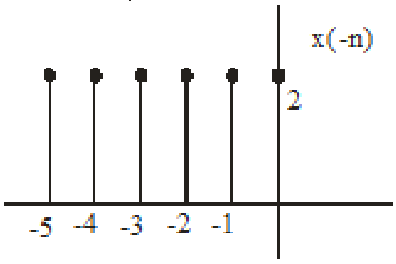
$$x_0(t) = \frac{x(t) - x(-t)}{2} =$$



(c)



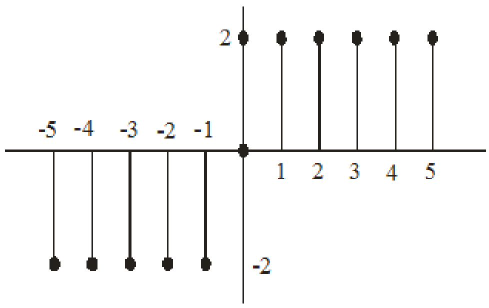




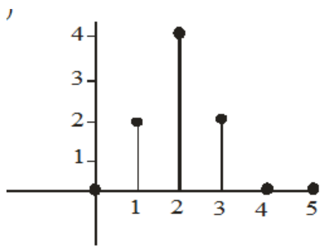
x(n)

$$x_e(n) = \frac{x(n) - x(-n)}{2}$$

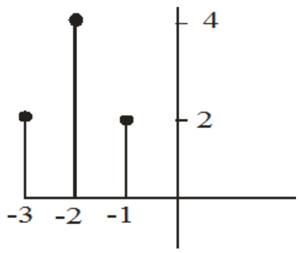
$$x_0(n) = \frac{x(n) - x(-n)}{2}$$



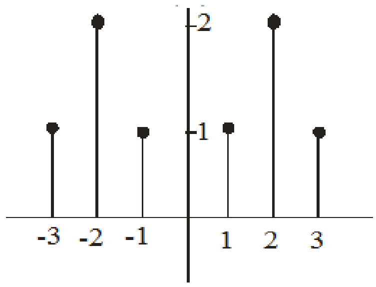
(d)



x(n)

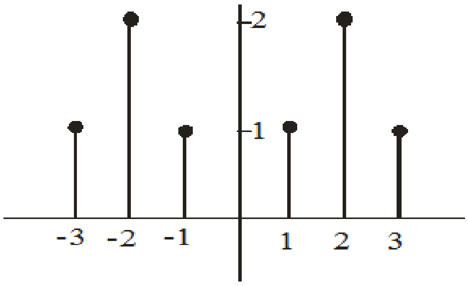


x(-n)



$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$x_0(n) = \frac{x(n) - x(-n)}{2}$$



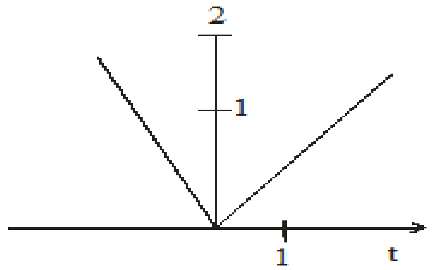
Example

Find even and odd compound of x(t)

Solution

x(t) = - 2t for t < 0

x(t) = t fort > 0



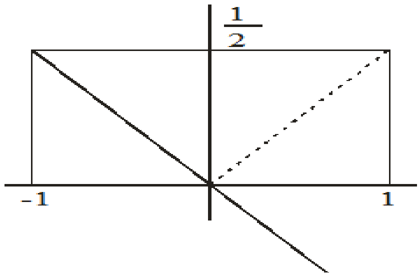
$$x_e(t) = \begin{cases} \frac{3t}{2} & t > 0 \\ \frac{-3t}{2} & t < 0 \end{cases}$$

$$x(t) = \begin{cases} t & t > 0 \\ -2t & t < 0 \\ 0 & t > 0 \end{cases}$$

or

$$x(-t) = \begin{cases} -t & t < 0 \\ 2t & t > 0 \\ 0 & t > 0 \end{cases}$$

$$x_0(t) = \begin{cases} \frac{-t}{2} & t > 0 \\ \frac{-t}{2} & t < 0 \end{cases}$$



1.6 PERIODIC AND NON PERIODIC SIGNALS

A Continuous signal x(t) is said to be periodic with period t
x[t ± mT] = x(t) all t
x[n ± mN] = x(n) all n
non periodic (a periodic) (not periodic)
T = where T is min. value of t after which f(t) repeats is called fundament period.
$f(t) = f_1(t) \pm f_2(t) \pm f_3(t)$
↓ ↓ ↓ ↓
T T_1 T_2 T_3

$\frac{T_1}{T_2} = \text{rational} = \frac{T_2}{T_2} = \frac{T_2}{T_1}$

T = LCM [T_1 & T_2 & T_3]

Note:- Addition or subtraction of DC will not affect the fundamental period because DC is periodic signal,. Its period is not define.

Periodicity of discrete signal

x(n) = x(n ± N)
N – integral No.

$x(n) = e^{j\omega_0 n}$

$x(n+N) = e^{j\omega_0 (n+N)}$

$= e^{j\omega_0 n} e^{j\omega_0 N}$

$\Rightarrow e^{j\omega_0 n} = 1 = e^{j2\pi}$

$\omega_0 N = 2\pi$

x(n) = $e^{j\omega_0 n}$

N = $\frac{2\pi}{\omega_0} \times m$ M smallest integer which

connects $\frac{2\pi}{\omega_0}$ to integer no.

$$x(n) = \begin{matrix} x_1(n) \pm & x_2(n) \pm & x_3(n) \\ \downarrow & \downarrow & \downarrow \\ N_1 & N_2 & N_3 \end{matrix}$$

N = LCM [N_1 & N_2 & N_3]

Note:- Sum & difference of discrete periodic signal is periodic and sum & difference of continuous periodic signal is periodic under certain condition.

Example: If x(t), check, periodic or a periodic, find fundamental period

a) $2\sin t + 4\sin 3t$ Sol: Periodic, $T = 2\pi$
b) $\sin 5\pi t + \sin 7t$ Sol: Aperiodic
c) $\sin\sqrt{2}\pi t + \sin t$ Sol: Aperiodic
d) $2 + \sqrt{3}\cos \pi t$ Sol: Periodic, $T = 2$
e) je^{j10t} Sol: Periodic, $T = \pi/5$
f) $e^{(-1+j)t}$ Sol: Aperiodic
g) $e^{j[(\pi/2)t-1]}$ Sol: Periiodic, $T = 4$
h) $e^{j(t+\pi/4)}u(t)$ Sol: Aperiodic
i) $e^{j7\pi n}$ Sol: Periodic, $N = 2$
j) $3e^{j3\pi(n+1/2)15}$ Sol: Periodic, N=10
k) $3e^{j3/5(n+1/2)}$ Sol: Periodic, $N = 10$
l) $2\cos(10t+1) - \sin(4t-1)$
Sol: Periodic, $T = \pi$
m) $e^{j[(\pi/2)t-1]}$ Sol: Periodic, $T = 4$
n) $\cos^2(\pi/8)n$ Sol: Periodic, $N = 8$
o) $u(n) + u[-n]$ Sol: Aperiodic
p) $\sum_{n=-\infty}^{\infty} \begin{bmatrix} \delta(n-4n) \\ -\delta(n-1-4k) \end{bmatrix}$ Sol: Periodic, $N = 4$
q)
$u(-t) + u(-t) = 1$ Sol: Periodic, T = not defined

1.7 ENERGY AND POWER SIGNALS

Signals can also be classified as those having finite energy or finite average power. However, there are some signals which can neither be classified as energy signals nor power signals.
The **energy signal** is one which has finite energy and zero average power, i.e. x(t) is an energy signal if 0 < E <∞, and P = 0. The

power signal is one which has finite average power and indefinite energy, i.e. $0 < P < \infty$, and $E = \infty$. If the signal does not satisfy any of these two conditions, then it is neither an energy nor a power signal.
Power Density Spectrum of Periodic Signals
The average power of a discrete-time signal with period N is given by

$$P_x = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt,\ E_\infty = \sum_{n=-\infty}^{\infty} |x(t)|^2$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-Th}^{Th} |x(t)|^2 dt, P$$

$$= \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} [x(n)]^2$$

For energy signal $0<E<\infty$ and $P = 0$
Power Signal $0 < P < \infty$, $E = \infty$
otherwise neither energy signals nor power signals

Example: Determines the values of P & Efor each of following

a) $x(t) = e^{-at}u(t) a > 0$

Solution: Energy Signal, E = 1/2a

b) $x(t) = A\cos(\omega_0 t + \theta)$

Solution: Power Signal, $P = A^2/2$

c) $x(t) = +u(t)$

Solution: Power Signal, P = 1/2

d) $x(t) = e^{j(2t+\frac{\pi}{4})}$

Solution: Power Signal, P = 1

e) $x[n] = \left(\frac{1}{2}\right)^n u[n]$

Solution: Energy Signal, E = 4/3

f) $x[n] = ej((\pi/2)n + \pi/8)$

Solution: Power Signal, P = 1

g) $x(n) = \cos(\pi/4)n$

Solution: Power Signal, P = 1/2

h) $x(n) = A$

Solution: Power Signal, $P = A^2$

i) $x[n] = \delta(t)\ (t)$

Solution: Neither Energy nor Power Signal

k) $1/2\ \mathrm{Sgn}(t)$

Solution: Power Signal, P = ¼

l) $5u(-t)$

Solution: Power Signal, P = 25/2

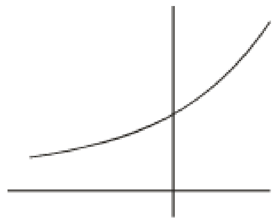
(m) 5cos 3t + 6 sin 4t

Solution: Power Signal, P = 25/2 + 18 = 61/2

All periodic Signals are power signal, but vice, versa is not true.

Example: Find the energy/power signal?

a) $x(t) = e^{3t}\ u(t) \rightarrow E = \infty, P = \infty$
Neither E nor P

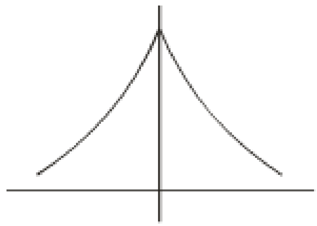


b) $x(t) = e^{-a|t|}$

$= e^{-at}u(t) + e^{at}u(-t)$

$\downarrow \quad \downarrow$

$E \rightarrow \frac{1}{2a} + \frac{1}{2a} \quad E = \frac{1}{a}$



Example : if x(t) = E, then x(2t) = ?

$$E_t = \int_{-\infty}^{\infty} |x(t)|^2 dt, E_{2t} = ?$$

$2t = p$

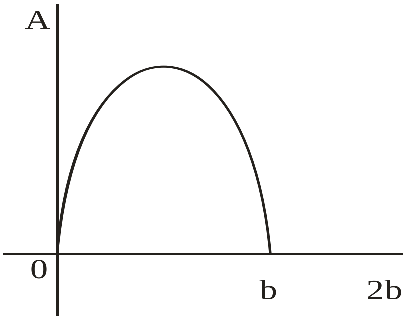
$2dt = dp$

$dt = \frac{dp}{2}$

$$E_{2t} = \int_{-\infty}^{\infty} |x(2t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |x(p)|^2 \frac{dp}{2} = \frac{E}{2}$$

Example :



Find the energy of $x(t)$.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$A\sin\frac{\pi}{b}t \qquad 0 < t < b$

$0 \qquad$ Otherwise

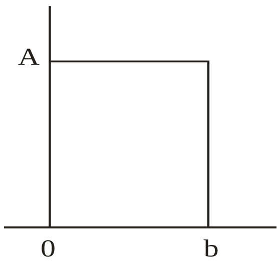
$$\int_0^b \left|A\sin\left(\frac{\pi}{b}t\right)\right|^2$$

$$= \int_0^b \left[\frac{A^2}{2} - \frac{A^2}{2}\cos\left(\frac{2\pi}{b}t\right)\right]dt$$

$$= \frac{A^2}{2}(b-0) - \frac{A^2}{2}\left[\frac{\sin\frac{2\pi t}{b}}{\frac{2\pi}{b}}\right]$$

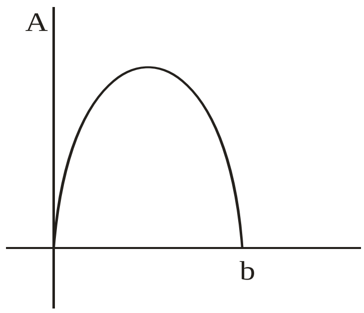
$$= \frac{A^2}{2}b - \frac{A^2}{2}\left[\frac{\sin\frac{2\pi}{b}\times b}{\frac{2\pi}{b}}\right] = \frac{A^2}{2}b$$

1)



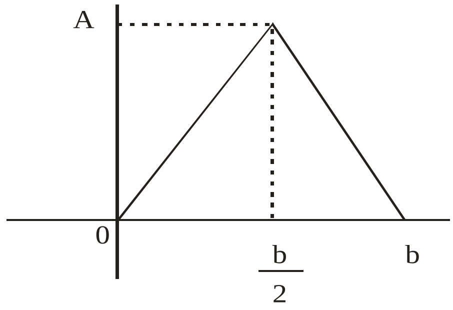
$E = A^2 b$

2)



$E = A^2 b/2$

3)



$E = A^2 b/3$

Signal Energy is directly proportional to with of signal and square of Amplitude. It is independent of the location of the signal

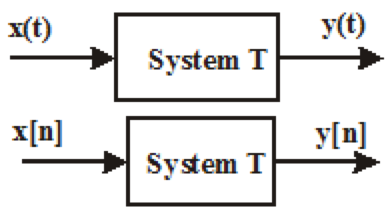
1.8 CLASSIFICATION OF SYSTEMS

As with signals, systems are also broadly classified into continuous-time and discrete-time systems. In a continuous-time system, the associated signals are also continuous, i.e. the input and output of the system are both continuous-time signals. On the other hand, a discrete-time system handles discrete-time signals. Here, both the input and output signals are discrete-time signals. Both continuous and discrete-time systems are further classified into the following types.

i) Static and dynamic systems
ii) Linear and non-linear systems
iii) Time-variant and time-invariant systems
iv) Causal and non-causal system
v) Stable and unstable systems.

A system is a mathematical model of a physical process that relates the input (or excitation) signal to the output (response) signal.

A continuous time & Discrete time systems



i)System with memory and without Memory:
Static and Dynamic Systems :

The output of a static system at any specific time depends on the input at that particular time. It does not depend on past or future values of the input. Hence, a static system can be considered as a system with on memory or energy storage elements. A simple resistive network is an example of a static system. The input/output relation of such systems does not involve integrals or derivatives.
The output of a dynamic system, on the other hand at any specified time depends on the inputs at that specific time and at other time. Such systems have memory or energy storage elements. The equation characterizing a dynamic system will always be a differential equation for continuous-time system or a difference equation for a discrete-time system. Any electrical circuit consisting of a capacitor or an inductor is an example of a dynamic system.
The following equations characterize dynamic systems.

i) $\frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}+3y(t)=\frac{dx(t)}{dt}+x(t)$

ii) y(n - 1) + 2y(n) = 4x(n) – x(n – 1)
A system is said to be memoryless if the output at any time depends on only the input at that some time

$y(t)=Rx(t)-$ memoryless

$y(t)=\int_{-\infty}^{t}x(t)dt-$ memory

$y[n]=\sum_{k=-\infty}^{n}x[k]-$ memory

1.9 STATIC[MEMORYLESS] & DYNAMIC [WITH MEMORY]

Static :- output and input should depend on the same instant, not on past & future.

1) $y(t)=(t+3)^3x(t)\rightarrow$ static

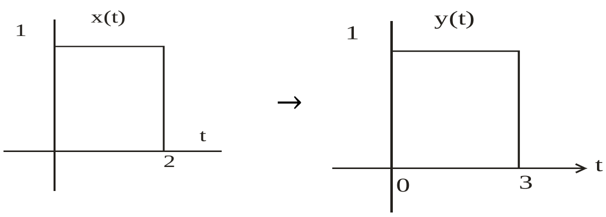
2) $y(t)=e^{-3t}x(t)\rightarrow$ static

3) $y(t)=\frac{d}{dt}x(t)\rightarrow$ rate of change have dynamic

4) $y(n)=g(n)-x(n-1)$

5) $y(n)=x[3n]\rightarrow$ dynamic

All static systems are causal (not vice versa)



output starting at 0 same as input hence causal. Duration of input and output is not same hence dynamic

1.10 LINEAR SYSTEM & NON LINEAR SYSTEM

A linear system is one in which the principle of superposition holds. For a system with two inputs x1(t) and x2(t). Thus, a linear system is a defined as one whose response to the sum of the weighted inputs is same as the sum of the weighted responses.

Example :

a) $\frac{dy(t)}{dt}+2y(t)=x(t)$ is linear

b) $\frac{dy(t)}{dt}+y(t)+4=x(t)$ is linear

Linearity $\rightarrow$1) Superposition property
2) Homogeneity (or scaling)

$$\text{Super position}\begin{cases}y_1=T[x_1]\\y_2=T[x_2]\\T[x_1+x_2]=y_1+y_2\end{cases}$$

Homogeneity (or scaling) {a zero input yields a zero output}
Otherwise non – linear

output due to scaled input must be scaled version of output.

$\therefore \propto x_1(t) + \beta x_2(t) \to \propto y_1(t) + \beta y_2(t)$

output due to sum of inputs must equal to sum of individual outputs.

Example : $y(t) = x(t) \sin 3\pi t$

Solution : Since sin $3\pi t$ is a predefined signal hence system is linear.

Product of arbitrary signal makes the system non linear.

Example : $x_1(t) \to y_1(t)$, $x_2(t-4) \to y_2(t)$, where $x_1(t)$ & $x_2(t-4)$ are inputs and $y_1(t)$ & $y_2(t)$ are outputs

Solution:

$y = y_1(t) + y_2(t) \to x_1(t) + x_2(t-4)$

$y(t) = x(t)\, x(t-4)$

$x_1(t) \to y_1(t) = x_1(t)\, x_1(t-4)$

$x_2(t) \to y_2(t) = x_2(t)\, x_2(t-4)$

$y(t) \to y_1(t) + y_2(t)$ for linear

$y(t) \neq y_1(t) + y_2(t)$ non linear

Important Points :

I) Integration, differentiation is linear system

II) Modules is non linear system

III) Adding of const to signal makes system non linear

IV) Exponential is a non – linear

V) Trigonometric function is non linear

VI) Arbitrary signal division is non linear

VII) Conjugate signal leads non linear

VIII)Real & imaginary part also leads to non linear

IX) Sampling is a linear system (variant)

Example :

i) $y(n) = a^{x(n)}$ Sol: $\to$ non linear

ii) $y(n) = x^2(n)$ Sol: $\to$ non linear

iii) $y(n) = x[n^2]$ Sol: $\to$ linear

iv) $y(n) = x[n^2 - 2]$ Sol: $\to$ linear

v) $y(n) = x[k_0 n]$ Sol :$\to$ linear

vi) $y(n) = nx[n]$ Sol :$\to$ linear

1.11 TIME INVARIANT AND TIME – VARYING SYSTEMS

A time-invariant system is one whose input-output relationship does not vary with time.

$H[x(t-T)] = y(t-T)$ or

A system is called time invariant if a time shift (delay or advance) in the input signal causes the same time shift in the output signal system is time Invariant

$T[x(t-\tau)] = y(t-\tau)$

$T[x(n-k)] = y[n-k]$

Otherwise time varying

Example : $y(t) = tx(t) + 2$

Solution :

delay input $y(t) = t[x(t-t_0)] + 2$...... (1)

delay output $y(t-t_0) = (t-t_0)[x(t)] + 2$ .... (2)

since (1) ≠ (2)

∴ system is time variant.

Example : $y(t) = e^{-x(t)}$

Solution : → time invariant

Example : $y(t) = x^2(t)$

Solution : → time invariant

Example : $y(t) = x(t) \cos t$,

Solution: → time variant, modulation system are time variant

Example : $y(t) = \frac{d}{dt}x(t)$

Solution: delay input : $\frac{d}{dt}x(t - t_0)$

delay output : $\frac{d}{d(t-t_0)}x(t - t_0)$,

constant

$\frac{d}{dt}\text{const} = 0$ time invariant

Example : $y(t) = x(3t)$

Solution: delay input

$y(t) = x(3t - t_0)$ ....(1)

delay output

$y(t - t_0) = x[3(t - t_0)]$ ....(2)

(1) & (2) are not same hence time variant

Example : $y(t) = (\cos 3t)\, x(t)$
Solution: → time variant
delay input → $y(t) = \cos 3t\, x(t - t_0)$
delayed output → $y(t - t_0) = \cos 3(t - t_0)\, x(t - t_0)$

Example : $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$
Solution : input delayed by p – unit
$\int_{-\infty}^{2t-p} x(\tau - p)d\tau$ $\tau - p = k$
$d\tau = dk$
$= \int_{-\infty}^{2t-p} x(k)dk$
Delayed output $y(t-p) = \int_{-\infty}^{2(t-p)} x(\tau)d\tau$
$= \int_{-\infty}^{2t-p} x(\tau)d\tau \rightarrow$ not equal
time variant

1.12 CAUSAL & NON CAUSAL SYSTEM

A continuous time signal is said to be causal if its amplitude is zero for negative time,
i.e., $x(t) = 0$ for $t < 0$
For a discrete time signal, the condition for causality is
$x(n) = 0$ for $n < 0$
$x(t) = u(t)$, the unit step function is a good example for a causal signal.
A signal is said to be anti-causal if its amplitude is zero for positive time,
i.e., for a continuous time signal,
$x(t) = 0$ for $t > 0$
for a discrete time signal,
$x(n) = 0$ for $n > 0$
A signal which is neither causal nor anti causal is called a non-causal signal.
A system is called causal if its output y(t) at an arbitrary time $t = t_0$ depends on only the input x(t) for $(t \le t_0)$ not causal = non causal,

Example : $y(t) = x(t + 1)$
Solution : non causal

Example : $y(t) = x[-n]$
Solution : causal
$y(t) = x(t) \cos (t + 1)$

Note: - All memory less system are causal but not vice versa.

Example : $y(t) = (3t + 5)\, x(t)$
Solution : → causal (slating has no effect)

Example: $y(t) = |x(t)|$ All practical systems are causal system
Solution : → causal

Example : $y(t) = x(2t)$, $y(1) = x(2)$, $t = 1$
Solution : → non causal

Example : $y(t) = \sin [x(t)]$
Solution : → causal

Example : $y(t) = x [\sin t]$
Solution : → non causal
$y[-\pi] = x(0)$
Past out put future input

Example : $y[n] = \sum_{k=no}^{n} x[k]$ no is finite

Solution :
case : 1 $n_0 > n$ → non causal
case : 2 $n_0 \le n$ → non causal

Example : $y(n) = \sum_{k=0}^{n} x[k]$

Solution : → non causal
$y(-1) = x[0] + x[-1]$

Example : $y[n] = \sum_{k=-\infty}^{n} x[k]$ [accumulator]

Solution : → causal
Adding all past input's and present input

1.13 STABLE AND UNSTABLE SYSTEMS

A system is bounded input/bounded output BIBO stable if for any bound input $(x) \le k$, $(y) \le k_2$
Response without input = Natural Response
Response with input = Force Response

"A system is said to be stable if it produces the bounded output for bounded input (BIBO stable system)"

To ensure the stability the output must be bounded for the bounded input.
Therefore input

Discrete System	Continuous System
$\lvert y(n) \rvert < \infty$	$\lvert y(t) \rvert < \infty$
$\left\lvert \sum_{k=-\infty}^{\infty} x(k)h(n-k) \right\rvert < \infty$	$\int_{-\infty}^{\infty} \lvert x(t)h(t-\tau)\, dt \rvert < \infty$
When input is Bounded $M_x = \lvert x(n) \rvert < \infty$	When input is Bounded $M_x = \lvert x(t) \rvert < \infty$
$\sum_{k=-\infty}^{\infty} \lvert h(n-k) \rvert < \infty$	$\int_{-\infty}^{\infty} \lvert h(t-\tau)\, dt \rvert < \infty$

If $x(t) \le m(x) < \infty$, then $y(t) \le m$ $y < \infty$ system is stable.

Example : $y(t) = x^2(t)$
Solution : $\therefore y(t) = u^2(t) \rightarrow$ stable
$|y(t)| = |x(t)|^2 \rightarrow$ finite $\rightarrow y(t)$ is also finite

Example : $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

Solution : $y(t) = \int_{-\infty}^{t} u(\tau)\, dt = \int_{-\infty}^{t} dt = t$

unbounded, since integration of unit step is ramp as $t \rightarrow \infty$, $y(t) \rightarrow \infty$

Example : $y(t) = x(2t)$
Solution: = stable, amplitude remains same

Example: $y(t) = x(t-4)$
Solution: = stable, amplitude remains same

Example : $y(t) = \frac{d}{dt} x(t)$

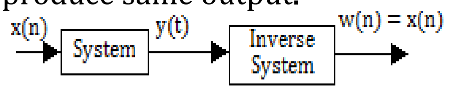
Solution : $\rightarrow$ Unstable, $\frac{d}{dt}$ of step is impulse which is unbounded

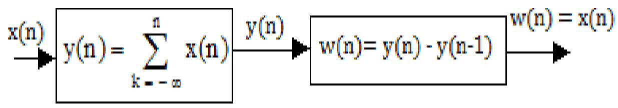
Example: $y[n] = e^{-|x[n]|}$
Solution : $\rightarrow$ stable
$y(n) = e^{-|x[n]|}$ if $x(n) = u(n)$ then

$y(n) = e^{\text{finite}} =$ finite, stable for unbounded input

1.14 INVERTIBLE & INVERSE SYSTEM

A system is invertible if different inputs leads to different outputs. i.e. for a given system two different inputs should not produce same output.





Example : $y(t) = x^2(t)$
Solution : $\rightarrow$ [apply standard inputs]
when $x(t) = -2u(t)$, $y(t) = 4u(t)$
$x(t) = 2u(t)$, $y(t) = 4u(t)$
same outputs with different input, non invertible

Example : $y(t) = |x(t)|$
Solution : $x(t) \rightarrow y(t)$, $-x(t) \rightarrow y(t)$
$\rightarrow$ non invertible

Example : $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$

Solution: Invertible system, Integration – Linear

$\therefore$ Inverse of system is $\left[\frac{dy(t)}{dt}\right]$

Example : $y(t) = x(t-5)$
Solution : $\rightarrow$ Invertible System
$\therefore$ Inverse of system is $y(t+5) = x(t)$

Example : $y(t) = \frac{d}{dt} x(t)$

Solution : $\rightarrow$ Non invertible system
$\frac{d}{dt}$ (const is zero) $\rightarrow$ Inverse is not same
$\frac{d}{dt} c = 0, \frac{d}{dt} 2c = 0$

Example : $y(n) = x(n)\, x(n-4) \rightarrow$ i. e. Multiplier systems

Solution :→ Non invertible

Example : $y(t) = x(2t)$ → Invertible
Solution : $x(t) = y(t/2)$

Example : $y[n] = x[2n]$ → Non invertible
Solution :
$x_1(n) = \delta(n) + \delta(n-1)$ and $x_2(n) = \delta(n)$
given $y(n) = \delta(n)$

$$y(n) = \begin{cases} x(n/2) & n\text{-even} \\ 0 & n\text{-odd} \end{cases}$$

Invertible $y(n) = x(2n)$
$x(n) = y(2n)$

GATE QUESTIONS(EC)

Q.1 Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty}\delta(t)\cos\left(\frac{3t}{2}\right)dt$ is

a) 1 b) -1

c) 0 d) $\frac{\pi}{2}$

[GATE-2001]

Q.2 If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to

a) E b) $\frac{E}{2}$

c) 2E d) 4E

[GATE-2001]

Q.3 Let P be linearity, Q be time – invariance, R be causality and S be stability .A discrete- time system has the input- output relationship.

$$y(n)\begin{cases} x(n) & n \geq 1 \\ 0, & n = 0 \\ x(n+1), & n \leq -1 \end{cases}$$

Where $x(n)$ is the input and $y(n)$ is the output. The above system has the properties.

a)P,S but not Q,R b)P,Q,S but not R

c)P,Q,R,S d)Q,R,S but not P

[GATE-2003]

Q.4 Consider the sequence

$x[n] = [-4, j5 \underset{\uparrow}{1+j2} \quad 4]$

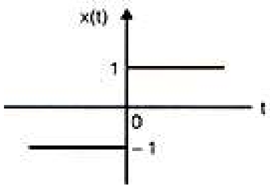
The conjunction anti-symmetric part of the sequence is

a)$[-4-j2.5\ j2\ 4-j2.5]$ b)$[-j2.5\ 1\ j2.5]$

c)$[-j5 \quad j2 \quad 0]$ d)$[-4 \quad 1 \quad 4]$

[GATE-2004]

Q.5 The function $x(t)$ is shown in the figure .Even and odd parts of a unit – step function $u(t)$ are respectively.



a)$\frac{1}{2},\frac{1}{2}x(t)$ b)$-\frac{1}{2},\frac{1}{2}x(t)$

c) $\frac{1}{2},-\frac{1}{2}x(t)$ d)$-\frac{1}{2},-\frac{1}{2}x(t)$

[GATE-2005]

Q.6 The power in the signal $s(t) = 8\cos(20\pi t - \frac{\pi}{2}) + 4\sin(15\pi t)$ is

a) 40 b) 41

c) 42 d) 82

[GATE-2005]

Q.7 The Dirac delta function δ(t) is defined as

a) $\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{otherwise} \end{cases}$

b) $\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases}$

c) $\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)dt = 1$

d) $\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)dt = 1$

[GATE-2006]

Q.8 A system with input $x[n]$ and output $y[n]$ is given as $y[n] = \left(\sin\frac{5}{6}\pi n\right) \times (n)$

The system is

a) linear, stable and invertible

b) non-linear stable and non-invertible

c) linear, stable and non- invertible

d) linear, unstable and invertible

[GATE-2006]

Q.9 The input and output of a continuous time system are respectively denoted by x(t) and y(t). Which of the following descriptions corresponds to a causal system?

a) $y(t) = x(t-2)x(t+4)$

b) $y(t) = (t-4)x(t+1)$

c) $y(t) = (t+4)x(t-1)$

d) $y(t) = (t+5)x(t+5)$

[GATE-2008]

Q.10 Let x(t) be the input and y(t) be the output of a continuous time system. Match the system properties $P_1, P_2,$ and $P_3,$ with system relations R_1, R_2, R_3, R_4

Properties	Relation
P_1: Linear but NOT time –invariant	R_1: $y(t) = t^2\|x(t)\|$
	R_2: $y(t) = t\|x(t)\|$
P_2: Time –invariant but NOT linear	R_3: $y(t) = \|x(t)\|$
P_3: Linear and time – invariant	R_4: $y(t) = x(t-5)$

a) $(P_1, R_1), (P_2, R_3)(P_3, R_4)$

b) $(P_1, R_2), (P_2, R_3)(P_3, R_4)$

c) $(P_1, R_3), (P_2, R_1)(P_3, R_2)$

d) $(P_1, R_1), (P_2, R_2)(P_3, R_3)$

[GATE-2008]

Q.11 The input x(t) and output y(t) of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\, d\tau$. The system is

a) time-invariant and stable

b) stable and not time-invariant

c) time-invariant and not stable

d) not time-invariant and not stable

[GATE-2012]

Q.12 For a periodic signal $v(t) = 30\sin 100t + 100\cos 300t + 6\sin(500t + \pi/4)$ the fundamental frequency in rad/s is

a) 100 b) 300

c) 500 d) 1500

[GATE-2013]

Q.13 The impulse response of a continuous time system is given by $h(t) = \delta(t-1) + \delta(t-3)$. The value of the step response at $t = 2$ is

a) 0 b) 1

c) 2 d) 3

[GATE-2013]

Q.14 A discrete-time signal $x[n]=\sin(\pi^2 n)$, n being an integer, is

a) periodic with period π

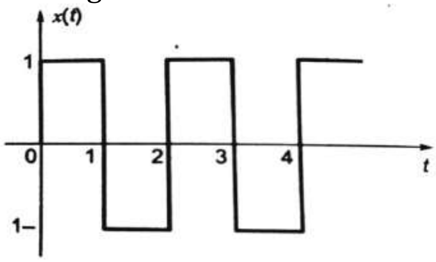
b) periodic with period π^2

c) periodic with period $\pi/2$

d) not periodic

[GATE-2014, Set1]

Q.15 Consider the periodic square wave in the figure shown



The ratio of the power in the 7th harmonic to the power in the 5th harmonic for this waveform is closest in value to................

[GATE-2014, Set2]

Q.16 A stable linear time invariant (LTI) system has a transfer function

$$H(s) = \frac{1}{s^2 + s - 6}$$

To make this system casual it needs to be cascaded with another LTI system having a transfer function $H_1(s)$ among the following options is

a) S+3 b) s-2

c) S-6 d) s+1

[GATE-2014, Set4]

Q.17 A continuous time function x(t) is periodic with period T. The function is sampled uniformly with a sampling period T_S. In which one of the following cases is the sampled signal periodic?

a) $T=\sqrt{2}\,T_S$ b) $T=1.2T_s$

c) Always d) Never

[GATE-2016, Set-1]

Q.18 The input x(t) and the output y(t) of a continuous time system are related as

$$y(t)=\int_{t-T}^{t} x(u)du$$

The system is

a) linear and time-variant
b) linear and time-invariant
c) non-linear and time-variant
d) non-linear and time-invariant

[GATE-2017, Set-2]

Q.19 Let the input be u, the output be y of a system, and the other parameters are real constants. Identify which among the following systems is not a linear system.

a)

$$\frac{d^3y}{dt^3}+a_1\frac{d^2y}{dt^2}+a_2\frac{dy}{dt}+a_3y=b_3u+b_2\frac{du}{dt}+b_1\frac{d^2u}{dt^2}$$

b) $y(t)=\int_0^t e^{\alpha(t-\tau)}\beta u(\tau)d\tau$

c) $y=au+b, b\neq 0$

d) $y=au$

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(a)	(b)	(a)	(a)	(a)	(a)	(d)	(c)	(c)	(a)	(d)	(a)	(b)	(d)
15	16	17	18	19									
0.51	(b)	(b)	(b)	(c)									

EXPLANATIONS

Q.1 (a)

$\int_{-\infty}^{\infty} \delta(t)\left(\frac{3t}{2}\right) dt = f(0)$

$= \cos\left(\frac{3\times 0}{2}\right) = \cos 0 = 1$

Q.2 (b)

$E = \int_{-\infty}^{\infty} f(t)^2 dt$

$E' = \int_{-\infty}^{\infty} f(2t)^2 dt$

$= \int_{-\infty}^{\infty} f(p)^2 \frac{dp}{2} \left(2t = p;\ dt = \frac{dp}{2}\right)$

$E' = \frac{E}{2}$

Q.3 (a)

$y(n - n_0)x(n - n_0 + 1)$

(time varying)

$y(n) = x(n + 1)$

(depends on future)

ie., $y(1) = x(2)$ (non causal)

For bounded input, system has bounded output, So it is stable.

$y(n) = x(n), n \geq 1$

$= 0, n = 0$

$= x(n + 1), n \leq -1$

So, system is linear.

$= Lt_{s\to\infty'} \frac{5\times s}{s(s^2+3s+2)} = 0$

Q.4 (a)

$x(n) = [-4 - j5, 1 + 2j, 4]$

$x^*(-n) = [4 \ \ 1 - 2j, -4 + j5]$

$x_{CAS}(n) = \frac{x(n)-x^*(-n)}{2}$

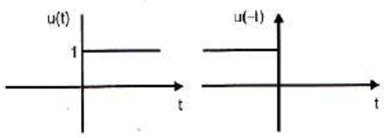
$= [-4 - 2.5j, 2j, 4 - j2.5]$

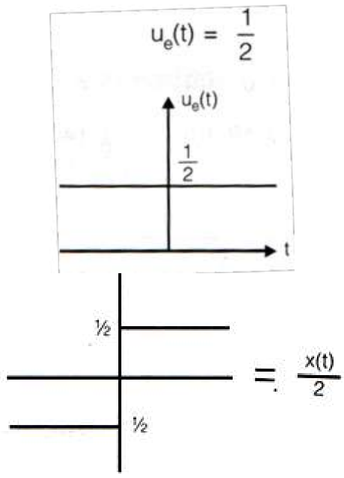
Q.5 (a)

Even part $= \frac{\alpha(t)+\alpha(-t)}{2}$

Odd part $= \frac{\alpha(t)+\alpha(-t)}{2}$

Here $\alpha(t) = u(t)$





Q.6 (a)

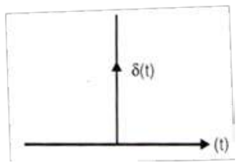
$s(t) = 8\cos\left(\frac{\pi}{2} - 20\pi t\right)$

$4 \sin 15\pi t$

$= 8 \sin 20\pi t + 4 \sin 15\pi t$

$P = \frac{8^2}{2} + \frac{4^2}{2} = 32 + 8 = 40$

Q.7 (d)



Q.8 (c)

$y[n] = \left(\sin\frac{5}{6}\pi n\right) \times (n)$

Let $x[n] = \delta(n)$

$\therefore y[n] = \sin 0 = 0(bounded)$

BIBO stable

Q.9 (c)

A system is causal if the output at any time depends only on values of the input at the present time and in the past.

Q.10 (a)

Q.11 (d)

$y = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\, d\tau$

$y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\cos(3\tau)\,d\tau$

$y'(t)$ for input $x(t-t_0)$ is

$y'(t) = \int_{-\infty}^{t} x(\tau - t_0)\cos 3\tau\,d\tau$

$y'(t) = \int_{-\infty}^{(t-t_0)} x(\tau)\cos 3(\tau + t_0)d\tau$

$y'(t) \neq y(t-t_0)$ so system is not time invariant for input $x(\tau) = \cos(3\tau)$ bounded input

$y'(t) = \int_{-\infty}^{t} \cos^2(3\tau)\,d\tau$

$\rightarrow \infty$ as $t \rightarrow \infty$

So for bounded input, output is not bounded therefore system is not stable.

Q.12 (a)

$\omega_1 = 100$

$\omega_2 = 300$

$\omega_3 = 500$

H.C.F. of ω_1, ω_2 and ω_3 = H.C.F (100, 300, 500)

$\omega = 100\ rad/sec$

Q.13 (b)

Step response = Integration of impulse response

$\int \delta(t-1) = u(t-1)$

$\int \delta(t-3) = u(t-3)$.

At t = 2

$y(t) = 1$

Q.14 (d)

$x[n] = \sin(\pi^2 n)$

$\omega_0 = \pi^2$

$N = \frac{2\pi}{\omega_0} m$

Where m is the smallest integer that converts $\frac{2\pi}{\omega_0}$ into a integer value.

$N = \frac{2\pi}{\pi^2} m = \frac{2}{\pi} m$

So there exists no such integer value of m which could make the N integer, so the system is not periodic

Q.15 0.51

Q.16 (b)

$H(s) = \frac{1}{(s^2 + s - 6)}$

The system is said to be causal if the output at any time depends only on present and/or past values of input.

$H(s) = \frac{1}{(s+3)(s-2)}$

The system is causal

Q.17 (b)

A signal is said to be periodic if $\frac{T}{T_s}$ is a rational number.

Here, $T = 1.2T_s$

$T = 1.2T_s$

Q.18 (b)

The system is linear and time-invariant

Q.19 (c)

$T\{u_1\} = au_1 + b$

$pT\{u_1\} = pau_1 + pb$

$T\{u_2\} = au_2 + b$

$qT\{u_2\} = qau_2 + qb$

$T\{pu_1 + qu_2\} = a[pu_1 + qu_2] + b$

$T\{pu_1 + qu_2\} = apu_1 + aqu_2 + b$

Equation 1 + equation 2 ≠ equation 3

Principle of superposition is NOT being followed.

a linear system.

GATE QUESTIONS(EE)

Q.1 A current of $5\delta(t)$, is forced through a capacitor C. The voltage $v_c(t)$, across the capacitor is given by

a) 5 t   b) 5 u(t) − C

c) $\frac{5}{C}t$ d) $\frac{5u(t)}{C}$

[GATE-2002]

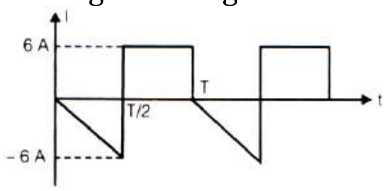
Q.2 The rms value of the resultant current in a wire which carries a dc current of 10A and a sinusoidal alternating current of peak value 20 A is

a) 14.1 A   b) 17.3 A

c) 22.4 A   d) 30.0 A

[GATE-2004]

Q.3 The rms value of the periodic waveform given in figure is

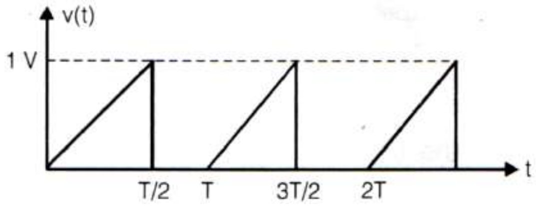


a) $2\sqrt{6}$ A   b) $6\sqrt{2}$ A

c) $\sqrt{4/3}$ A   d) 1.5 A

[GATE-2004]

Q.4 For the triangular wave from shown in the figure, the RMS value of the voltage is equal to



a) $\sqrt{\frac{1}{6}}$ V   b) $\sqrt{\frac{1}{3}}$ V

c) $\frac{1}{3}$ V   d) $\sqrt{\frac{2}{3}}$ V

[GATE-2005]

Q.5 The running integrator, given by $y(t) = \int_{-\infty}^{t} x(\alpha)d\alpha$

a) has no finite singularities in its double sided Laplace Transform Y(s)

b) produces a bounded output for every causal bounded input

c) produces a bounded output for every anti causal bounded input

d) has no finite zeroes in its double sided Laplace Transform Y(s)

[GATE-2006]

Q.6 A continuous-time system is described by $y(t) = e^{-|x(t)|}$, where $y(t)$ is the output and $x(t)$ is the input. $y(t)$ is bounded

a) only when x(t) is bounded

b) only when x(t) is non-negative

c) only for $t \le 0$ if x(t) is bounded for $t \ge 0$

d) even when x(t) is not bounded

[GATE-2006]

Q.7 Which of the following is true?

a) A finite signal is always bounded

b) A bounded signal always possesses finite energy

c) A bounded signal is always zero outside the interval $[-t_0, t_0]$ form some t_0

d) A bounded signal is always finite

[GATE-2006]

Linked Answer Questions 8 & 9.

Q.8 A signal is processed by a causal filter with transfer function G(s). For a distortion free output signal waveform, G(s) must

a) provide zero phase shift for all frequency

b) Provide constant phase shift for all frequency

c) provide linear phase shift that is proportional to frequency

d) provide a phase shift that is inversely proportional to frequency

[GATE-2007]

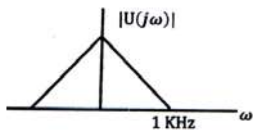
Q.9 $G(Z) = \alpha z^{-1} + \beta z^{-3}$ is allow pass digital filter with a phase characteristics same as that of the above question if

a) $\alpha = \beta$ b) $\alpha = -\beta$

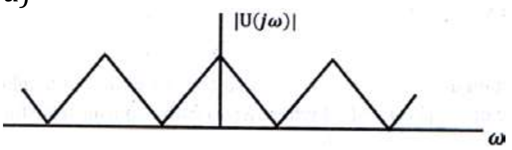
c) $\alpha = \beta^{(1/3)}$ d) $\alpha = \beta^{(-1/3)}$

[GATE-2007]

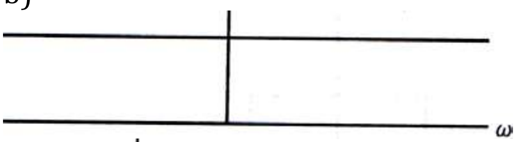
Q.10 The frequency spectrum of a signal is shown in the figure. If this is ideally sampled at intervals of 1 ms, then the frequency spectrum of the sampled signal will be

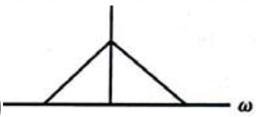


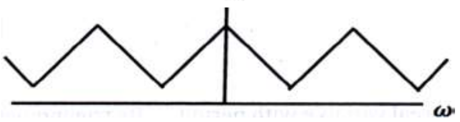
a)



b)



c) 

d) 

[GATE-2007]

Q.11 Given a sequence $x[n]$, to generate the sequence $y[n] = x[3 - 4n]$, which one of the following procedures would be correct?

a) First delay $x[n]$ by 3 samples to generate $Z_1[n]$, then pick every 4^{th} sample of $Z_1[n]$ to generate $Z_2[n]$, and then finally time reverse $Z_2[n]$ to obtain $y[n]$

b) First advance $x[n]$ by 3 samples to generate $Z_1[n]$ then pick every 4th sample of $Z_1[n]$ to generate $Z_2[n]$, and then finally time reverse $Z_2[n]$ to obtain $y[n]$

c) First pick every fourth sample of $x[n]$ to generate $v_1[n]$, time-reverse $v_1[n]$ to obtain $v_2[n]$, and finally advance $v_2[n]$, by 3 samples to obtain $y[n]$

d) First pick every fourth sample of $x[n]$ to generate $v_1[n]$, time-reverse $v_1[n]$ to obtain $v_2[n]$, and finally delay $v_2[n]$, by 3 samples to obtain $y[n]$

[GATE-2008]

Q.12 A zero mean random signal is uniformly distributed between limits –a and +a and its mean square value is equal to its variance. Then the r.m.s. value of the signal is

a) $\frac{a}{\sqrt{3}}$ b) $\frac{a}{\sqrt{2}}$

c) $a\sqrt{2}$ d) $a\sqrt{3}$

[GATE-2011]

Q.13 An input signal $x(t)=2 + 5\sin(100\pi t)$ is sampled with a sampling frequency of 400Hz and applied to the system whose transfer function is represented by

$$\frac{Y(z)}{X(z)} = \frac{1}{N}\left(\frac{1-Z^{-N}}{1-Z^{-1}}\right)$$

Where, N represents the number of samples per cycle. The output y(n) of the system under steady state is

a) 0 b) 1 c) 2 d) 5

[GATE-2014]

Q.14 A sinusoid x(t) of unknown frequency is sampled by an impulse train of period 20ms. The resulting sample train is next applied to an ideal low-pass filter output is seen to be a sinusoid of frequency 20Hz. This means that x(t) has a frequency of

a) 10Hz b) 60Hz

c) 30Hz d)90Hz

[GATE-2014]

Q.15 The value of $\int_{-\infty}^{+\infty} e^{-t}\,\delta(2t-2)\,dt$, where $\delta(t)$ is the Dirac delta function, is

a) $\frac{1}{2e}$ b) $\frac{1}{2e}$

c) $\frac{1}{e^2}$ d) $\frac{1}{2e^2}$

[GATE-2016]

Q.16 Consider the system with following input-output relation $y[n]=(1+(-1)^n)\,x[n]$, Where x[n] is the input and y[n] is the output. The system is

a) invertible and time invariant

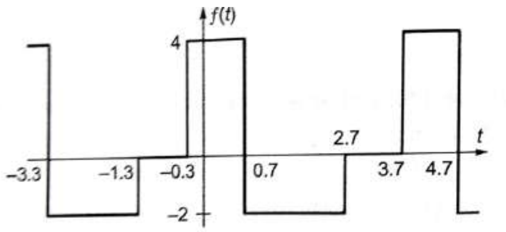
b) invertible and time varying

c) non-invertible and time invariant

d) non-invertible and time varying

[GATE-2017]

Q.17 The mean square value of the given periodic waveform f(t) is________



Q.18 Let *f* be the real valued function of a real variable defined $f(x)=x-[x]$ where [x] denotes the largest integer less than or equal to x . The value of $\int_{0.25}^{1.25} f(x)\,dx$ is _____(up to 2 decimal places)

[GATE-2018]

Q.19 Consider the two continuous-time signals defined below :

$$x_1(t)=\begin{cases}|t| & -1\le t\le 1\\ 0 & \text{otherwise}\end{cases}$$

$$x_2(t)=\begin{cases}1-|t| & -1\le t\le 1\\ 0 & \text{otherwise}\end{cases}$$

These signals are sampled with a sampling period of T = 0.25 seconds to obtain discrete-time signals $x_1[n]$ and $x_2[n]$, respectively. Which one of the following statements is true?

a) The energy of $x_1[n]$ is greater than the energy of $x_2[n]$.

b) The energy of $x_2[n]$ is greater than the energy of $x_1[n]$

c) $x_1[n]$ and $x_2[n]$ have equal energies.

d) Neither $x_1[n]$ and $x_2[n]$ is a finite-energy signal.

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9
d	b	a	a	c	d	c	c	a
10	11	12	13	14	15	16	17	18
b	b	a	c	c	a	d	6	0.5
19								
a								

EXPLANATIONS

Q.1 (d)

$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$

$= \frac{5}{C}u(t)$

Q.2 (b)

rms value of D.C. current = 10A

rms value of alternating current

$= 20/\sqrt{2} = 10\sqrt{2}$ A

∴ rms value of resultant current

$= \sqrt{10^2 + (10\sqrt{2})^2}A = 17.32$ A

Q.3 (a)

$I_{rms} = \left[\frac{I}{T}\int_0^T i^2(t)\right]^{1/2}$

$\therefore I_{rms}2 = [\frac{I}{T}[\int_0^{T/2}\left(-\frac{12t}{T}\right)^2 dt + \int_{T/2}^{T} 6^2\,dt]]$

$= \left[\frac{I}{T}\int_0^{T/2}\frac{144(t)^2}{T^2}dt + \frac{36}{T}\int_{T/2}^{T}dt\right]$

$= \frac{144}{T^3}\left[\frac{t^3}{3}\right]_0^{T/2} + \frac{36}{T}\left[\frac{T}{2}\right]$

$= \frac{144}{8\times 3} + \frac{36}{2} = 24$

$\therefore I_{rms} = 2\sqrt{6}$ A

Q.4 (a)

From the wave symmetry

$V_{rms} = \left[\frac{I}{T}\left[\int_0^{T/2}\left(\frac{2t}{T}\right)^2 dt + \int_{T/2}^{T} 0\,dt\right]\right]^{1/2}$

$V_{rms}^2 = \frac{I}{T}\int_0^{T/2}\frac{4t^2}{T^2} = \frac{4}{3T^3}\left[\frac{T^3}{8}\right] = \frac{1}{6}$

$\therefore \quad V_{rms} = \sqrt{\frac{1}{6}}$ V

Q.5 (c)

$y(t) = \int_{-\infty}^{t} x(\alpha)d\alpha$

So for bounded input and for bounded duration (finite duration) y(t) will be bounded.

So if signal will be causal and bounded then y(t) may be bounded or not since integration is from t = 0 to t = the time (maximum time) for which x(t) has finite values.

So if x(t)=u(t) which is bounded, output will be unbounded.

If signal is bounded and anti-causal then integration will only for time t = 0 for other time t > 0 signal will be zero so output will be finite.

Q.6 (d)

$y(t) = e^{-|x(t)|}$

e^{-x} is always convergent even when 'x' is not bounded.

∴ e^{-x} is bounded even through 'x' is not bounded.

Q.7 (c)

If the amplitude of a signal have some finite boundaries for all values of time then it is called as bounded signal.

i. e. $|f(t)| < \infty$

or $|f(t)| < M$ (*a finite +ve value*)

for all t.

So a bounded signal may possess finite energy or infinite energy.

For example u(t) is bounded signal but it possess infinite energy because it is a power signal.

It can be zero or nonzero outside a finite interval $(-t_0, t_0)$.

But it is always true that it will be always finite for any value of time t.

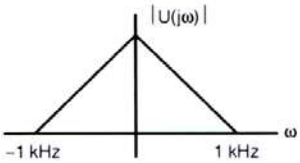
Q.8 (c)

For distortion free output phase shift must be linear function of frequency i.e. proportional to frequency, this is because delay to all frequency component will be equal

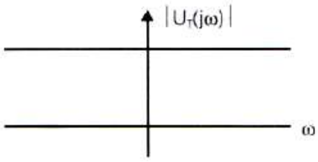
Q.9 (a)

For distortion free output phase shift must be linear function of ω as will as all the frequency component must be amplified by same amount so $z^{-1} = e^{-j\omega}$ corresponds to frequency ω. While $z^{-3} = e^{-3j\omega}$ corresponds to frequency 3ω.
In order to have same amplification of frequency component at ω and 3ω, $\alpha = \beta$.

Q.10 (b)

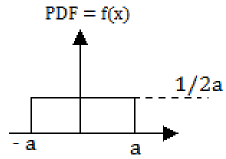


Given that sampling interval = msec
i.e.T_s = 1msec = 10^{-3}sec
Therefore sampling frequency
$f_s = \frac{1}{T_s} = \frac{1}{10^{-3}} = 1\text{kHz}$
After sampling new signal in frequency domain
$U_T(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} U(f - nf_s)$
∴ spectrum of sampled signal will be



Q.11 (b)
$y[n] = x[3 - 4n] = x[-4n + 3]$
So to obtain $y[n]$ we first advance $x[n]$ by 3 unit.
i.e. $Z_1[n] = x[n + 3]$
Now we will take every fourth sample of $Z_1[n]$ i.e. $Z_2[n] = Z_1[4n]$ = $x[4n + 3]$ Now reverse (time reverse) $Z_2[n]$ will give $y[n] = Z_2[-n] = x[-4n + 3]$

Q.12 (a)

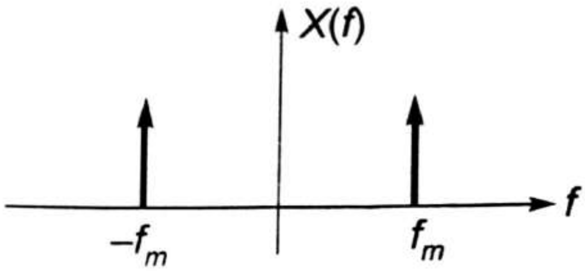


Mean square value
$= \int_{-\infty}^{\infty} x^2 f(x)\,dx$
$= \frac{1}{2a}\int_{-a}^{a} x^2\,dx = \frac{a^2}{3}$
RMS value = $\frac{a}{\sqrt{3}}$

Q.13 (c)

Q.14 (c)

Given, impulse train of period 20ms.
Then, Sampling frequency
$= \frac{1}{20\times10^{-3}} = 50\text{Hz}$
If the input signal x(t) $= \cos\omega_m(t)$ having spectrum



The filtered out sinusoidal signal has 20Hz frequency the sampling must be under sampling.
The output signal which is an under sampled signal with sampling frequency 50Hz is

And $50 - f_m = 20$ Hz
$f_m = 30$ Hz

Q.15 (a)

To find value of $\int_{-\infty}^{\infty} e^{-t}\delta(2t-2)dt$

Since, $\delta(2t-2)=\frac{1}{2}\delta(t-1)$

Above integral can be written as

$$\int_{-\infty}^{\infty} e^{-t}\frac{1}{2}\delta(t-1)dt=\frac{1}{2}e^{-1}=\frac{1}{2e}$$

Q.16 (d)

Given relationship,

$y(n)=[1+(-1)^n]x(n)$

Time invariance test:

Since, y(n-1) ≠ y'(n)

So, the system is time variant.

Invariability Test:

Thus, we are getting many to one mapping between input and output. So, the system is non-invertible.

Q.17 (6)

Mean square value = Power of f(t)

Mean square value=

$$\frac{1}{T_0}\int_{T_0}|f(t)|^2 dt$$

$$=\frac{1}{4}\left[4^2\times 1+2^2\times 2\right]$$

$$=\frac{16+8}{4}=6$$

Q.18 0.5

Given: $f(x)=x-[x]$

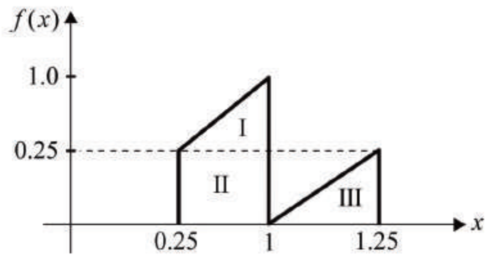
Where, [x] represents the largest integer less than or equal to *x.*

For $0\le x<1 \Rightarrow f(x)=x-[x]=x-0=x$

For $1\le x<2 \Rightarrow f(x)=x-[x]=x-1$

So,

$$f(x)=\begin{cases} x & 0.25\le x<1 \\ x-1 & 1\le x<1.25 \end{cases}$$



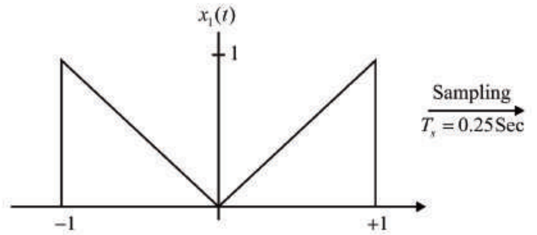
$\int_{0.25}^{1.25} f(x)dx=$ Area or triangle I + Area of Rectangle II + Area of triangle III

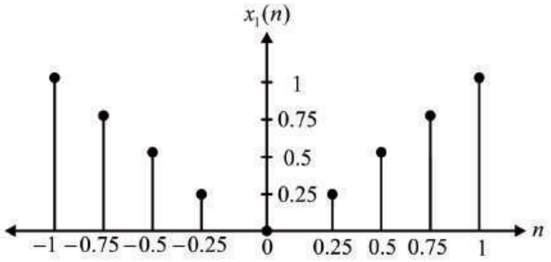
$$=\left(\frac{1}{2}\times 0.75\times 0.75\right)+(0.75\times 0.25)+\left(\frac{1}{2}\times 0.25\times 0.25\right)$$

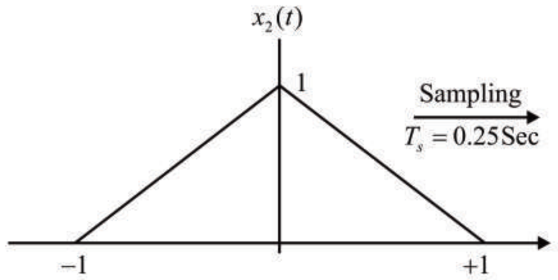
$=0.5$

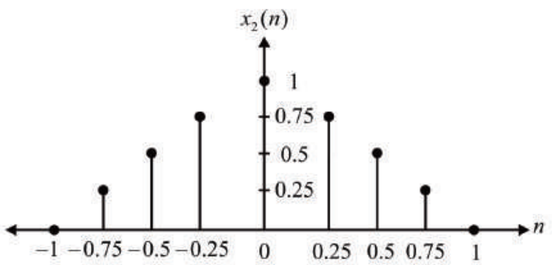
Hence, the correct answer is 0**.5.**

Q.19 (a)









$$E_1 = \sum_{n=-\infty}^{\infty} |x_1(n)|^2 = 0^2 + 2\left[1^2 + 0.75^2 + 0.5^2 + 0.25^2\right]$$

$$E_2 = \sum_{n=-\infty}^{\infty} |x_2(n)|^2 = 1^2 + 2\left[+0.75^2 + 0.5^2 + 0.25^2 + 0^2\right]$$

From above equations

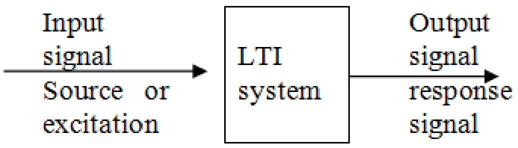
$E_1 > E_2$

Hence, the correct option is (A).

2 LINEAR TIME INVARIANT SYSTEM & CONVOLUTION

2.1 LTI SYSTEM

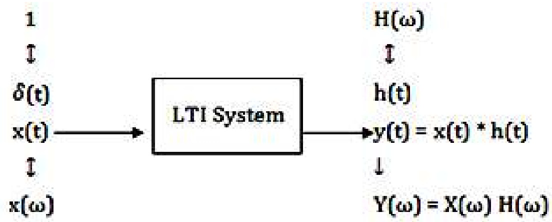
Signal transmission is a process whereby a message (or information - bearing) signal is transmitted over a communication channel. Signal filtering purpose fully alters the spectral content of signal so that a letter transmission and reception can be achieved.



Linearity → additive + homogeneity property

Time invariant → delayed input gives delayed output

LTI System = Linear + Time invariant System



Response y(t) of Linear Time Invariant is convolution of x(t) & impulse response h(t) of system.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

The convolution is commutation.

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Impulse response h(t) is → response of system when input is $\delta(t)$

$$h(t) = \delta(t) * h(t) = \int_{-\infty}^{\infty} \delta(t)\, h(t-\tau)\, d\tau$$

If h(t) = 0 for t < 0, then system is causal

Frequency response y(ω) = x(ω) H(ω)

y(ω) = Fourier transform Output

x(ω) = Fourier Transform Input

H(ω) = Fourier Transform of Impulse response

$$H(\omega) = \frac{y(\omega)}{x(\omega)}$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)H(\omega)e^{-j\omega t}d\omega$$

Inverse Fourier Transform

$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)d\tau$	Shifting property

According to shifting property energy, signal can be produced as superposition of impulses

$$X[n] = \sum_{k=-\infty}^{\infty} x(k)\delta[n-k]$$

The output of any Linear Time Invariant system is convolution of the input with the impulse response of the system.

Continuous System	Discrete System
$x(t) = \int_{-\infty}^{\infty} x(\tau) = \delta = (t-\tau)d\tau$	$x[n] = \sum_{k=-\infty}^{\infty} x(k)\delta[n-k]$
$y(t) = x(t) * h(t)$	$y(n) = x(n) * h(t)$
$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$	$x[n] = \sum_{k=-\infty}^{\infty} x(k)h[n-k]$

Impulse Response

$$h(t) = T[\delta(t)]$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

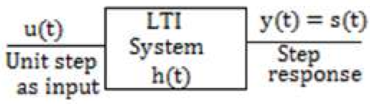
$$y(t)=T\left[x(t)\right]=T\left[\int_{-\infty}^{\infty} x(\tau)\ \delta(t-\tau)\,d\tau\right]$$

$y(t)=\int_{-\infty}^{\infty} x(\tau)\ T[\delta(t-\tau)]d\tau$ (Use of linearity property)

$y(t)=\int_{-\infty}^{\infty} x(\tau)\ h(t-\tau)\,d\tau$ (Use of Time invariance property)

$$y(t)=x(t)*=\ h(t)*x(t)=\int_{-\infty}^{\infty} h(\tau)\ x(t-\tau)\,d\tau$$

Step Response



s(t) = u(t)*h(t)

$$=\int_{-\infty}^{\infty} u(\tau)\ h(t-\tau)\,d\tau=\int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau$$

$$=\int_{-\infty}^{t} h(\tau)\,d\tau \ \Rightarrow h(t)=\frac{ds(t)}{dt}$$

Example

$s(t) = (1 - e^{-\alpha t})\ u(t)$,

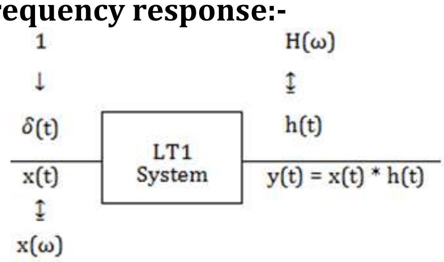
Find impulse response h(t)?

Solution

Impulse response h(t) = Differentiation of step response solve your self

2.2 THE FREQUENCY RESPONSE OF CONTINUOUS-TIME LTI SYSTEMS

A. Frequency response:-



y(t) = x(t) * h(t)

Y(ω) = X(ω) H(ω)

y(t) output of continuous time Linear Time invariant system equals : → Convolution of input x(t) & impulse response h(t)

y(t) = x(t) * h(t)

$$H(\omega)=\frac{Y(\omega)}{X(\omega)}=|H(\omega)|\,e^{j\theta x(\omega)}\text{phase resopnse}$$

↓

Magnitude response

$X(\omega)|X(\omega)e^{j\theta x(\omega)}Y(\omega)=|Y(\omega)|e^{j\theta y(\omega)}$

$|Y(\omega)| = |X(\omega)|\,|H(\omega)|$

$\theta_y(\omega) = \theta_x(\omega) + \theta_H(\omega)$

if input $x(t)=e^{j\omega ot}$

$X(\omega) = 2\pi\delta(\omega - \omega_o)$

$Y(\omega)=2\pi\ H(\omega)\delta(\omega-\omega_0)$

$Y(t) = H(\omega_o)e^{jwot} \rightarrow$ eigen function

↓

eigen value

B. Distortion less Transmission:

For distortion less Transmission → exact input signal shape reproduced at output (may Amplitude different, delay in time)

$y(t) = kx(t- t_d)$

$y(\omega) = ke^{-j\omega td}\ X^*(\omega)$

$H(\omega)e^{j\theta H\omega}=ke^{-j\omega td}$

$H(\omega) = k$, ⊾ $H(\omega)=e^{-j\omega td}$

Amplitude of H(ω) → const for entire frequency range

Phase of H(ω) → linearly vary with frequency

Amplitude distortion:-

$|H(\omega)|$ is not constant within frequency band i.e. frequency component of input are transmitted with a different amount of gain or attenuation → Amplitude distortion.

Phase distortion:-

$|\theta_H(\omega)|$ is not linear with frequency i.e. output has a different waveform than input signal because of different delays for frequency components of input signal.→ phase distortion.

2.3 FILTERING

Filtering is a process by which relative amplitudes of frequency components in a signal are change or some frequency components are suppressed. It exhibits some sort of frequency selection.

2.3.1 IDEAL FREQUENCY SELECTIVE FILTERS:-

It exactly passes signal at one set of frequency and completely rejects the rest band of frequency passed by filter – pass band of frequency rejected by filter – stop band

1. Ideal Low pass filter:-

$$|H(\omega)| = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \rightarrow \text{cut off frequency} \end{cases}$$

2. Ideal High pass filter:-

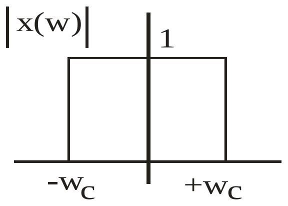
$$|H(\omega)| = \begin{cases} 0 & |\omega| < \omega_c \\ 1 & |\omega| > \omega_c \rightarrow \text{cut off frequency} \end{cases}$$

3. Ideal Band pass filter:-

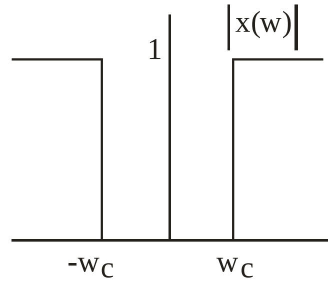
$$|H(\omega)| = \begin{cases} 1 & \omega_1 < |\omega| < \omega_c \\ 0 & \text{Otherwise} \end{cases}$$

4. Ideal Band stop filter:-

$$|H(\omega)| = \begin{cases} 0 & \omega_1 < |\omega| < \omega_c \\ 1 & \text{Otherwise} \end{cases}$$

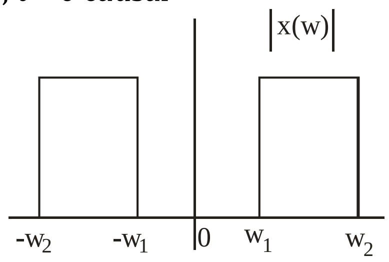


LPF

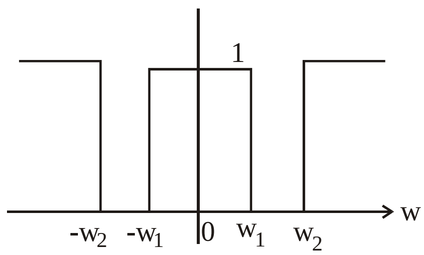


HPF

$h(t) = 0, t < 0$ causal



BPF



BSF

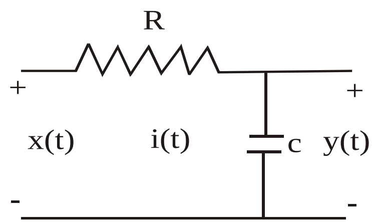
A filter should have a linear phase characteristic for pass band of filter.

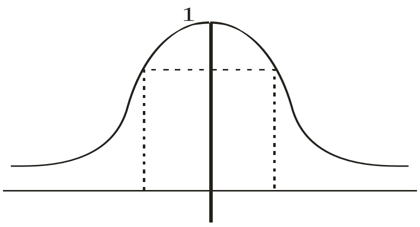
$\angle H(\omega) = -\omega t_d$

All ideal → frequency selective filters are non causal filter. It is not possible to realize physically.

2.3.2 NON IDEAL FREQUENCY SELECTIVE FILTERS

Continuous causal frequency selective filters.

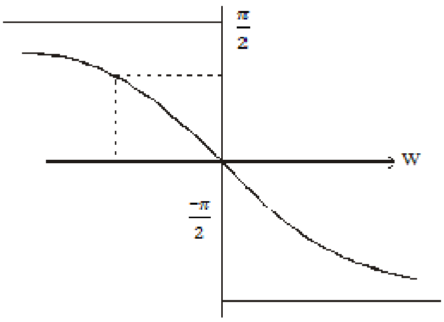




$$RC\frac{dy(t)}{dt} + y(t) = x(t)$$

$$H(\omega) = \frac{y(\omega)}{x(\omega)} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\left(\frac{\omega}{\omega_0}\right)}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)}}, \theta H(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$



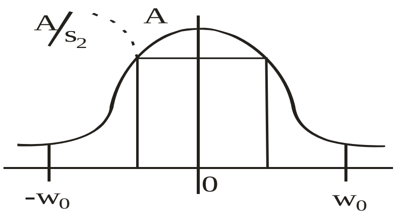
Works as LPF.

1. Absolute Band Width:-

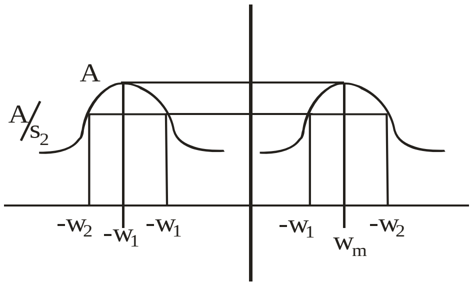
Filter Band Width (BW)
BW = ω_c (LPF)
BW = $\omega_2 - \omega_1$ (BPF)
(HPF), (BSF) × → BW not possible

(1) 3-db (Half Power) Band width:-

Positive frequency at which amplitude spectrum $|H(\omega)|$ drops to a value equal to $H(\omega)/\sqrt{2}\ |H(\omega)|$



$2w_0$ LPF



$BW = w_2 - w_1$

Signal Band W Range of Positive frequency in which 90% of the energy or power lies within signal band.

3-dB Band width:-
Same as filter, for signal

Band Limited Signal:-
A signal x(t) is called a band limited signal if $|x(\omega)| = 0,\ |\omega| > \omega_m$

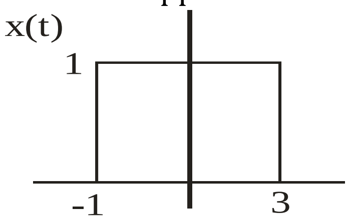
2.4 CONVOLUTION

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)dz = x(t) * \delta(t)$$

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau$$

Steps in Convolution
(1) Obtain limits of y(t). Sum of lower limits $< t <$ sum of upper limits



$2 < t < \infty$, -1 to 3
2 to ∞
2 to ∞

(2) $t \to \tau$
$x(t) \to x(\tau)$
$h(t) \to h(\tau)$

(3) Folding (flipping)

x(-τ) or h(-τ)
Shifting
x(t – τ) | h(t - τ)
(4) Multiplication
(5) Integration

for all values of τ, fixed t, signal y(t) output Repeat 4 & 5 by varying t [-∞ to ∞] the entire output y(t)

2.5 PROPERTIES OF CONVOLUTION

1) Limit of convolution Signals:

$$x(t)*h(t)=\int_{\infty}^{\infty}x(\tau)h(t-\tau)d\tau$$

For general x(t) & h(t)

$$=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau$$

For causal x(t) & general h(t)

$$=\int_{-\infty}^{t}x(\tau)h(t-\tau)d\tau$$

For general x(t) & causal h(t)

$$=\int_{0}^{t}x(\tau)h(t-\tau)d\tau \quad \text{for causal } h(t), x(t)$$

2) Area property:-

Area of the convolved signal is same as the product of the Areas of the signal to be convolved.

$$y(t) = x(t) * h(t)$$

$$\int_{-\infty}^{\infty}y(t)dt=\left(\int_{-\infty}^{\infty}x(t)dt\right)\left(\int_{-\infty}^{\infty}h(t)dt\right)$$

$$A_y = A_x \times A_h$$

3) Scaling property

If y(t) = x(t) * h(t)

$$\frac{1}{|a|}y(at) = x(at) * h(at)$$

y(-t) = x(-t) * h(-t)

4) Cascade connection:

$x(t) \rightarrow h_1(t) \rightarrow h_2(t) \rightarrow h_3(t) \rightarrow h_n(t) \rightarrow y(t)$

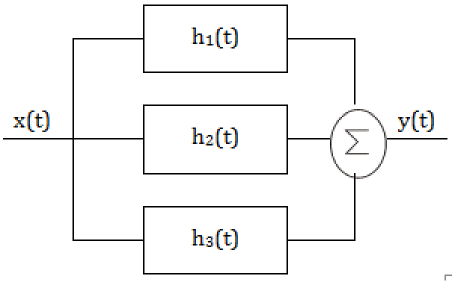
y(t) = x(t) * h(t)

$h(t) = h_1(t) * h_2(t) * h_3(t) * \ldots\ldots\ h_n(t)$

5) Parallel connection:

y(t) = x(t)*h(t)

$h(t) = h_1(t) + h_2(t) + h_3(t)$



6) Shifting Property

x(t – α) * h(t- β) = y(t – α – β)

7) Differential Property:

y(t) = x(t) * h(t) then

y` (t) = x(t) * h` (t)

or

y` (t) = x` (t) * h(t)

In general,

$y^{(m+n)th}(t) = y^{mth}(t) * h^{nth}(t)$

$y^{(m+n)th}(t) = (m+n)^{th}$ derivative of y(t)

$y^{mth}(t) = m^{th}$ derivative of x(t)

$h^{nth}(t) = n^{th}$ derivative of h(t)

2.6 CONVOLUTION OF STANDARD SIGNAL

1) $$x(t)*\delta(t)=\int_{-\infty}^{\infty}x(\tau)\,\delta(t-\tau)d\tau$$

$$=\int_{-\infty}^{\infty}x(\tau)\,\delta[-(t-\tau)]d\tau$$

$$=\int_{-\infty}^{\infty}x(\tau)\,\delta(\tau-t)dz=x(t)$$

2) x(t) * δ(t – T) = x(t – T)

Example:

a) δ(t – 2) * δ (t) = δ (t – τ)

b) δ (t–2)* δ (t+5)= δ (t + 5-2)= δ (t +3)

c) u(t–3)*δ (2t-3) =u (t-3) *δ [2(t–1.5)]

$$= u(t-3) * \frac{1}{|2|}\delta\ (t-3/2)$$

$$=\frac{1}{2}\ u\left(t-\frac{3}{2}-3\right)$$

$= \frac{1}{2}u\left(t-\frac{9}{2}\right)$

d) $e^{-3t}u(t) * \delta(t-1)$

$= e^{-3t}(t-1)\,u(t-1)$

$= e^{-3t}e^{3}\,u(t-1)$

e) $e^{-5t}u(t) * \delta(t) = e^{-5t}u(t)$

$e^{-5t}u(t) * \delta^{1}(t) = \frac{d}{dt}\,[e^{-5t}u(t)]$

$= e^{-5t}u`(t) + (-5)e^{-5t}\,\delta\, u(t)$

$= -5e^{-5t}u`(t) + e^{-5t}(t)$

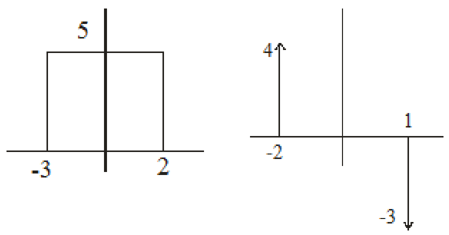
$= -5e^{-5t}u(t) + \delta(t)$

f) $e^{-5t}u(t) \times \delta``(t) = \frac{d^2}{dt^2}\,[e^{-5t}u(t)]$

$= 25e^{-5t}u(t) - 5 + \delta'(t)$

g) $e^{-5t}u(t) \times \delta^{m}(t) = \frac{d^m}{dt^m}\,[e^{-5t}u(t)]$

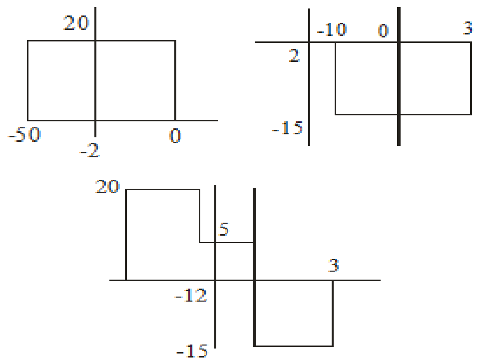
Example:



Solution :

$x(t) = [5u(t+3) - 5u(t-2)]$

$h(t) = [4\,\delta(t+2) - 3\delta(t-1)]$



Example :

If $x(t) = u(t)$, $h(t) = e^{-at}u(t)$,

Find $y(t) = h(t) * x(t) = ?$

Solution :

$$y(t) = \int_{-\infty}^{\infty} u(\tau)e^{-a(t-\tau)}u(t-\tau)d\tau$$

Since both are causal have a

$$y(t) = \int_{-\infty}^{\infty} 1e^{-a(t-\tau)}d\tau$$

$$= e^{-at}\int_0^t e^{-a\tau}d\tau$$

$$= \frac{e^{-at}}{a}\left[e^{-a\tau}\right]_0^t = \frac{e^{-at}}{a}[e^{at}-1]$$

$$= \frac{1}{a}[1-e^{-at}];\, t>0$$

$$y(t) = \frac{1}{a}[1-e^{-at}]u(t)$$

i. Convolution of impulse with other = Impulse response

ii. Convolution of step with other = Step responseiii. Convolution of ramp with other = Ramp
 response

iv. Convolution of two causal signal is causal

v. Convolution of two anti causal signal is anti causal.

Example

$$y(t) = e^{-at}u(t) \times e^{-bt}u(t)$$

$$\downarrow \qquad \downarrow$$

$$x(t) \qquad h(t)$$

Solution

$$\int_{-\infty}^{\infty} e^{-a\tau}u(\tau)e^{-b(t-\tau)}u(t-\tau)d\tau$$

$$= e^{-bt}\int_0^t e^{-a\tau}e^{-a\tau}d\tau = e^{-bt}$$

$$\int_0^t e^{-b\tau-a\tau}d\tau$$

$$= \frac{e^{-bt}}{(b-a)}\left[e^{(b-a)\tau}\right]_0^t$$

$$= \frac{e^{-bt}}{(b-a)}\left[e^{(b-a)t}-1\right]$$

$$= \frac{e^{-at}-e^{-bt}}{(b-a)} \text{for } a \neq 0,\ b \neq 0,\ t>0 \quad \text{if } a=b$$

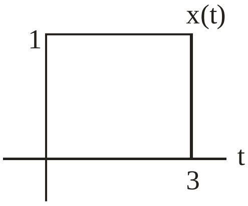
$$e^{-bt}\int_0^t e^{0}d\tau = te^{-bt}u(t);\ a=b$$

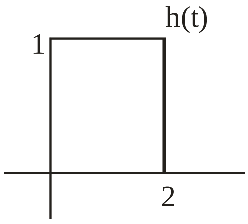
$$e^{-at}u(t)+e^{-bt}u(t) = \frac{e^{-at}-e^{-bt}}{(b-a)}u(t);\ a \neq b$$

$$\boxed{e^{-at}u(t)+e^{-bt}u(t)=\left(\frac{e^{-at}-e^{-bt}}{(b-a)}\right)u(t);\ a\neq b}$$

Example:

Evaluate y(t) = x(t) × h(t)





Solution :

x(t) = u(t) – u(t - 3),

h(t) = u(t) – u(t – 2)

$$y(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau$$

$$=\int_{-\infty}^{\infty}[u(\tau)-u(t-3)][u(t-\tau)-u(t-\tau-2)]d\tau$$

$$=\int_{-\infty}^{\infty}[u(\tau)-u(t-\tau)]d\tau-\int_{-\infty}^{\infty}[u(\tau)-u(t-2-\tau)]d\tau$$

$$-\int_{-\infty}^{\infty}u(\tau-3)u(t-\tau)d\tau+$$

$$=\int_{-\infty}^{\infty}u(\tau-3)u(t-2-\tau)d\tau$$

Since (1) $u(\tau)u(t-\tau)=\begin{cases}1 & 0<\tau<t\\ 0 & \text{Otherwise}\end{cases}$

(1) $=\int_0^t 1d\tau=[\tau]_0^t=tu(t)$

(2) $u(\tau)u(t-2-\tau)=\begin{cases}1 & 0<\tau<t-\tau,t>2\\ 0 & \text{Otherwise}\end{cases}$

$$-\left(\int_0^{t=2}d\tau\right)u(t-2)$$

$$-(t-2)u(t-2)$$

(3) $u(\tau-3)u(t-\tau)=\begin{cases}1 & 3<\tau<t,t>3\\ 0 & \text{Otherwise}\end{cases}$

$$-\left(\int_3^t d\tau\right)u(t-3)$$

$$-(t-3)u(t-3)$$

(4)Since (1)

$u(\tau)u(t-\tau)=\begin{cases}1 & 3<\tau<t-\tau,t>5\\ 0 & \text{Otherwise}\end{cases}$

$u(\tau-3)u(t-2-\tau)(t-5)u(t-5)$

y(t) = tu(t)– (t –2) u(t–2)–(t–3)u(t–3) + (t – 5) u(t – 5)

Discrete Convolution :-

1) Linear Convolution

2) Circular Convolution

x(n) → [h(n)] → y(n) = x(n) * h(n)

x(n) = input sequence, h(n) = impulse sequence & y(n) = output sequence

y(n) = x(n) * h(n)

$$y(n)=\sum_{K=-\infty}^{\infty}x(k)\,h(n-k)=\sum_{K=-\infty}^{\infty}x(n-k)\,h(k)$$

Properties :

1) Commutative

x(n) * h(n) = h(n) * x(n)

$$\sum_{K=\infty}^{k=\infty}x(k)\,h(n-k)=\sum_{K=-\infty}^{\infty}h(k)\,x(n-k)$$

2) Associative :-

$x(n) * [h_1(n) * h_2(n)]$

$= [x(n) * h_1(n)] * h_2(n)$

3) Distributive :-

$x[n]* h_1(n) + x(n) * h_2(n)]$

$= x(n)* h_1(n) + x(n)* h_2(n)]$

4) Width :-

if x(n) * h(n) = y(n)

$x(n-N_1) * x(n-N_2) = y(n-N_1-N_2)$

$L\rightarrow$ length of $x(n)$

$M\rightarrow$ length of $h(n)$ $\boxed{\rightarrow N=L+M-1}$

$N\rightarrow$ length of $y(n)$

$N_L=M_L+L_L$

$=M_U+L_U$

5) Limits :-

$x(n) * h(n) = \sum_{K=-\infty}^{\infty} x(k)\, h(n-k)$ for general

x(n) and h(n)

$= \sum_{K=0}^{\infty} x(k)\, h(n-k)$

$x(n) \rightarrow$ causal

$h(n) \rightarrow$ general

$= \sum_{K=-\infty}^{\infty} x(k)\, h(n-k)$

for $x(n) \rightarrow$ general

$h(n) \rightarrow$ causal

$= \sum_{K=-\infty}^{n} x(k)\, h(n-k)$

for $x(n) \rightarrow$ causal

$h(n) \rightarrow$ causal

Procedure of compute linear convolution of finite time Sequence

Example :

Given that $x(n) = [4,\ -3,\ 2,\ 1]$ (arrow under 2)

$h(n) = [1,\ 2,\ 3,\ 4]$ (arrow under 2)

$y(n) = x(n) * h(n)$

Solution :

$x(n) \rightarrow (-1, 2)$

$h(n) \rightarrow (-2, 1)$

$y(n) \rightarrow (-3, 3)$

$y(n) = \sum_{K=-\infty}^{\infty} x(k)\, h(n-k)$

putting all value of n, one by one, we get result

$y(-3) = \sum_{K=-\infty}^{\infty} x(k)\, h(-3-k) \quad 0 \quad 0$

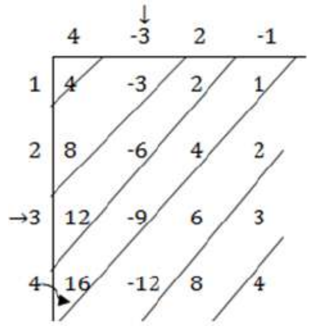
Z Z

$= x(-1)\, h(-2) + x(0)\, h(-3) + x(1)\, h(-4)$

$k=-1 \qquad k=0 \qquad k=1$

y(-3) = 4 × 1 = 4 repeat for all value of n

$y(n) = [4,\ 5,\ 8,\ 12, -4,\ 11,\ 4]$ (arrow under 12)



$y(n) = [4,\ 5,\ 8,\ 12, -4,\ 11,\ 4]$ (arrow under 12)

Example :

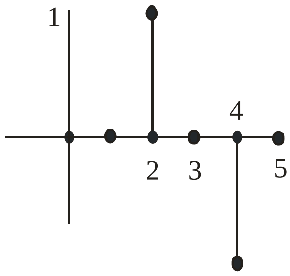
x(n) = [-2, 1, -1, 5, 4] h(n) = [3, -2, 1, 2]

Answer :

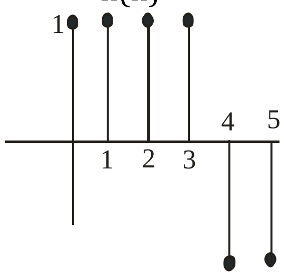
y(n) = [-6, 7, -7, 14, 3, -5, 14, 8]

Example :

x(n) =



h(n) =



Solution :

$y(n) \rightarrow$ [0, 0, 0,1, 1, -2, -2, 1, 1]

GATEFLIX

GATE QUESTIONS(EC)

Q.1 The impulse response function of four linear systems S_1 S_2,S_3 S_4 are given respectively by

$h_1(t) = 1$ $\quad$ $h_2(t) = u(t)$

$h_3(t) = \frac{u(t)}{t+1}$ $\quad$ $h_4(t) = e^{-3t}u(t)$

Where $u(t)$ the unit step function .Which of these system is is time invariant, causal, and stable?

a) S_1 $\quad$ b) S_2

c) S_3 $\quad$ d) S_4

[GATE-2001]

Q.2 Convolution of x(t+5) with impulse function δ(t - 7) is equal to

a) $x(t-12)$ $\quad$ b) $x(t+12)$

c) $x(t-2)$ $\quad$ d) $x(t+2)$

[GATE-2002]

Q.3 The impulse response h[n] of a linear time-invariant system is given by

$h[n] = u[n+3] + u[n-2] - 2u[n-7]$

where u[n] is the unit step sequence. The above system is

a) Stable but not causal

b) Stable and causal

c) Causal but unstable

d) Unstable and not causal

[GATE-2004]

Q.4 The impulse response h[n] of a linear time invariant system is given as

$$h[n] = \begin{cases} -2\sqrt{2} & n=1, \quad -1 \\ 4\sqrt{2} & n=2, \quad -2 \\ 0, & \text{otherwise} \end{cases}$$

If the input to the above system is the sequence $e^{j\pi n/4}$, then the output is

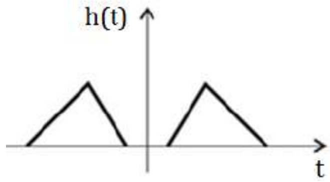
a) $4\sqrt{2}\ e^{j\pi n/4}$ $\quad$ b) $4\sqrt{2}\ e^{j\pi n/4}$

c) $4\ e^{j\pi n/4}$ $\quad$ d) $-4\ e^{j\pi n/4}$

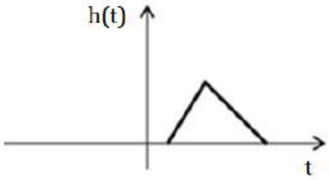
[GATE-2004]

Q.5 Which of the following can be impulse response of a causal system?

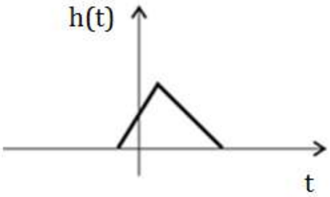
a)



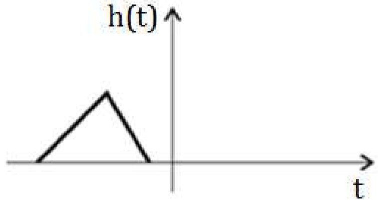
b)



c)



d)



[GATE-2005]

Q.6 The impulse response h(t) of a linear time-invariant continuous time system is described by h(t)=exp(αt)u(t)+exp(βt)u(-t),where u(t) denotes the unit step function and α and β are real constants. This system is stable if

a) α is positive and β is positive

b) α is negative and β is negative

c) α is positive and β is negative

d) α is negative and β is positive

[GATE-2008]

Q.7 A discrete time linear shift-invariant system has an impulse response $h[n]$ with $h[0] = 1, h[1] = -1, h[2] = 2,$ and zero otherwise. The system is given an input sequence $x[n]$ with $x[0] = x[2] = 1$, and zero otherwise. The number of nonzero samples in the output sequence y[n], and the value of $y[2]$ are, respectively

a) 5, 2 b) 6, 2
c) 6, 1 d) 5, 3

[GATE-2008]

Q.8 A system is defined by its impulse response $h(n) = 2^n u(n-2)$. The system is

a) Stable and causal
b) Causal but not stable
c) Stable but not causal
d) Unstable and non-causal

[GATE-2011]

Q.9 Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals

a) 0 b) 1/2
c) 1 d) 3/2

[GATE-2012]

Q.10 Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by

a) product of $h_1(t)$ and $h_2(t)$
b) Sum of $h_1(t)$ and $h_2(t)$
c) Convolution of $h_1(t)$ and $h_2(t)$
d) Subtraction of $h_2(t)$ from $h_1(t)$

[GATE-2013]

Q.11 Which one of the following statements is not true for a continuous time casual and stable LTI system?

a) All the poles of the system must lie on the left side of iω axis.

b) Zeroes of the system can lie anywhere in the s-plane.

c) All the poles must lie within |s|=1

d) All the roots of the characteristics equation must be located on the left side of the iω axis.

[GATE-2013]

Q.12 A continuous, linear time-invariant filter has an impulse response h(t) described by

$$h(t) = \begin{cases} 3 & \text{for } 0 \le t \le 3 \\ 0 & \text{otherwise} \end{cases}$$

When as constant input of value 5 is applied to this filter, the steady state output is..........

[GATE-2011, Set 1]

Q.13 The sequence $x[n] = 0.5^n u[n]$ is the unit step sequence, is convolved with itself to obtain y[n]. Then

$\sum_{n=-\infty[n]}^{\infty} y[n]$ is................

[GATE-2014, Set 4]

Q.14 The input $-3e^{-2t}u(t)$, where u(t) is the unit step function, is applied to a system with transfer function $\frac{s-2}{s+3}$.
If the initial value of the output is -2. Then the value of the output at steady state is..................

[GATE-2014, Set 3]

Q.15 The result of the convolution

$x(-t)*\delta(-t-t_0)$ is

a) $x(t+t_0)$ b) $x(t-t_0)$
c) $x(-t+t_0)$ d) $x(-t-t_0)$

[GATE-2015, Set 1]

Q.16 The impulse response o an LTI system can be obtained by
a) differentiating the unit ramp response
b) differentiating the unit step response
c) integrating the unit ramp response
d) integrating the unit step response

[GATE-2015, Set 3]

Q.17 Which one of the following is an eigen function of the class of all continuous-time, linear, time-invariant systems? {u(t) denotes the unit-step function).
a) $e^{i\omega_0 t}u(t)$ b) $\cos(\omega_0 t)$
c) $e^{i\omega_0 t}$ d) $\sin(\omega_0 t)$

[GATE-2016, Set 1]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10
d	c	a	d	b	d	d	b	a	c
11	12	13	14	15	16	17			
C	45	4	0	D	B	c			

EXPLANATIONS

Q.1 **(d)**

$h_1(t) \neq 0 \ for \ t < 0$

Therefore S_1 is non causal

$h_2(t) = u(t)$

$\int_{-\infty}^{\infty} h_2(t)dt = \int_{-\infty}^{\infty} u(t)dt =$

$\int_{0}^{\infty} dt = \infty$

Therefore S_2 is unstable

$h_3(t) = \frac{u(t)}{t+1}$

At t=-1

$h_3(t) = \infty$

Therefore S_3 is unstable

$h_4(t) = e^{-3t}u(t)$

S_4 is time invariant, causal and stable.

Q.2 **(c)**

$x(t+5)^*\delta(t-7) = x(t+5-7)$

$= x(t-2)$

Q.3 **(a)**

$\sum_{k=-\infty}^{\infty} h(k) =$

$\sum_{k=-3}^{\infty} u(k+3) + \sum_{k=2}^{\infty} u(k-2)$

$-2\sum_{k=7}^{\infty} u(k-7)$

$= \sum_{k=-3}^{6} 1 + \sum_{k=2}^{6} 1$

$= 10 + 5 = 15 < \infty$

For bounded input , output .So system is stable.

Q.4 **(d)**

$h(n) = 4\sqrt{2}\delta(n+2) - 2\sqrt{2}\delta(n+1)$

$-2\sqrt{2}\delta(n-1) + 4\sqrt{2}\delta(n-2)$

$x(n) = e^{j\pi n/4}$

$y(n) = x(n) * h(n)$

$= 4\sqrt{2}\delta(n+2)^* e^{\delta n\pi/4}$

$+4\sqrt{2}\delta(n-2)^* e^{jn\pi/4}$

$-2\sqrt{2}[\delta(n+1)^* e^{jn\pi/4}$

$+\delta(n-1)e^{jn\pi/4}]$

$x(n)^*\delta(n-2) \rightarrow x(n-a)$

$y(n) = 4\sqrt{2}[e^{j\pi/4(n+2)} + e^{j\pi/4(n-2)}]$

$-2\sqrt{2}[e^{j\pi/4(n+1)} + e^{j\pi/4(n-1)}]$

$= e^{jn\pi/4}4\sqrt{2}[e^{j\pi/2} + e^{-j\pi/2}]$

$-2\sqrt{2}(e^{j\pi/4} + e^{-j\pi/4})$

$= e^{jn\pi/4}[0 - 2\sqrt{2} \times 2\cos\pi/4]$

$y(n) = -4e^{jn\pi/4}$

Q.5 **(b)**

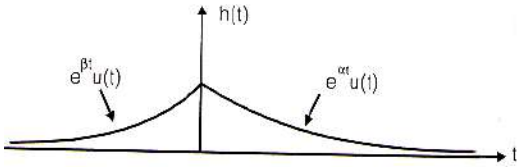
For a causal system

$h(t)0 \ for \ t < 0$

Q.6 **(d)**

$h(t) = e^{\alpha t}u(t) + e^{\beta t}u(-t)$

For the system to be stable, $\int_{-\infty}^{\infty} h(t)dt < \infty$ For the above condition, h(t) should be as shown below.



Therefore $\alpha < 0 \ \& \ \beta > 0$

Q.7 **(d)**

$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$n < 0, y[n] = 0$

$n = 0, y[n] = 1$

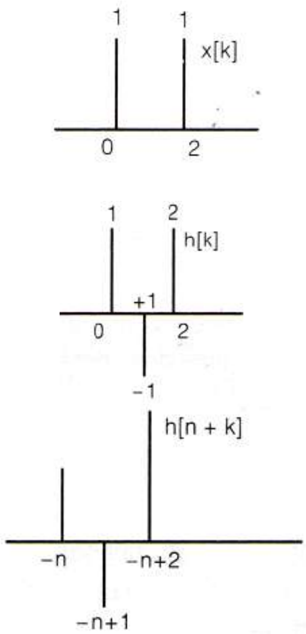
$n = 1, y[n] = -1$

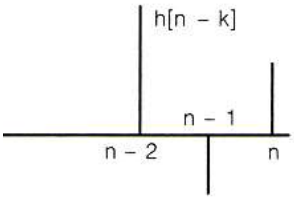
$n = 2, y[n] = 3$

$n = 3, y[n] = -1$

$n = 4, y[n] = 2$

$n = 5, y[n] = 0$





Q.8 (b)

$h(n) = 2^n u(n-2)$

For causal system

$h(n) = 0 \text{ for } n < 0$

Hence given system is causal

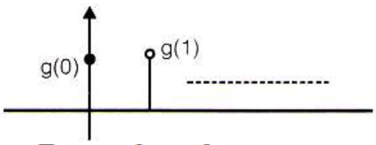
For stability:

$\sum_{n=2}^{\infty} 2^n = \infty$, so given system is not stable.

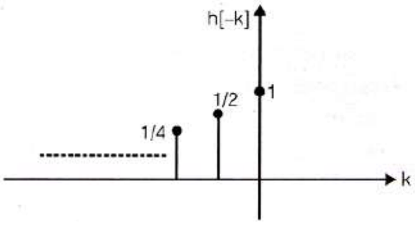
Q.9 (a)

$h[n] = \left(\frac{1}{2}\right)^n u[n]$

$g[n] = ??$



$y[n] \sum_{k=-\infty}^{\infty} h[n-k] g[k]$



$y[0] \sum_{k=-\infty}^{\infty} h[-k] g[k]$,

$y[0] = h[0] g[0]$

$1 = 1\, g[0]$

$g[0] = 1$

$y[1] \sum_{k=-\infty}^{\infty} h[1-k] g[k]$

$y[1] = h[1]g[0] + h[0]g[1]$

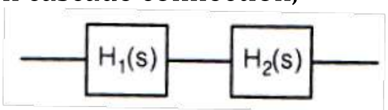
$h[1-k]$ will be zero for $k > 1$ and $g[k]$ will be zero for $k < 0$ as it is causal sequence.

$\frac{1}{2} = \frac{1}{2} \times 1 + 1 g[1]$

$g[1] = 0$

Q.10 (c)

In cascade connection,



$H(s) = H_1(s).\ H_2(s)$

$\Rightarrow h(s) = h_1(t) * h_2(f)$

Q.11 (c)

Q.12 45

$$h(t) = \begin{cases} 3 & \text{for } 0 \le t \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$$h(t) = 3[u(t) - u(t-3)]$$

$$H(s) = 3\left[\frac{1}{s} - \frac{e^{-3s}}{s}\right]$$

$$x(t) = 5$$

$$X(s) = \frac{5}{s}$$

Now,

$$Y(s) = H(s)X(s)$$

$$Y(s) = 3\left[\frac{1}{s} - \frac{e^{-3s}}{s}\right]\frac{5}{s}$$

Steady state output

$$= \lim_{s \to 0} s.Y(s)$$

$$= \lim_{s \to 0} s \frac{15}{s} \frac{(1 - e^{-3s})}{s}$$

$$= 45$$

Q.13 4

Q.14 0

$$x(t) = -3e^{-2t}$$

Laplace transform

$$x(t) \leftrightarrow X(s)$$

$$X(s) = \frac{-3}{s+2}$$

$$H(s) = \frac{s-2}{s+3}$$

$$Y(s) = X(s)H(s)$$

$$Y(s) = \frac{-3(s-2)}{(s+3)(s+2)}$$

Using final value theorem, the steady state value of Y(s).

$$y(\infty) = \lim_{s \to 0} sY(s)$$

$$= 0$$

Q.15 **(d)**

$$x(-t)*\delta(-t-t_0) = x(-t)*\delta(t+t_0)$$
$$= x(-t-t_0)$$

Q.16 **(b)**

$$IR = \frac{d}{dt}(SR)$$

Q.17 **(c)**

If the input to the system is eigen signal output also the same eigen signal

GATE QUESTIONS(EE)

Q.1 Given the relationship between the input u(t) and the output y(t) to be
$y(t) = \int_0^t (2+t-\tau)\, e^{-3(t-\tau)} u(\tau) d\tau$
The transfer function Y(s)/U(s) is
a) $\frac{2e^{-2s}}{s+3}$
b) $\frac{s+2}{(s+3)^2}$
c) $\frac{2s+5}{s+3}$
d) $\frac{2s+7}{(s+3)^2}$
[GATE-2001]

Q.2 Let s(t) be the step response of a linear system with zero initial conditions. Then the response of this system to an input u(t) is
a) $\int_0^t s(t-\tau) u(\tau) d\tau$
b) $\frac{d}{dt}\left[\int_0^t s(t-\tau) u(\tau) d\tau\right]$
c) $\int_0^t s(t-\tau)\left[\int_0^t u(\tau_1) d\tau_1\right] d\tau$
d) $\int_0^t s(t-\tau)^2 u(\tau) d\tau$
[GATE-2002]

Q.3 The Fourier series for the function $f(x) = \sin^2 x$ is
a) $\sin x + \sin 2x$
b) $1 - \cos 2x$
c) $\sin 2x + \cos 2x$ (d)
d) $0.5 - 0.5\cos 2x$
[GATE-2005]

Q.4 $x[n] = 0; n < -1, n > 0, x[-1] = -1, x[0] = 2$ is the input and
$y[n] = 0; n < -1, n > 2,\ y[-1] = -1,$
$y[1], y[0] = 3, y[2] = -2$
is the output of a discrete time LTI system. The system impulse response h[n] will be
a) $h[n] = 0; n < 0, n > 2,$
$h[0] = 1, h[1] = h[2] = -1$
b) $h[n] = 0; n < -1, n > 1,$
$h[-1] = 1, h[0] = h[1] = 2$
c) $h[n] = 0; n < 0, n > 3,$
$h[0] = -1, h[1] = 2, h[2] = 1$
d) $h[n] = 0; n < -2, n > 1, h[-2]$
$= h[1] = h[-1] = -h[0] = 3$
[GATE-2006]

Q.5 Let a signal $a_1 \sin(\omega_1 t + \phi_1)$ be applied to a stable linear time invariant system. Let the corresponding steady state output be represented as $a_2 F(\omega_2 t + \phi_2)$. Then which of the following statements is true?
a) F is not necessarily a "sine" or "cosine" function but must be periodic with $\omega_1 = \omega_2$
b) F must be a "sine" or "cosine" function with $a_1 = a_2$
c) F must be a "sine" function with $\omega_1 = \omega_2$ and $\phi_1 = \phi_2$
d) F must be a "sine" or "cosine" function with $\omega_1 = \omega_2$
[GATE-2007]

Q.6 If $u(t), r(t)$ denote the unit step and unit ramp functions respectively and $u(t) * r(t)$ their convolution, then the function $u(t+1) * r(t-2)$ is given by
a) $(1/2)(t-1)(t-2)$
b) (1/2)(t-1)(t-2)
c) $(1/2)(t-1)^2 u(t-1)$
d) none of the above
[GATE-2007]

Q.7 A signal $e^{-\alpha t}\ \text{sinc}(\omega t)$ is the input to a real Linear Time Invariant system. Given K and ϕ are constants, the output of the system will be of the from $K e^{-\beta t}\ \text{sinc}(vt + \phi)$ where
a) β need not be equal to α but v equal to ω

b) V need not be equal to ω but β equal to α
c) β equal to α and v equal to ω
d) β need not be equal to α and v need not be equal to ω

[GATE-2008]

Q.8 The impulse response of a causal linear time-invariant system is given as h(t). Now consider the following two statements:
Statements (I): principle of superposition holds
Statement (II): $h(t) = 0$ for $t < 0$.
Which one of the following statements is correct?
a) Statements (I) is correct and statement (II) is wrong
b) Statements (II) is correct and statement (I) is wrong
c) Both statements (I) and statement (II) are wrong
d) Both statements (I) and statement (II) are correct

[GATE-2008]

Q.9 A signal $x(t) = \text{sinc}(\alpha t)$ where α is a real constant $\left(\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}\right)$ is the input to a Linear tie invariant system whose impulse response $h(t) = \text{sinc}(\beta t)$. where β is a real constant. If min (α, β) denotes the minimum of α and β, similarly max (α, β) denotes the maximum of α and β and K is a constant, which one of the following statements is true about the output of the system?
a) It will be of the form $K\,\text{sinc}(\gamma t)$ where $\gamma = \min(\alpha, \beta)$
b) It will be of the $k\,\text{sinc}(\gamma t)$ where $\gamma = \max(\alpha, \beta)$
c) It will be of the form $k\,\text{sinc}(\alpha t)$
d) It cannot be a sinc type of signal

[GATE-2008]

Q.10 A system with x(t) and output y(t) is defined by the input-output relation:
$y(t) = \int_{-\infty}^{-2t} x(t)\,d\tau$
The system will be
a) Causal, time-invariant and unstable
b) Causal, time-invariant and stable
c) Non-causal, time-invariant and unstable
d) Non-causal, time-variant and unstable

[GATE-2008]

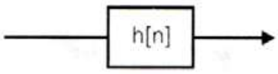
Q.11 A cascade of 3 Linear Time Invariant systems is causal and unstable. From this, we conclude that
a) Each system in the cascade is individually causal and unstable
b) At least one system is unstable and at least one system is causal
c) At least one system is causal and all systems are unstable
d) The majority are unstable and the majority are causal

[GATE-2009]

Q.12 A Linear Time Invariant system with an impulse response h(t) produces output y(t) when input x(t) is applied. When the input $x(t-\tau)$ is applied to a system with impulse response $h(t-\tau)$, the output will be
a) $y(\tau)$
b) $y(2(t-\tau))$
c) $y(t-\tau)$
d) $y(t-2\tau)$

[GATE-2009]

Q.13 Given the finite length input x[n] and the corresponding finite length output y[n] of an LTI system as shown below, the impulse response h[n] of the system is



a) $h[n] = \{1, 0, 0, 1\}$
b) $h[n] = \{1, 0, 1\}$
c) $h[n] = \{1, 1, 1, 1\}$
d) $h[n] = \{1, 1, 1\}$

[GATE-2010]

Q.14 The system represented by the input-output relationship
$y(t) = \int_{-\infty}^{5t} x(\tau)\,d\tau, t > 0$ *is*
a) Linear and causal

b) Linear but not causal
c) Causal but not linear
d) Neither linear nor causal
[GATE-2010]

Q.15 Given two continuous time signals $x(t) = e^{-t}$ and $y(t) = e^{-2t}$ which exist for $t > 0$, the convolution $z(t) = x(t) * y(t)$ is
a) $e^{-t} - e^{-2t}$
b) e^{-3t}
c) e^{+t}
d) $e^{-t} + e^{-2t}$
[GATE-2011]

Q.16 Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals
a) 0
b) 1/2
c) 1
d) 3/2
[GATE-2012]

Q.17 The input x(t) and output y(t) of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$. The system is
a) time-invariant and stable
b) stable and not time-invariant
c) time-invariant and not stable
d) not time-invariant and not stable
[GATE-2012]

Q.18 Consider the differential equation
$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t) \text{ with}$$
$y(t)|_{t=0^-} = -2$ and $\left.\frac{dy}{dt}\right|_{t=0^-} = 0$.
The numerical value of $\left.\frac{dy}{dt}\right|_{t=0^+}$ is
a) -2
b) -1
c) 0
d) 1
[GATE-2012]

Q.19 Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by
a) product of $h_1(t)$ and $h_2(t)$
b) sum of $h_1(t)$ and $h_2(t)$
c) convolution of $h_1(t)$ and $h_2(t)$
d) subtraction of $h_2(t)$ from $h_1(t)$
[GATE-2013]

Q.20 The impulse response of a system is $h(t) = tu(t)$. For input $u(t-1)$, the output is.
a) $\frac{t^2}{2}u(t)$
b) $\frac{t(t-1)}{2}u(t-1)$
c) $\frac{(t-1)^2}{2}u(t-1)$
d) $\frac{t^2-1}{2}u(t-1)$
[GATE-2013]

Q.21 The impulse response of a continuous time system is given by $h(t) = \delta(t-1) + \delta(t-3)$. The value of the step response at $t = 2$ is
a) 0
b) 1
c) 2
d) 3
[GATE-2013]

Q.22 x(T) is non-zero only for $T_x < t < T'_x$, and similarly, y(t) is non-zero only for $T_y < t < T'_y$. Let z(t) be convolution of x(t) and y(t). Which one of the following statements is TRUE?

a) Z(t) can be non-zero over an unbounded interval
b) Z(t) is non-zero for $t < T_x + T'_y$
c) Z(t) is zero outside of $T_x + T_y < T'_x + T'_y$
d) Z(t) is non-zero for $t > T'_x > T'_y$
[GATE-2014]

Q.23 For a periodic square wave, which one of the following statement is TRUE?

a) The Fourier series coefficients do not exist
b) The Fourier series coefficient exist but the reconstruction converges at most point.
c) The Fourier series coefficient exist but the reconstruction converges at no point.
d) The Fourier series coefficient exist but the reconstruction converges at every point.

[GATE-2014]

Q.24 Consider an LTI system with impulse response $h(t)=e^{-5t}\ u(t)$. If the output of the system is $y(t) = e^{-3t}u\ (t) - e^{-5t}\ u(t)$ then the input, x(t) is given by

a) $e^{-3t}u\ (t)$ b) $2e^{-3t}u\ (t)$
c) $e^{-5t}u\ (t)$ d) $2e^{-5t}u\ (t)$

[GATE-2014]

Q.25 For the signal $f(t)=3\sin 8\pi t + 6\sin 12\pi t + \sin 14\pi t$, the minimum sampling frequency (in Hz) satisfying the Nyquist criterion is___________

[GATE-2014]

Q.26 A moving average function is given by $y\ (t) = \frac{1}{T}\int_{t-T}^{t} u(\tau)d\tau$. If the input u is a sinusoidal signal of frequency $\frac{1}{2T}$ Hz, then in steady state, the output y will lag u (in degree) by__________.

[GATE-2015]

Q.27 For a linear time invariant systems that are Bounded Input Bounded Output stable, which one of the following statement is TRUE?

a) The impulse response will be integrable, but may not be absolutely integrable.
b) The unit impulse response will have finite support.
c) The unit step response will be absolutely integrable.
d) The unit response will be bounded.

[GATE-2015]

Q.28 Consider continuous-time system with input x(t) and output y(t) given by

$$Y(t)=x(t)\cos(t)$$

This system is

a) linear and time-invariant
b) non-linear and time-invariant
c) linear and time varying
d) non-linear and time varying

[GATE-2016]

Q.29 Let z(t)=x(t)*y(t), where " * " denotes convolution. Let c be a positive real-valued constant. Choose the correct expression for z(ct).

a) C x(ct) * y(ct)
b) b) x(ct) * y(ct)
c) C x(t) * y(ct)
d)C x(ct) * y(t)

[GATE-2017]

Q.30 A continuous time input signal $x(t)$ is an eigen function of an LTI system, if the output is

a) $kx(t)$,where k is an eigen value

b) $ke^{j\omega t}x(t)$,where k is an eigen value and $e^{j\omega t}$ is a complex exponential signal.

c) $x(t)e^{j\omega t}$ where $e^{j\omega t}$ is a complex exponential signal

d) $kH(\omega)$ where k is an eigen value and $H(\omega)$ is a frequency response of the system.

[GATE-2018]

Q.31 The signal energy of the continuous time signal

$$x(t)=[(t-1)u(t-1)]$$
$$-[(t-2)u(t-2)]$$
$$-[(t-3)u(t-3)]$$
$$+[(t-4)u(t-4)]$$

a) $\frac{11}{3}$ b) $\frac{7}{3}$

c) $\frac{1}{3}$ d) $\frac{5}{3}$

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(d)	(b)	(d)	(a)	(d)	(c)	(a)	(d)	(a)	(d)	(b)	(d)	(c)	(b)
15	16	17	18	19	20	21	22	23	24	25	26	27	28
(a)	(a)	(d)	(d)	(c)	(c)	(b)	(c)	(b)	(b)	(d)	90^0	(d)	(d)
29	30	31											
(a)	(a)	(d)											

EXPLANATIONS

Q.1 **(d)**

$y(t) = \int_0^t (2 + t - \tau)\, e^{-3(t-\tau)} u(\tau) d\tau$

$y(t) = [(2 + t)e^{-3t}] * u(\tau)$

$\therefore H(s) = \frac{2}{s+3} + \frac{1}{(s+3)^2}$

$\therefore T.F. = H(s) = \frac{2s+7}{(s+3)^2}$

Q.2 **(b)**

Q.3 **(d)**

$f(n) = \sin^2 x$

for finding the fourier series expansion

$f(x) = \sin^2 x = \frac{1-\cos 2x}{2}$

$= 0.5 - 0.5 \cos 2x$

Q.4 **(a)**

For infinite duration convolution

X[n] have M terms

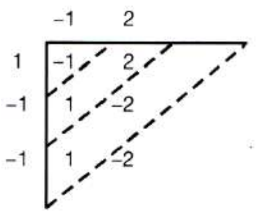
h[n] have N terms

they, y[n] should have terms (M+N-1)Here, $x[n] = \{-1, 2\}$ ↑

$y[n] = \{-1, 3, -1, -2\}$ ↑

So, h[n] should have only 3 terms and h[n] have value starting from origin or $[n = 0]$ only because y[n] have value start from $[n = -1]$

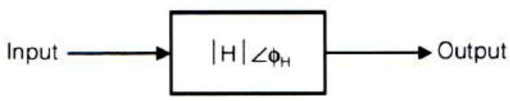
∴ Consider option (I)



$y[n] = \{-1, 3, -1, -2\}$

So option (a) is correct.

Q.5 (d)



The effect of linear time invariant system is best understood by considering that, at any given input frequency ω_1 the output is sinusoidal at same frequency, as the input applied, with amplitude given by output $= |H| \times$ input and phase equal to $\phi_2 = \phi_H + \phi_1$, where $|H|$ is the magnitude of the frequency response and ϕ_H is its phase angle. Both $|H|$ & ϕ_H are functions of frequency.

Q.6 **(c)**

$u(t + 1) * r(t - 2)$

$= \int_{-\infty}^{\infty} r(\tau - 2) u(t + 1 - \tau) d\tau$

$z(t) = \int_{-\infty}^{(t+1)} r(\tau - 2) d\tau$

$z(t) = \int_{2}^{(t+1)} r(\tau - 2) d\tau$

for $(t + 1) \geq 2$

$z(t) = \left(\frac{\tau^2}{2} - 2\tau\right)_2^{(t+1)}$

$= \frac{1}{2}[-4\tau]_2^{(t+1)}$

for $t \geq 1$

$= \frac{1}{2}[(t + 1)^2 - 4(t + 1) - 4 + 8]$

$z(t) = \frac{1}{2}[t^2 + 2t + 1 - 4t - 4 + 4]$

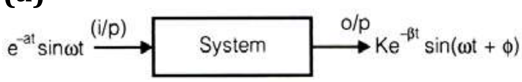
for $t \geq 1$

$z(t) = \frac{1}{2}(t - 1)^2$

for $t \geq 1$

$\Rightarrow \quad z(t) = \frac{1}{2}(t - 1)^2 u(t - 1)$

Q.7 **(a)**



Frequency (ω) to (output) must be equal to input frequency (ω) while 'β' will depend on system parameters and need not be equal to α.

Q.8 **(d)**

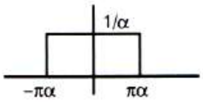
Since system is causal therefore its impulse response will be zero for $t < 0$.

So statement –II is correct.

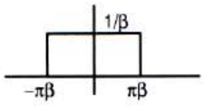
$\Rightarrow$ since system is linear it also satisfies the principle of superposition.
So both the statement are correct.

Q.9 (a)

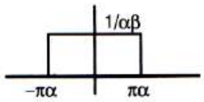
$x(t) = \text{sinc}(\alpha t) = s_a[\pi\alpha t]$
$h(t) = \text{sinc}(\beta t) = s_a[\pi\beta t]$
$x(t) = s_a[\pi\alpha t]$

$\therefore$ $X(j\omega) =$


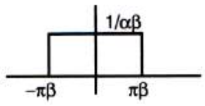
$h(t) = s_a[\pi\beta t]$

$\Rightarrow$ $H(j\omega) =$


$y(t) = h(t) * x(t)$
$\Rightarrow$ $Y(j\omega) = X(j\omega)\,H(j\omega)$

$Y(j\omega) =$


if $\alpha < \beta$

or $Y(j\omega) =$


if $\beta < \alpha$

So $y(t) = \frac{1}{\beta} s_a(\pi\alpha t)$ if $\alpha < \beta$

$y(t) = \frac{1}{\beta}\text{sinc}(\alpha t)$ if $\alpha < \beta$

or $y(t) = \frac{1}{\alpha}\text{sinc}(\beta t)$ if $\beta < \alpha$

so output is of the form $k\,\text{sin c}(\gamma t)$
where $\gamma = \min(\alpha, \beta)$

Q.10 (d)

First of all we will check for causality
$y(t) = \int_{-\infty}^{-2t} x(t)\,d\tau$
so for $t = -2$
$y(-2) = \int_{-\infty}^{4} x(\tau)\,d\tau$

so output depends on future values of input along with past and present values of input so system is non-causal.
Let us find output for shifted input $x(t - t_0)$
$y'(t) = \int_{-\infty}^{-2t} x(\tau - t_0)\,d\tau$

$y'(t) = \int_{-\infty}^{(2t-t_0)} x(\tau)\,d\tau \ldots(i)$

Now shift the output by t_0 then
$y(t - t_0) = \int_{-\infty}^{(2t-t_0)} x(\tau)\,d\tau \ldots(ii)$
so from eq. (i) and (ii)
$y'(t) \neq y(t - t_0)$
Therefore system is time variant.
Non-causal and time variant is present only in option (d).

Q.11 (b)

Since is cascade overall impulse response $h(t) = h_1(t) * h_2(t) * h_3(t)$
$h_1(t), h_2(t), h_3(t)$ are impulse responses of individual systems.
Since initial point where h(t) is nonzero is $t \geq 0$ and since in convolution initial point.
$= t_1 + t_2 + t_3$
where, t_1, t_2, t_3 are initial points of $h_1(t), h_2(t), h_3(t)$ respectively.
So, for it to be greater than or equal to zero at least one of them t_1, t_2, t_3 must be +ve i.e. greater than zero so atleast one of them must be causal.
Similarly if one (atleast) of the system become unstable then overall system will become unstable.

Q.12 (d)

Case 1 : $Y(s) = H(s).X(s)$
Case 2 : input
$x(t - \tau) \Rightarrow X(s).e^{-s\tau}$
Impulse response
$h(t - \tau) \Rightarrow H(s).e^{-s\tau}$
Output $Y(s) = X(s).e^{-s\tau}.H(s)e^{-s\tau}$
$= X(s).H(s).e^{-s2\tau}$
$\Rightarrow$ $y(t - 2\tau)$

Q.13 (c)

$x[n] = \{1, -1\},\ M = 2$
$\uparrow$
$y[n] = \{1, 0, 0, 0, -1\},$
$\uparrow$
$N_1 = 5$
Since output has $\rightarrow$ no. of elements
$N_1 = M + N - 1$
Where N is number of elements in impulse response h[n]

$\therefore 5 = 2 + N - 1$

$N = 4$

Let it be a_1, a_2, b_1, b_2

$y[n] = [a_1, (a_2 - a_1),$
$(b_1 - a_2), (b_2 - b_1), -b_2]$
$\uparrow$

Comparing with

$y[n] = [1, 0, 0, 0, -1]$
$\uparrow$

$a_1 = 1$

$a_2 = a_1 = 1$

$b_1 = a_1 = 1$

$b_2 = b_1 = 1$

Therefore

$h[n] = \{1, 1, 1, 1\}$
$\uparrow$

Q.14 (b)

Integrator is always a linear system.

Since

$y(t) = \int_{-\infty}^{5t} x(\tau)d\tau \quad t > 0$

for $t = 1$

$y(t) = \int_{-\infty}^{5t} x(\tau)d\tau$

here value at t = 1 depends on future values like at t = 2, 3, ……. Of input x(t).

So it is noncausal system.

Q.15 (a)

$x(s) = \frac{1}{s+1} \quad y(s) = \frac{1}{s+2}$

$z(s) = x(s).\ y(s)$

$= \frac{1}{s+1} \times \frac{1}{s+2}$

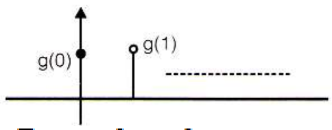
$= \frac{1}{s+1} - \frac{1}{s+2}$

$z(t) = e^{-t} - e^{-2t}$

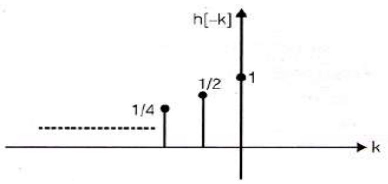
Q.16 (a)

$h[n] = \left(\frac{1}{2}\right)^n u[n]$

$g[n] = ??$



$y[n] \sum_{k=-\infty}^{\infty} h[n-k] g[k]$



$y[0] \sum_{k=-\infty}^{\infty} h[-k] g[k],$

$y[0] = h[0] g[0]$

$1 = 1\ g[0]$

$g[0] = 1$

$y[1] \sum_{k=-\infty}^{\infty} h[1-k] g[k]$

$y[1] = h[1] g[0] + h[0] g[1]$

$h[1-k]$ will be zero for $k > 1$ and $g[k]$ will be zero for $k < 0$ as it is causal sequence.

$\frac{1}{2} = \frac{1}{2} \times 1 + 1 g[1]$

$g[1] = 0$

Q.17 (d)

$y = \int_{-\infty}^{t} x(\tau) \cos(3\tau)\, d\tau$

$y(t - t_0) = \int_{-\infty}^{t-t_0} x(\tau) \cos(3\tau)\, d\tau$

$y'(t)$ for input $x(t - t_0)$ is

$y'(t) = \int_{-\infty}^{t} x(\tau - t_0) \cos 3\tau\, d\tau$

$y'(t) = \int_{-\infty}^{(t-t_0)} x(\tau) \cos 3(\tau + t_0) d\tau$

$y'(t) \neq y(t - t_0)$ so system is not time invariant for input $x(\tau) = \cos(3\tau)$ bounded input

$y'(t) = \int_{-\infty}^{t} \cos^2(3\tau)\, d\tau$

$\rightarrow \infty$ as $t \rightarrow \infty$

So for bounded input, output is not bounded therefore system is not stable.

Q.18 (d)

$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$

$\Rightarrow s^2 Y(s) - sy(0)$

$= y'(0) + 2[sY(s) - y(0)] + Y(s) = 1$

$\Rightarrow Y(s)[s^2 + 2s + 1] - (s \times -2)$

$- y'(0) - (2 \times -2) = 1$

$\Rightarrow Y(s)[s^2 + 2s + 1] = -2s - 3$

$\Rightarrow \quad Y(s) = \frac{-2s-3}{s^2+2s+1}$

$= \frac{-2}{s+1} - \frac{1}{(s+1)^2}$

$\Rightarrow Y(t) = -2e^{-t} - te^{-t}$

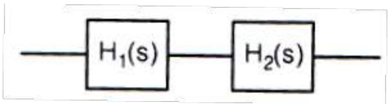
$\frac{dy(t)}{dt} = -2e^{-t} - (-t\, e^{-t} + e^{-t})$

$= e^{-t} + e^{-t}$

$\left.\frac{dy(t)}{dt}\right|_{t=0^+} = 1 + 0 = 1$

Q.19 (c)

In cascade connection,



$H(s) = H_1(s).\ H_2(s)$

$\Rightarrow h(s) = h_1(t) * h_2(f)$

Q.20 (c)

$h(t) = tu(t)$

$H(s) = \frac{1}{s^2}$

$\Rightarrow \frac{Y(s)}{U(s)} = \frac{1}{s^2}$

$Y(s) = \frac{1}{s^2}\frac{e^{-s}}{s}$

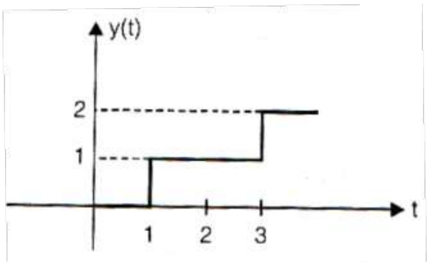
$\Rightarrow y(t) = \frac{(t-1)^2}{2}u(t-1)$

Q.21 (b)

Step response =Integration of impulse response

$\int \delta(t-1) = u(t-1)$

$\int \delta(t-3) = u(t-3)$



At $t = 2$

$y(t) = 1$

Q.22 (C)

x(t) is non-zero for $T_x < t < T'_x$

and y(t) is non-zero for $T_y < t < T'_y$

$\because z(t) = x(t) \otimes y(t)$

Then the limits of the resultant signal is

$T_x + T_y < t < T'_x + T'_y$

Q.23 (B)

Q.24 (B)

Impulse response of LTI system

= transfer function

$= \frac{Y(s)}{X(s)} = H(s)$

Where, $y(t) = e^{-3t}u(t) - e^{-5t}u(t)$

$\therefore Y(s) = \frac{1}{s+3} - \frac{1}{s+5}$

$= \frac{2}{(s+3)(s+5)}$

Also, $h(t) = e^{-5t}$

$\therefore H(s) = \frac{1}{(s+5)}$

Therefore, $X(s) = \frac{Y(s)}{H(s)}$

$= \frac{2}{(s+3)(s+5)} \times (s+5)$

$= \frac{2}{(s+3)}$

$\therefore$ Input $= x(t) = 2e^{-3t}u(t)$

Q.25 14

F_{m1}=4Hz

F_{m2} = 6 Hz

F_{m3} = 7 Hz

Then minimum sampling frequency satisfying the nyquist criterion is = 14 Hz

Q.26 90^0

Q.27 (D)

Q.28 (C)

Q.29 (A)

Time scaling property of convolution.

If, $x(t) * y(t) = z(t)$

then, $x(ct) * y(ct) = \frac{1}{c}z(ct)$

$\Rightarrow z(ct) = c \times x(ct) * y(ct)$

Q.30 (a)

If the output signal is a scalar multiple of input signal, the signal is refereed as an eigen function (or characteristic function) and the multiplier is referred as an eigen value (or characteristic value).

If $x(t)$ is the eigen function and λ is the eigen value, then output $y(t)=\lambda x(t)$.

Hence, the correct option is (A).

Q.31 (d)

We know that,

$r(t)=tu(t)$

Put $t=t-1 \Rightarrow r(t-1)=(t-1)u(t-1)$

Put $t=t-2 \Rightarrow r(t-2)=(t-2)u(t-2)$

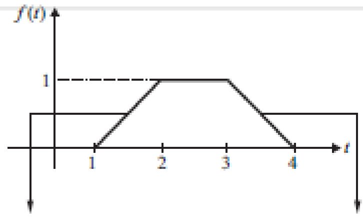
Put $t=t-3 \Rightarrow r(t-3)=(t-3)u(t-3)$

Put $t=t-4 \Rightarrow r(t-4)=(t-4)u(t-4)$

Given:

$$f(t)=(t-1)u(t-1)-(t-2)u(t-2) -(t-3)u(t-3)+(t-4)u(t-4)$$

$$f(t)=r(t-1)-r(t-2)-r(t-3)+r(t-4)$$



$$f(t)-0=\frac{1-0}{2-1}(t-1)$$
$$f(t)=t-1$$

$$f(t)-1=\frac{0-1}{4-3}(t-3)$$
$$f(t)=-t+3+1$$
$$f(t)=-t+4$$

$$E=\int_{-\infty}^{\infty}|f(t)|^2\,dt=\int_1^2(t-1)^2\,dt+\int_2^3 1^2\,dt+\int_3^4(4-t)^2\,dt$$

$$E=\int_1^2\left(t^2+1-2t\right)dt+\int_2^3 1^2\,dt+\int_3^4\left(16+t^2-8t\right)^2 dt$$

$$E=\left(\frac{t^3}{3}+t-t^2\right)_1^2+(t)_2^3+\left(16t+\frac{t^3}{3}-4t^2\right)_3^4$$

$$E=\left(\frac{8}{3}+2-4-\frac{1}{3}-1+1\right)+(3-2)$$
$$+\left(64+\frac{64}{3}-64-48-9+36\right)$$

$$E=\frac{5}{3}$$

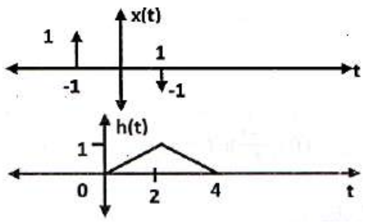
GATE QUESTIONS(IN)

Q.1 Given,
X(t) * x(t) = t exp (-2t) u(t)
The function x(t) is
(a) exp (-2t)u(t)
(b) exp (-t)u(t)
(c) t exp(-t) u(t)
(d) 0.5 exp(-t)u(t)
[GATE-2006]

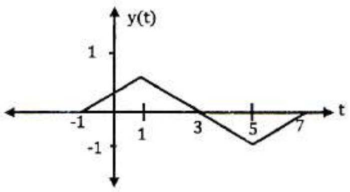
Q.2 The Fourier series for a periodic signal is given as
$x(t) = \cos(1.2\pi t) + \cos(2\pi t) + \cos(2.8\pi t)$.
The fundamental frequency of the signal is
a) 0.2Hz b) 0.6Hz
c) 1.0Hz d) 1.4Hz
[GATE-2006]

Q.3 Consider the periodic signal $x(t) = (1 + 0.5cos40\pi t.)$ where t is in seconds. Its fundamental frequency, in Hz is
a) 20 b) 40
c) 100 d) 200
[GATE-2007]

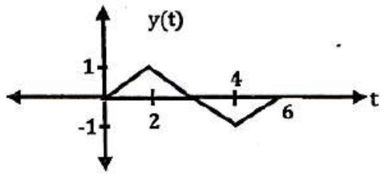
Q.4 The signals x(t) and h(t) shown in the figures are convolved to yield y(t).



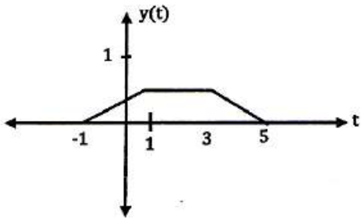
Which one of the following figures presents the output y(t)?



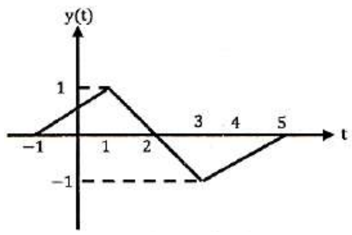
a)



b)



c)



d)

[GATE-2007]

Q.5 The step response of a linear time invariant system is y(t)=5 e^{-10t} u(t) is the unit step function. If the output of the system corresponding to an impulse input δ(t) is h (t), then h(t) is
(a) -50e^{-10t} u (t)
(b) 5 e^{-10t} $\delta(t)$
(c) 5u (t) – 50 e^{-10t} u (t)
(d) 5 $\delta(t)$ – 50 e^{-10t} u (t)
[GATE-2008]

Q.6 The fundamental period of
$x(t) = 2\sin 2\pi t + 3\sin 3\pi t$,
with expressed in seconds , is
a) 1s b) 0.67s
c) 2s d) 3s
[GATE-2009]

Q.7 For input x(t), an ideal impulse sampling system produces the output
y(t)=$\sum_{k=-\infty}^{0} x(kT)\delta(t-kT)$
where $\delta(t)$ is the Dirac delta function.
The system is

a) Non-linear and time invariant
b) Non-linear and time varying
c) Linear and time invariant
d) Linear and time varying

[GATE-2009]

Q.8 The integral $\int_{-\infty}^{t} \delta(t-\frac{\pi}{6})$ 6sin(t) dt evaluated to
a) 6 b) 3 c) 1.5 d) 0
[GATE-2010]

Q.9 The input x(t) and the corresponding output y(t) of a system are related by
$y(t) \int_{-\infty}^{5t} x(\tau)d\tau$ The system is
a) time invariant and causal
b) time invariant and non-causal
c) time variant and non- causal
d) time variant and causal
[GATE-2010]

Q.10 The continuous –time signal $x(t) = sin\omega_0 t$ is a periodic signal .However, for its discrete time counterpart $x[n] = sin\omega_0 n$ to be periodic , the necessary condition is
a) $0 \le \omega_0 < 2\pi$
b) $\frac{2\pi}{\omega_0}$ to be an integer
c) $\frac{2\pi}{\omega_0}$ to be a ratio of integers
d) none
[GATE-2011]

Q.11 The integral $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t^2 e^{-t^2/2} \delta(1-2t)dt$ is equal to
a) $\frac{1}{8\sqrt{2\pi}} e^{-1/8}$ b) $\frac{1}{4\sqrt{2\pi}} e^{-1/8}$
c) $\frac{1}{\sqrt{2\pi}} e^{-1/2}$ d) 1

[GATE-2011]

Q.12 Consider a system with input x(t) and output y(t) related as follows
$y(t) = \frac{d}{dt}\{e^{-t}x(t)\}$
Which one of the following statements is TRUE?
a) The system is nonlinear
b) The system is time –invariant
c) The system is stable
d) The system has memory
[GATE-2011]

Q.13 The input x(t) and output y(t) of a system are related as $y(t)= \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$. The system is
a) time-invariant and stable
b) stable and not time-invariant
c) time-invariant and not stable
d) not time-invariant and not stable

[GATE-2012]

Q.14 Two system with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by
a) product of $h_1(t)$ and $h_2(t)$
b) sum of $h_1(t)$ and $h_2(t)$
c) convolution of $h_1(t)$ and $h_2(t)$
d) subtraction of $h_1(t)$ and $h_2(t)$

[GATE-2013]

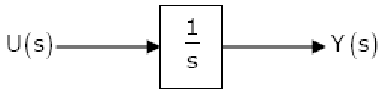
Q.15 For a periodic signal v (t) 30sin100t 10cos300t +6 sin $\left(500+\frac{\pi}{4}\right)$, the fundamental frequency in rad/s is
a) 100 b) 300
c) 500 d) 1500
[GATE-2013]

Q.16 Which of the following statements is NOT TRUE for a continuous time causal and stable LTI system?
a) All the poles of the system must lie on the left side of the $j\omega$-axis
b) Zeroes of the system can lie anywhere in the s-plane
c) All the poles must lie within |S|= 1

d) All the roots of the characteristic equation must be located on the left side of the $j\omega$-axis.

[GATE-2013]

Q.17 Assuming zero initial condition, the response y (t) of the system given below to a unit step input u(t) is



a) u(t) b) t u(t)

c) $\frac{t^2}{2}u(t)$ d) $e^{-t}u(t)$

[GATE-2013]

Q.18 The impulse response of a system is h(t) = tu(t) . For an input u(t − 1) , the output is

a) $\frac{t^2}{2}u(t)$ b) $\frac{t(t-1)}{2}u(t-1)$

c) $\frac{(t-1)^2}{2}u$ d) $\frac{t^2-1}{2}u$ (t-1)

[GATE-2013]

Q.19 The impulse response of a continuous time system is given by h(t) = δ(t − 1) + δ (t − 3) . The value of the step response at t = 2 is

a) 0 b) 1

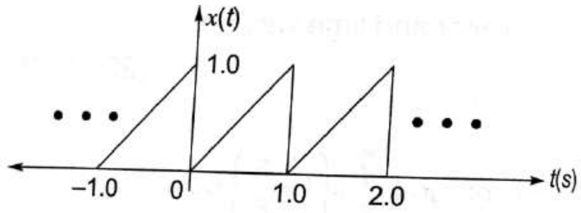
c) 2 d) 3

[GATE-2013]

Q.20 The fundamental period of the signal $x(t)=2\cos\left(\frac{2\pi}{3}t\right)2+\cos(\pi t)$, in second, is ___ s.

[GATE-2015]

Q.21 A periodic signal x(t) shown in the figure. The fundamental frequency of the signal x(t) in Hz is________.



[GATE-2017]

Q.22 Consider signal $x(t)=\begin{cases}1 & |t|\le 2\\ 0 & |t|>2\end{cases}$.

Let δ(t) denote the unit impulse (Dirac-delta) function. The value of the integral $\int_0^5 2x(t-3)\delta(t-4)\,dt$ is

(a) 2 (b) 1

(c) 0 (d) 3

[GATE-2018]

Q.23 Two periodic signals x(t) and y(t) have the same fundamental period of 3 seconds. Consider the signal z(t) = x(−t) + y(2t + 1). The fundamental period of z(t) in seconds is

a) 1 b) 1.5

c) 2 d) 3

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10
(a)	(a)	(a)	(d)	(d)	(c)	(d)	(b)	(c)	(c)
11	12	13	14	15	16	17	18	19	20
(a)	(c)	(d)	(c)	(a)	(c)	(b)	(c)	(b)	6
21	22	23							
1	(a)	(d)							

EXPLANATIONS

Q.1 (a)

Given,

x(t) * x(t) = t exp(-2t) u(t)

taking Laplace transform on both the sides we get,

$X(s). X(s) = \frac{1}{(s+2)^2}$

$X(s)=\frac{1}{(s+2)}$

Taking inverse Laplace transform of X(s) is

X(t)=exp(-2t) u(t)

Q.2 (a)

$f_1 = 0.6Hz; f_2 = 1Hz; f_3 = 1.4Hz$

∴ Fundamental frequency of

$x(t) = f_0 = 0.2Hz$

$f_1 = 3, f_0 = 3^{rd}$ harmonic ;

$f_2 = 5\ f_6 = 5$th hormonic

Q.3 (a)

$X(t) = (1 + 0.5cos40\pi t)cos200\pi t$

$= cos200\pi t + 0.5cos40\pi t\ cos200\pi t$

$= cos200\pi t + 0.25cos160\pi t +$

$0.25\ cos240\pi t$

$= x_1(t) + x_2(t) + x_3(t)$

Where, $x_1 = cos200\pi t$

$x_2 = 0.25cos160\pi t$

$x_3 = 0.25\ cos240\pi t$

Now, T_1 = time period of $x_1(t)$

$= \frac{2\pi}{200\pi} = \frac{1}{100}s$

$T_2 = \frac{1}{80}s$

$T_3 = \frac{1}{120}s$

Now, fundamental period of x(t0 is the LCM of $T_1, T_2\ and\ T_3$, which is $\frac{1}{20}s$

So, fundamental frequency =20Hz.

Q.4 (d)

$y(t) = x(t) * h(t)$

$x(t)$and $h(t)$ are shown in Fig .1 and Fig. 2

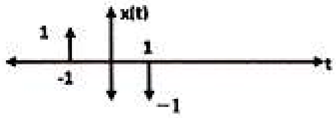


Fig. 1

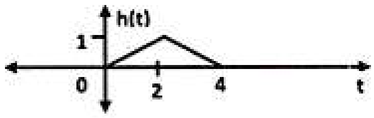


Fig. 2

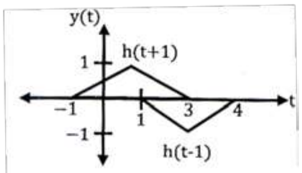


Fig. 3

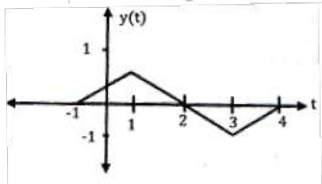


Fig. 4

$x(t) = 1\delta(t+1) - 1\delta(t-1), y(t)$

$= \delta(t+1) * h(t) - \delta(t-1) * h(t)$

Use the properties of convolution:

$\delta(t) * h(t) = h(t)$

$\delta(t \mp 1) * h(t) = h(t \mp 1)$

$y(t) = h(t+1) - h(t-1)$

The resultant of the triangles in Fig. 3 is shown in Fig. 4

Q.5 (d)

Step response s(t) =5u(t)e^{-10t}

Impulse response h(t)=$\frac{ds(t)}{dt}$

$\therefore h(t) = \frac{ds}{dt} = \frac{d\{5e^{-10t}u(t)\}}{dt}$

=-50e^{-10t}u(t) + 5 e^{-10t} du(t)/dt

=-50 e^{-10t} u(t) + 5δ(t)dt

Q.6 (c)

$x(t) = 2sin2\pi t + 3sin3\pi t$

Period of $2sin2\pi t$ is 1

Period of $3sin3\pi t$ is 2/3

∴ Period is 2 sec

Q.7 (d)

Given signal is

y(t)=$\sum_{k=-\infty}^{c} x(kT)\delta(t-kT)$
This is the representation of a signal x(t) in weighed and sum form and it obeys the principle of superposition and homogeneity. So, this is linear but it is time varying as for as x(t-t_0),
y'(t) = $\sum x(kT-t_0)\delta(t-kT)$
and y(t-t_0)= $\sum x(kT)\delta(t-t_0-kT)$

Therefore y(t-t_0) ≠ y'(t), so time varying.

Q.8 (b)

Given signal is

x(t)=$\int_{-\infty}^{\infty} \delta(t-\frac{\pi}{6})$ 6sin(t)dt

By shifting property of unit impulse function

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = \begin{cases} x(t_0); & t_1 < t_0 < t_2 \\ 0; & elsewhere \end{cases}$$

$\int_{-\infty}^{\infty} \delta(t-\frac{\pi}{6})$6sin(t)dt = 6sin$\frac{\pi}{6}$ = 6 x ½ = 3

Q.9 (c)

$x(t) \to y(t) = \int_{\tau=-\infty}^{5t} x(\tau)d\tau \dots\dots\dots (1)$

Value of y at 't' depends on value of x for times from $-\infty$ to 5t

As output is depending on future value of input, the system is Non causal

For the input

$x_1(t) = x(t-t_0), y_1(t) = \int_{\tau=-\infty}^{5t} x(\tau)d \dots\dots (2)$

$y_1(t) = \int_{\tau=-\infty}^{5t} x(\tau-t_0)d\tau \dots\dots\dots\dots\dots (3)$

From (1) ,

$y(t-t_0) = \int_{\tau=-\infty}^{5(t-t_0)} x(\tau)d\tau \dots\dots\dots\dots\dots (4)$

Let $\tau_1 = \tau - t_0$ in (3)

From (3),

$y_1(t) = \int_{\tau=-\infty}^{5t} x(\tau_1)d\tau_1 \dots\dots\dots\dots\dots (5)$

As (4) & (5) are not same, the system is Time variant.

∴ The system is Time variant and Non causal.

Q.10 (c)

If x(n) is periodic with period N, then the condition to be satisfied is

$x(n) = x(n+N)$

If $x[n] = sin\omega_0 n$ Then the necessary condition is

$sin(\omega_0 n) = sin[\omega_0(n+N)]$

Or $\frac{2\pi}{\omega_0} = \frac{N}{m}$ = ratio of integers Or ω_0 should be expressible as $2\pi\left(\frac{m}{N}\right)$

Q.11 (a)

$$I = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} t^2 e^{\frac{-t^2}{2}}\delta(1-2t)dt$$

By using property

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$$

Here , $t_0 = \frac{1}{2}$

$$I = \frac{1}{\sqrt{2\pi}}\times\frac{1}{2}\times\left(\frac{1}{2}\right)^2\times e^{\frac{(-1/2)^2}{2}}$$

$$= \frac{1}{\sqrt{2\pi}}\times\frac{1}{8}\times e^{\frac{-1}{8}}$$

Q.12 (c)

Given the system,

$y(t) = \frac{d}{dt}\{e^{-t}x(t)\}$

$y(t) = e^{-t}[x(t)] + [-x(t)e^{-t}]$

$= e^{-t}\left[\frac{d}{dt}x(t) - x(t)\right]$

As y(t) is obtained from linear operation on x(t) , the system is linear. As the input x(t) is multiplied by a time varying function e^{-t},the system is Time–varying. For a bounded input $x(t), e^{-t}x(t)$ is bounded, $y(t)$ is also bounded, Therefore the system is stable .As y(t) depends only on x(t), the system has no memory.

Q.13 (d)

Q.14 (c)

Q.15 (a)

$\omega_o = 100$ rad / sec fundamental

$3\omega_o = 3$ 300 rad / sec third harmonic

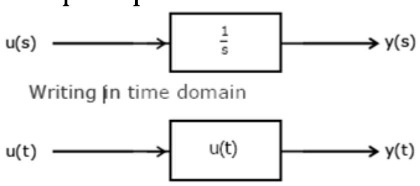
$5\omega_o =$ 500 rad / sec fifth harmonic

Q.16 (c)

For an LTI system to be stable and causal all poles or roots of characteristic equation must lie on LHS of s-plane i.e., left hand side of j ω-axis. [Refer Laplace transform].

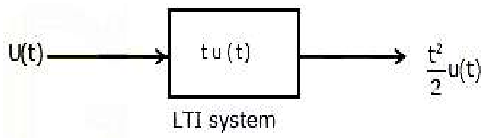
Q.17 (b)

Integration of unit step function is ramp output



y(t) = u(t) * u(t) = tu(t)

Q.18 (c)

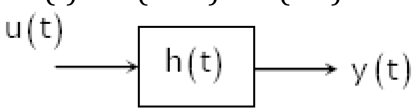


For LTI system, if input gets delayed by one unit, output will also get delayed by one unit.

$u(t-1) \rightarrow \frac{(t-1)^2}{2} u(t-1)$

Q.19 (b)

h(t) = δ (t – 1) + δ (t –)



y(t)= u(t - 1)+ u(t- 3)

y(2)= u(1)+ u(-1) =1

Q.20 (6)

$$\omega_0 = \frac{\text{H.C.F}(2\pi, \pi)}{\text{L.C.M}(3,1)} = \frac{\pi}{3}$$

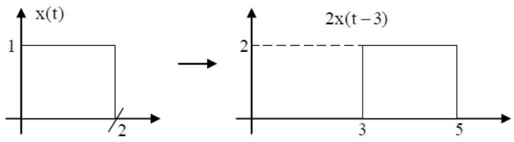
$$\frac{2\pi}{T} = \frac{\pi}{3} \Rightarrow T = 6\text{sec}$$

Q.21 (1)

$$f = \frac{1}{T} = \frac{1}{1} = 1\text{Hz}$$

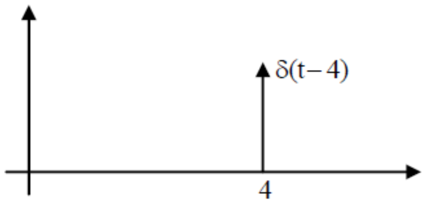
Q.22 (a)

$$\text{Given, } x(t) = \begin{cases} 1 & |t| \le 2 \\ 0 & |t| > 2 \end{cases}$$



$$2x(t-3)\delta(t-4) = 2\delta(t-4)$$

$$\int_0^5 2x(t-3)\delta(t-4)\,dt = 2$$



Q.23 (d)

$x(t) \rightarrow T = 3$seconds

$x(-t) \rightarrow T = 3$seconds

Time period does not change due to time reversal

$y(t) \rightarrow T = 3$seconds

$y(t+1) \rightarrow T = 3$seconds

Due to shifting time period does not change

$y(2t+1) \rightarrow T = 1.5$seconds

Signal will get compress by factor of 2.

Thus $x(-t) + y(2t+1)$ will have time period

T = LCM$(1.5, 3) = 3$

GATE QUESTIONS(IN)

Q.1 Given x[n]=$\frac{\sin\omega_c n}{\pi n}$, the energy of the signal given by $\sum_{n=-\infty}^{\infty}|x[n]|^2$ is
a) $\frac{\omega_c}{\pi}$ b) $\pi\omega_c$
c) Infinite d) $2\pi\omega_c$

[GATE-2003]

Q.2 Given h[n]=[1,2,2], f(n) is obtained
↑
By convolving h[n] with itself and g[n] by correlating h[n] with itself. Which of the following statement is true?
a) f[n] is causal and its maximum value is 9
b) f[n] is non-causal and its maximum value is 8
c) g[n] is causal and its maximum value is 9
d) g[n] is non-causal and its maximum value is 9

[GATE-2003]

Q.3 The fundamental period of the sequence x[n] = 3sin(1.3πn+0.5π) + 5 sin(1.2 πn) is
a) 20 b) $\frac{2\pi}{1.3\pi}$
c) $\frac{2\pi}{1.2\pi}$ d) 10

[GATE-2005]

Q.4 Consider the discrete – time signal $x(t)=\left(\frac{1}{3}\right)^n u[n]$ where $u[n]=\begin{cases}1, n\geq 0\\ 0, n<0\end{cases}$.
Define the signal $y[n]$ as $y[n]=x[-n], -\infty<n<\infty$ .Then $\sum_{n=-\infty}^{\infty} y[n]$ equals.
a)$-\frac{2}{3}$ b) $\frac{2}{3}$
c) $\frac{3}{2}$ d)3

[GATE-2007]

Q.5 The fundamental period of the discrete-time signal $X[n]=e^{j\left(\frac{5\pi}{6}\right)n}$ is
a) $\frac{6}{5\pi}$ b) $\frac{12}{5}$
c) 6 d) 12

[GATE-2008]

Q.6 Which one of the following discrete –time system is time invariant?
a)$y[n]=nx[n]$ b) $y[n]=x[3n]$
c) $y[n]=x[-n]$ d) $y[n]=x[n-3]$

[GATE-2008]

Q.7 Consider a discrete–time LTI system with input $x[n]=\delta[n]+\delta[n-1]$ and impulse response $h[n]=\delta[n]-\delta[n-1]$. The output of the system will be given by
a) $\delta[n]-\delta[n-2]$
b) $\delta[n]-\delta[n-1]$
c) $\delta[n-1]+\delta[n-2]$
d) $\delta[n]+\delta[n-1]+\delta[n-2]$

[GATE-2008]

Q.8 Consider a discrete –time system for which the input $x[n]$ and the output $y[n]$ are related as $y[n]=x[n]-\frac{1}{3}y[n-1]$. If $y[n]=0$ for $n<0$ and $x[n]=\delta[n]$ then $y[n]$ can be expressed in terms of the unit step $u[n]$ as
a)$\left(\frac{-1}{3}\right)^n u[n]$ b) $\left(\frac{1}{3}\right)^n u[n]$
c)$(3)^n u[n]$ d) $(-3)^n u[n]$

[GATE-2008]

Q.9 Let y[n] denote the convolution of h[n] and g[n], where h[n]=$(1/2)^n$u[n] and g[n] is a causal sequence. If y[0] = 1 and y[1]= ½ , then g[1] equals
a) 0 b)1/2 c) 1 d) 3/2

[GATE-2012]

Q.10 The discrete-time transfer function $\frac{1-2z^{-1}}{1-0.5z^{-1}}$ is

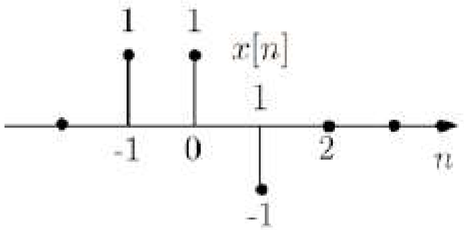
a) non-minimum phase and unstable
b) minimum phase and unstable
c) minimum phase and stable
d) non-minimum phase and stable
[GATE-2013]

Q.11 The impulses response of an LTI system is given as:

$$= \begin{cases} \frac{\omega_c}{\pi} & n = 0 \\ \frac{\sin \omega_c n}{\pi n} & n \neq 0 \end{cases}$$

It represents an ideal
a) non-causal, low-pass filter
b) causal, low-pass filter
c) non-causal, high-pass filter
d) causal, high-pass filter
[GATE-2014]

Q.12 The signal x[n] shown in the figure below is convolved with itself to get y[n]. The value of y[-1] is________.



[GATE-2016]

Q.13 The fundamental period N_0 of the discrete-time sinusoid x[n]=sin $\left(\frac{301}{4}\pi n\right)$ is_____

[GATE-2016]

ANSWER KEY:

1	2	3	4	5	6	7	8
a	d	a	c	d	d	a	a
9	10	11	12	13			
a	d	a	2	8			

EXPLANATIONS

Q.1 (a)

The Fourier transform of sinc function is gate signal shown below :

$$\text{Energy} = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} 1 d\omega$$

$$= \frac{1}{2\pi}\left[\omega_c - (-\omega_c)\right] = \frac{2\omega_c}{2\pi} = \frac{\omega_c}{\pi}$$

Q.2 (d)

$$h[n] * h[n] = f[n] = \{1, 4, 8, 8, 4\}$$

$$g[n] = h[n] * h[-n] = \{2, 6, \underset{\uparrow}{9}, 6, 2\}$$

So g[n] is non causal having maximum values 9.0

Q.3 (a)

Q.4 (c)

Given $x(n) = \left(\frac{1}{3}\right)^n u[n]$

and $y[n] = x(-n), -\infty < n < \infty$

$\sum_{n=-\infty}^{\infty} y[n] \sum \left(\frac{1}{3}\right)^n u(-n) =$

$\sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots \ldots$

$= \frac{\text{First term}}{1-\text{Common ratio}} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$

Q.5 (d)

$n(n) = e^{j5n\frac{\pi}{6}n} = e^{j5n\frac{2\pi}{12}n} = e^{jm\frac{2\pi}{N}n}$

Fundamental period =N=12

Q.6 (d)

$y[n] = x(n-3)$ represents a time delay system with delay =3 secs.

It is a Time invariant system as can be confirmed from the following Test. Test for time –invariance:

If $y_1(n) = x(n-n_0)$, then the sytem is time invariant For the given system $x(n) \to y(n0 = x(n-3)$.....................(1)

$x_1(n) = x(n-n_0) \to y_1(n)$

$= x_1(n-3) = x(n-3-n_0)$

From (1),

$y(n-n_0) = x(n-n_0-3)$

$= y_1(n)$

Q.7 (a)

$X(z) = 1 + z^{-1}$

and $H(z) = 1 - z^{-1}$

$\therefore Y(z) = X(z).H(z)$

$= 1 - z^{-2} \Rightarrow y[n]$

$= \delta[n] - \delta[n-2]$

Q.8 (a)

Given

$y(n) = x(n) - \frac{1}{3}y(n-1)y(n)$

$= 0, n < 0$

$y(n) + \frac{1}{3}y(n-1) = x(n)$

Take z-transform:

$y(z)\left[1 + \frac{1}{3}z^{-1}\right] = X(z)$

$\text{T.F} = H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+\frac{1}{3}z^{-1}} = \frac{z}{z+\frac{1}{3}}$

For the given

$x(n) = \delta(n), X(z) = 1$

$\therefore Y(z) = \frac{z}{z+\frac{1}{3}}$

Use the standard Z.T. pair:

$a^n u(n) \to \frac{z}{z-a}, |z| > |a|$

$y(n) = \left(-\frac{1}{3}\right)^n u[n], |z| > \frac{1}{3}$

Q.9 (a)

Given g [n] is a causal sequence

$\therefore$ g [n] will be zero for n < 0

$$y[n] = \sum_{k=-\infty}^{\infty} g[k]h[n-k]$$

and $h[n] = \left(\frac{1}{2}\right)^n u[n]$

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$

$h[0] = 1 \;,\; h[1] = \frac{1}{2},\; h[2] = \frac{1}{4}$

$y[0] = g[0]\,h[0] + g[1]h[-1] + \ldots\ldots$

$1 = g[0] \times 1 + 0 + 0 + \ldots\ldots$

$g[0] = 1$

$y[1] = g[0]h[1] + g[1]\,h[0] + g[2]h[-1] + \ldots.$

$\frac{1}{2} = 1 \cdot \frac{1}{2} + g[1] \cdot 1 + 0$

$\Rightarrow\; g[1] = 0$

Q.10 (d)

Q.11 (a)

It is non-causal, since

$h[n] \neq 0 \text{ for } h < 0$

It is an low-pass filter

Q.12 (2)

x(n) = {1,1,-1} $(-1 \leq n \leq 1)$

	1	1	−1
1	1	1	−1
1	1	1	−1
−1	−1	−1	1

↑

Y(n) = x(n)* x(n) ={1,2-1,-2,1}

Y(-1)=2

Q.13 (8)

In discrete case

$$\omega_0 N = 2\pi m \Rightarrow N = \frac{2\pi}{\omega_0} m$$

Where m is the smallest positive integer that makes integer.

$$\rightarrow N = 2\pi \times \frac{4}{301\pi} \times m \left[\frac{8}{301} m\right]$$

If m =301, then N=8

3 FOURIER SERIES & FOURIER TRANSFORM

3. 1 TRIGONOMETRIC FOURIER SERIES

A periodic function f(t) can be expressed in the form of trigonometric series as

$$f(t)=\frac{1}{2}a_0+a_1\cos\omega_0 t+a_2\cos 2\omega_0 t+a_3\cos 3\omega_0 t+....$$

$$+b_1\sin\omega_0 t+b_2\sin 2\omega_0 t+b_3\sin 3\omega_0 t+.........(3.1)$$

where $\omega_0=2\pi f=\frac{2\pi}{T}$, f is the frequency and a_n and b_n are the coefficients. The Fourier series exists only when the functions f(t) satisfies the following three conditions called **Dirichlet's conditions.**

i) f(t) is well defined and single-valued, except possibly at a finite number of points, i.e.,

ii) f(t) must have only a finite number of discontinuities in the period T.

iii) f(t) must have a finite number of positive and negative maxima in the period T.

Equation: May be expressed by the Fourier series

$$f(t)=a_0+\sum_{n=1}^{\infty}a_n\cos n\omega_0 t+\sum_{n=1}^{\infty}b_n\sin n\omega_0 t....(3.2)$$

where a_n and b_n are the coefficients to be evaluated.

Integrating equation 3.2 for a full period, we get

$$\int_{-T/2}^{T/2} f(t)dt=a_0\int_{-T/2}^{T/2}dt+\int_{-T/2}^{T/2}\sum_{n=1}^{\infty}(a_n\cos n\omega_0 t+b_n\sin n\omega_n t)dt$$

Integration of cosine or sine function for a complete period is zero.

Therefore, $\int_{-T/2}^{T/2} f(t)dt=a_0 T$

Hence, $a_0=\frac{1}{T}\int_{-T/2}^{T/2} f(t)dt$

or, equivalently $a_0=\frac{1}{T}\int_0^T f(t)dt$

Multiplying both sides of equation 3.2 by $\cos m\,\omega_0 t$ and integrating, we have

$$\int_{-T/2}^{T/2} f(t)\cos m\,\omega_0 t\,dt$$

$$=\frac{1}{T}\int_{-T/2}^{T/2} a_0\cos m\omega_0 tdt+\int_{-T/2}^{T/2}\sum_{n=1}^{\infty}a_n\cos n\omega_0 t\cos m\omega_0 tdt$$

$$+\int_{-T/2}^{T/2}\sum_{n=1}^{\infty}b_n\sin n\omega_0 t\cos m\omega_0 tdt$$

Here, $\frac{1}{2}\int_{-T/2}^{T/2} a_0\cos m\omega_0 tdt=0......(3.3)$

$$\int_{-T/2}^{T/2} a_n\cos n\omega_0 t\cos m\omega_0 dt$$

$$=\frac{a_n}{2}\int_{-T/2}^{T/2}[\cos(m+n)\omega_0 t+\cos(m-n)\omega_0 t]dt$$

$$=\begin{cases}0, & \text{for } m\neq n\\ \frac{T}{2}a_n, & \text{for } m=n\end{cases}$$

$$\int_{-T/2}^{T/2} b_n\sin n\omega_0 tdt=\frac{bn}{2}\int_{-T/2}^{T/2}[\sin(m+n)\omega_0 t$$

$$-\sin(m-n)\omega_0 t]dt=0$$

Therefore, $\int_{-T/2}^{T/2} f(t)\cos n\,\omega_0 t\,dt=\frac{Ta_n}{2}$

For m = n

Hence, $a_n=\frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\,\omega_0 t\,dt...(3.4)$

or, equivalently $a_n=\frac{2}{T}\int_0^T f(t)\cos n\,\omega_0 t\,dt$

Similarly, multiplying both sides of equation 3.2 by $\sin m\,\omega_0 t$ and integrating, we get

$$\int_{-T/2}^{T/2} f(t)\sin m\omega_0 tdt=\frac{1}{2}\int_{-T/2}^{T/2} a_0\sin m\omega_0 tdt$$

$$+\int_{-T/2}^{T/2}\sum_{n=1}^{\infty}a_n\cos n\omega_0 t\sin m\omega_0 tdt+$$

$$+\int_{-T/2}^{T/2}\sum_{n=1}^{\infty} b_n \sin n\omega_0 t \sin m\omega_0 t dt$$

Here, $\frac{1}{2}\int_{-T/2}^{T/2} a_0 \sin m\,\omega_0 t\, dt = 0$

$$\int_{-T/2}^{T/2} a_n \cos n\,\omega_0 t \sin m\,\omega_0 t\, dt = 0$$

$$\int_{-T/2}^{T/2} b_n \sin n\,\omega_0 t \sin m\,\omega_0 t\, dt = \begin{cases} 0, & \text{for } m \neq n \\ \frac{1}{2} b_n & \text{for } m = n \end{cases}$$

Therefore

$$\int_{-T/2}^{T/2} f(t) \sin m\ \omega_0\ t\, dt = \frac{T}{2} b_n, \text{for } m = n$$

Hence, $b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t) \sin n\,\omega_0 t\, dt$ (3.5)

or, equivalently,

$$b_n = \frac{2}{T}\int_0^T f(t) \sin n\,\omega_0 t\, dt$$

The number n = 1, 2, 3,……. Gives the values of the harmonic frequencies.

3.2 SYMMETRY CONDITION

i) If the function f(t) is even, then f(-t) = f(t). For example, cos t, t^2, t sin t, are all even. The cosine is an even function, since it may be expressed as the power series

$$\cos t = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \ldots\ldots$$

The waveforms representing the even functions of t are shown in figure. Geometrically, the graph of an even function will be symmetrical with respect to the y – axis and only cosine terms are present (d.c. term optional). When f(t) is even,

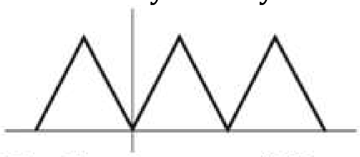
$$\int_{-a}^{a} f(t)\, dt = 2\int_{-a}^{a} f(t)\, dt$$

The sum of product of two or more even functions is an even function.

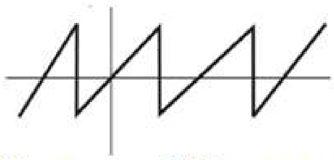
ii) If the function f(t) is odd, then f(-t)=-f(t) and only sine terms are present (d.c. term optional). For example, sin t, t^3, t cost are all odd. The waveforms shown in figure represent odd functions of t. The graph of an odd function is symmetrical about the origin. If f(t) is odd, $\int_{-a}^{a} f(t)\, dt = 0$. The sum of two or more odd functions is an odd function and the product of two odd functions is an even function.

iii) If f(t + T/2)=f(t), only even harmonics are present.

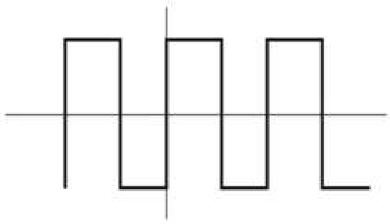
iv) If f(t+T/2)=-f(t), only harmonics are present and hence the waveform has half – wave symmetry.



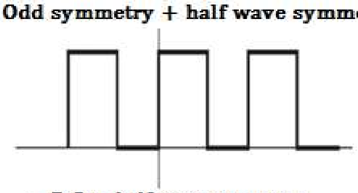
= D.C. + Even symmetry + Half wave symmetry



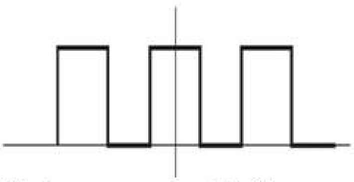
= Odd symmetry + Half wave symmetry



= Odd symmetry + half wave symmetry



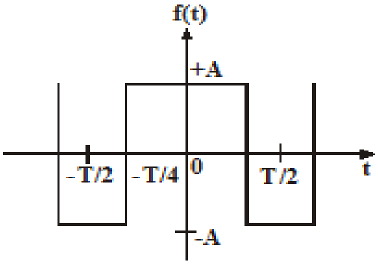
= D.C. + half wave symmetry



= D.C. + even symmetry + half wave symmetry

Example: Obtain the Fourier components of the periodic square wave signal which is symmetrical with respect to the vertical axis at time t = o, as shown in figure.

Solution: Since the given waveform is symmetrical about the horizontal axis, the average area is zero and hence the d.c. term $a_0 = 0$. In addition, f(t) = f(-t) and so only cosine terms are present, i.e., $b_n = 0$.



Now, $a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\,\omega_0 t\,dt$

where

$$f(t) = \begin{cases} -A, & \text{from } -T/2 < t < -T/4 \\ +A, & \text{from } -T/4 < t < +T/4 \\ -A, & \text{from } +T/4 < t < +T/2 \end{cases}$$

Therefore,

$$a_n = \frac{2A}{T}\left[\int_{-T/2}^{-T/4}(-\cos n\omega_0 t)dt + \int_{-T/4}^{T/4}\cos n\omega_0 t dt + \int_{T/4}^{T/2}(-\cos n\omega_0 t)dt\right]$$

$$= \frac{2A}{T}\left[\left(\frac{-\sin n\omega_0 t}{n\omega_0}\right)_{-T/2}^{-T/4} + \left(\frac{\sin n\omega_0 t}{n\omega_0}\right)_{-T/4}^{T/4} + \left(\frac{-\sin n\omega_0 t}{n\omega_0}\right)_{T/4}^{T/2}\right]$$

$$= \frac{2A}{n\omega_0 T}\left[-\sin\left(\frac{-n\omega_0 T}{4}\right) + \sin\left(\frac{-n\omega_0 T}{2}\right) + \sin\left(\frac{n\omega_0 T}{4}\right) - \sin\left(\frac{-n\omega_0 T}{4}\right) - \sin\left(\frac{n\omega_0 T}{2}\right) + \sin\left(\frac{n\omega_0 T}{4}\right)\right]$$

$$= \frac{8A}{n\omega_0 T}\sin\left(\frac{-n\omega_0 T}{4}\right) - \frac{4A}{n\omega_0 T}\sin\left(\frac{n\omega_0 T}{2}\right)$$

When $\omega_0 T = 2\pi$, the second term is zero for all integer values of n.

Hence,

$$a_n = \frac{8A}{2n\pi}\sin\left(\frac{n\pi}{2}\right) = \frac{4A}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$a_0 = 0$ (d.c. term)

$$a_1 = \frac{4A}{\pi}\sin\left(\frac{\pi}{2}\right) = \frac{4A}{n\pi}$$

$$a_2 = \frac{4A}{\pi}\sin(\pi) = 0$$

$$a_3 = \frac{4A}{3\pi}\sin\left(\frac{3\pi}{2}\right) = -\frac{4A}{3\pi}$$

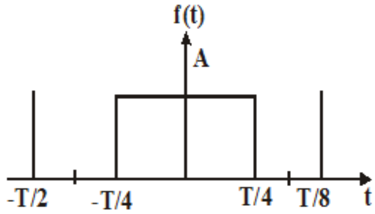
....

....

Substituting the values of the coefficients in Equation 3.2, we get

$$f(t) = \frac{4A}{\pi}\left[\cos(\omega_0 t) - \frac{1}{3}\cos(3\omega_0 t) + \frac{1}{5}\cos(5\omega_0 t) - .....\right]$$

Example: Obtain the Fourier Components of the periodic rectangular waveform shown in Figure.



Solution:- The given waveform for one period can be written as

$$f(t) = \begin{cases} 0, & \text{for } -T/2 < t < -T/4 \\ A, & \text{for } -T/4 < t < T/4 \\ 0, & \text{for } T/4 < t < T/2 \end{cases}$$

For the given waveform, f(-t) = f(t) and hence it is an even function and has $b_n = 0$. The value of the d.c. term is

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(t)dt \quad = \frac{2}{T}\int_{-T/4}^{T/4} A\,dt = \frac{2A}{T}\times\frac{T}{2} = A$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\,\omega_0 t\;dt$$

$$= \frac{2}{T}\int_{-T/4}^{T/4} A\cos n\omega_0 t\,dt$$

$$= \frac{2A}{T}\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_{-T/4}^{T/4}$$

$$= \frac{4A}{n\omega_0 T}\sin(n\omega_0 T/4)$$

When $\omega_0 T = 2\pi$, we have

$$a_n = \frac{2A}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

= 0, for n = 2, 4, 6,.......

$= \frac{2A}{n\pi}$, for $n = 1, 5, 9, 13$, ...

$= \frac{2A}{n\pi}$, for $n = 3, 7, 11, 15$, ...

Substituting the values of the coefficients in Equation 3.2, we obtain

$$f(t) = \frac{A}{2} + \frac{2A}{\pi}\left(\cos\omega_0 t - \frac{1}{3}\cos 3\omega_0 t + \frac{1}{5}\cos 5\omega_0 t - ......\right)$$

3.3 COMPLEX OR EXPONENTIAL FORM OF FOURIER SERIES

From equation 3.2, the trigonometric from of the Fourier series is

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

An alternative but convenient way of writing the periodic function f(t) is in exponential form with complex quantities. Since

$$\cos n\,\omega_0 t = \frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}$$

$$\sin n\,\omega_0 t = \frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}$$

Substituting these quantities in the expression for the Fourier series gives

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n\left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}\right) + \sum_{n=1}^{\infty} b_n\left(\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}\right)$$

$$= a_0 + \sum_{n=1}^{\infty}\left(\frac{(a_n - jb_n)e^{jn\omega_0 t}}{2}\right) + \left(\frac{(a_n + jb_n)e^{-jn\omega_0 t}}{-j2}\right)$$

Here, taking $c_n = \frac{1}{2}(a_n - jb\,n)$

$c_{-n} = \frac{1}{2}(a_n - jb_n)$

$c_0 = a_0$

Where c_{-n} is the complex conjugate of c_n. Substituting expressions for the coefficients a_n and b_n from Equations 3.4 and 3.5 gives

$$c_n = \frac{1}{T}\int_0^{T/2} f(t)[\cos n\omega_0 t - j\sin n\omega_0 t]dt$$

$$= \frac{1}{T}\int_0^{T/2} f(t)e^{-jn\omega_0 t}dt$$

and $c_{-n} = \frac{1}{T}\int_{-T/2}^{T/2} f(t)[\cos n\omega_0 t + j\sin n\omega_0 t]dt$

$$= \frac{1}{T}\int_0^{T/2} f(t)e^{jn\omega_0 t}dt$$

with $f(t) = c_0 + \sum_{n=1}^{\infty} c_n e^{jn\omega_0 t} + \sum_{n=-\infty}^{-1} c_n e^{jn\omega_0 t}$

where the values of n are negative in the last term and are included under the $\sum$ sign. Also, c_0 may be included under the $\sum$ sign by using the value of n= 0. Therefore,

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

PARSEVAL'S IDENTITY FOR FOURIER SERIES

A periodic function f(t) with a period T is expressed by the Fourier series as

$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\,\omega_0 t + b_n \sin n\,\omega_0 t)$ Now,

$$[f(t)]^2 = a_0 f(t) + \sum_{n=1}^{\infty}[a_n f(t)\cos n\omega_0 t + b_n f(t)\sin n\omega_0 t]$$

Therefore

$$\frac{1}{T}\int_{-T/2}^{T/2}[f(t)]^2 dt = \frac{(a_0/2)}{T}\int_{-T/2}^{T/2}[f(t)]dt + \frac{1}{T}\sum_{n=1}^{\infty}\left[a_n\int_{-T/2}^{T/2} f(t)\cos n\,\omega_0 t\,dt + b_n\int_{-T/2}^{T/2} f(t)\sin n\,\omega_0 t\,dt\right]$$

From Equations 3.2, 3.3 and 3.4, we have

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\,dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\,\omega_0 t\,dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\,\omega_0 t\,dt$$

Therefore, substituting all these values, we get

$$\frac{1}{T}\int_{-T/2}^{T/2}[f(t)]^2\,dt = \left(\frac{a_0}{2}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

This is the **Parseval's identity.**

POWER SPECTRUM OF A PERIODIC FUNCTION

The power of a periodic signal spectrum f(t) in the time domain is defined as

$$P = \frac{1}{T}\int_{-T/2}^{T/2}[F(t)]^2 dt$$

The Fourier series for the signal f(t) is

$$f(t) = \sum_{n=-\infty}^{\infty} C^n e^{jn\omega_0 t}$$

According to Parseval's relation, we have

$$P_{av} = \frac{1}{T}\int_{-T/2}^{T/2}[f(t)]^2 \, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2}[f(t)]^2 \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} dt$$

$$= \sum_{n=-\infty}^{\infty} c_n \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{jn\omega_0 t} dt$$

$$= \sum_{n=-\infty}^{\infty} c_n c_{-n}$$

$$= \sum_{n=-\infty}^{\infty} |c_n|^2 \text{, watts}$$

The above equation becomes

$$\left(\frac{a_0}{1}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right) = \sum_{n=0}^{\infty}|c_n|^2$$

Here, $c_0 = \left|\frac{a_0}{1}\right|$ and $c_n = \sqrt{a_n^2 + b_n^2}$, $(n \geq 1)$

Thus the power in f(t) is

$P = \ldots + |c_{-n}|^2 + \ldots + |c_{-1}|^2 + |c_0|^2 + \ldots + |c_n|^2 + \ldots$

$P = |c_0|^2 + 2|c_1|^2 + 2|c_2|^2 + \ldots + |c_n|^2 + \ldots$

Hence, for a periodic function, the power in a time waveform f(t) can be evaluated by adding together the powers contained in each harmonic, i.e., frequency component of the signal f(t). The power for the n^{th} harmonic component at $n\,\omega_0$ radians per sec is $|c_n|^2$ and that of $-n\,\omega_0$ is $|c_{-n}|^2$. For the single real harmonic, we have to consider both the frequency components $\pm\, n\,\omega_0$. Here, $c_n = c_{-n}$ and hence $|c_n|^2 = |c_{-n}|^2$. The power for the n^{th} real harmonic f(t) is $P_n = |c_n|^2 + |c_{-n}|^2 = 2|c_n|^2$

The effective or RMS value of f(t):

Using Equations the RMS value of the function f(t) expressed by Equation 3.1 is

$$F_{rms} = \sqrt{\left(\frac{a_0}{2}\right)^2 + \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 + \ldots + \frac{1}{2}b_1^2 + \frac{1}{2}b_2^2 + \ldots}$$

$$= \sqrt{c_0^2 + \frac{1}{2}c_1^2 + \frac{1}{2}c_2^2 + \ldots}$$

Example: Determine the complex exponential following signals.

a) $x(t) = \cos \omega_0 t$

b) $x(t) = \sin \omega_0 t$

c) $x(t) = \cos\left(2t + \frac{\pi}{4}\right)$

d) $x(t) = \cos 4t + \sin 6t$

e) $x(t) = \sin^2 t$

f) $x(t) = 2 + \cos\frac{2\pi t}{2} + 4\sin\frac{5\pi t}{3}$

Solution :

a) $x(t) = \cos \omega_o t \rightarrow \frac{1}{2}\left(e^{-j\omega ot} + e^{-j\omega ot}\right)$

$$x(t) = \sum_{h=-\infty}^{\infty} e^{-jn\omega ot}$$

$n = 0$, $c_o = 0$, $c_k = 0$, $|k| \neq 1$

$n = 1$ $c_1 = \frac{1}{2}$

$n = -1$ $c_{-1} = \frac{1}{2}$

b) $x(t) = \sin \omega_o t \rightarrow \frac{1}{2j}\left(e^{j\omega ot} - e^{-j\omega ot}\right)$

$$= -\frac{1}{2j}e^{-j\omega ot} + e^{j\omega ot}$$

$$C_1 = \frac{1}{2j}, C_{-1} = \frac{1}{2j}, C_k = 0, |K| \neq 1$$

c) $x(t) = \cos\left(2t + \frac{\pi}{4}\right)$ $\quad w_o = 2$

$$x[t] = \sum_{k=-\infty}^{\infty} e_k e^{jk\omega ot} = \sum_{k=-\infty}^{\infty} c_k e^{j2kt}$$

$$x[t] = \frac{1}{2}\left[e^{j\left(2t+\frac{\pi}{4}\right)} + e^{-j\left(2t+\frac{\pi}{4}\right)}\right]$$

$$= \sum_{k=-\infty}^{\infty} c_k e^{j2kt} = \frac{1}{2j}e^{-j\frac{\pi}{4}}e^{-j2t}$$

$$=\frac{1}{2j}e^{j\frac{\pi}{4}}e^{j2t}$$

$$C_1=\frac{1}{2j}e^{j\frac{\pi}{4}}=\frac{11+j}{2\sqrt{2}}=\frac{\sqrt{2}}{4}(1+j)$$

$$C_{-1}=\frac{1}{2}e^{j\frac{\pi}{4}}=\frac{11-j}{2\sqrt{2}}=\frac{\sqrt{2}}{4}(1-j)$$

$c_k = 0,\ |k| \neq 1$

d) $x(t) = \cos 4t + \sin 6t$

$\downarrow \quad \downarrow$

$4 \quad 6$

ω_0 = G.G.D. (4, 6) = 2

$$=\sum_{k=-\infty}^{\infty} c_k e^{jk\omega ot}=\sum_{k=-\infty}^{\infty} c_k e^{j2kt}$$

$$x(t)=\frac{1}{2}\left(e^{-j4t}+e^{-j4t}\right)+\frac{1}{2j}\left(e^{j6t}-e^{-j6t}\right)$$

$$=\frac{1}{2j}e^{-j6t}+e^{-j4t}+\frac{1}{2}e^{j4t}+\frac{1}{2j}e^{j6t}$$

$$=\sum_{k=-\infty}^{\infty} c_k e^{j2kt}$$

$$c_{-3}=-\frac{1}{2j},c_{-2}=\frac{1}{2},c_2=\frac{1}{2},c_{-3}=\frac{1}{2j}$$

and all other $c_k = 0$

e) $x(t)=\sin^2 t \rightarrow \frac{1-\cos 2t}{2}=\frac{1}{2}-\frac{1}{4}\left(e^{+j2t}+e^{-j2t}\right)$

$$=\sum_{n=-\infty}^{\infty} c_n e^{j\omega ot}=\sum_{k=-\infty}^{\infty} c_n e^{j2nt},\omega_o=2$$

$$C_0=\frac{1}{2},C_1=-\frac{1}{4},C_{-1}=\frac{-1}{4}$$

f) $x(t)=2+\cos\frac{2\pi t}{3}+4\sin\frac{5\pi t}{3}$

$$=\sum_{n=-\infty}^{\infty} c_n e^{jn\omega ot}$$

$$C_o=2,=\sum_{n=-\infty}^{\infty} C_k e^{jn}\frac{\pi}{3}t$$

$$=\frac{e^{jn\frac{\pi}{3}}+e^{-2\frac{\pi}{3}j}}{2}+\frac{4}{2j}e^{j\frac{2\pi}{3}t}-e^{-j\frac{5\pi}{3}t}$$

$$=\frac{e^{j\frac{\pi}{3}\times 2}+e^{j\frac{\pi}{3}\times 2}}{2}+\frac{2}{j}\left[e^{+j\frac{\pi}{3}\times 5}+e^{-j\frac{\pi}{3}\times 5}\right]$$

$C_0 = 2$, $C_2 = \frac{1}{2}$, $C_{-2} = \frac{1}{2}$, $C_5 = -2j$, $C_{-5} = 2j$

3.4 FOURIER TRANSFORM

FOURIER TRANSFORM PAIR

The function $X(\omega)$ defined by Equation is called the Fourier transform of x(t), and equation defines the inverse Fourier transform of $X(\omega)$. Symbolically they are denoted by

$$X(\omega)=F\{x(t)\}=\int_{\infty}^{\infty} x(t)e^{-j\omega}dt$$

$$x(t)=F^{-1}\{X(\omega)\}=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

and we say that x(t) and $X(\omega)$ form a Fourier transform pair denoted by

$x(t) \leftrightarrow X(\omega)$

$$X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \qquad \text{Fourier Transform}$$

$$X(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{-j\omega t}d\omega \qquad \text{Inverse Fourier Transform}$$

$x(t) \leftrightarrow X(\omega)$

$$x(f)=\int_{-\infty}^{\infty} x(f)e^{-j\omega ft}df$$

$$x(t)=\int_{-\infty}^{\infty} x(f)e^{-j\omega ft}df$$

FOURIER SPECTRA

The Fourier transform $X(\omega)$ of x(t) is, in general, complex, and it can be expressed as $X(\omega)=|X(\omega)|e^{j\varphi(\omega)}$

By analogy with the terminology used for the complex Fourier coefficients of a periodic signal x(t), the Fourier transform $X(\omega)$ of a non-periodic signal x(t) is the frequency domain specification of x(t) and is referred to as the spectrum (or Fourier spectrum) of x(t). The quantity $|X(\omega)|$ is called the magnitude spectrum of x(t), and $\varphi(\omega)$ is called the phase spectrum of x(t).

If x(t) is a real signal, then from above Eq. we get

$X(-\omega)=\int_{-\infty}^{\infty}x(t)e^{j\omega t}dt$

Then it follows that

$X(-\omega)=X^*(\omega)$

$|X(-\omega)|=|X(\omega)|$ even function of ω

$\varphi(-\omega)=-\varphi(\omega)$ odd function of ω

Hence, as in the case of periodic signals, the amplitude spectrum $|X(\omega)|$ is an even function and the phase spectrum $\varphi(\omega)$ is an odd function of ω.

CONVERGENCE OF FOURIER TRANSFORMS

Just as in the case of periodic signals, the sufficient conditions for the convergence of X(ω) are the following (again referred to as the Dirichlet's conditions):

1. x(t) is absolutely integrable, that is,

   $$\int_{-\infty}^{\infty}|f(t)|\,dt<\infty$$

2. x(t) has a finite number of maxima and minima within any finite interval.
3. x(t) has a finite number of discontinuities within any finite interval, and each of these discontinuities is finite.

Although the above Dirichlet's conditions guarantee the existence of the Fourier transform for a signal, if impulse functions are permitted in the transform, signals which do not satisfy these conditions can have Fourier transforms.

→ **Fourier Transform is defined for stable and energy signals.**

→ **Area under signal is observed :-**

$X(0)=\int_{-\infty}^{\infty}x(t)\,dt$ spectrum at zero, near (ω= 0)

→ **Area under Spectrum is observed :-**

Or $x(0)=\int_{-\infty}^{\infty}X(\omega)\,d\omega$ signal at (t = 0), value x

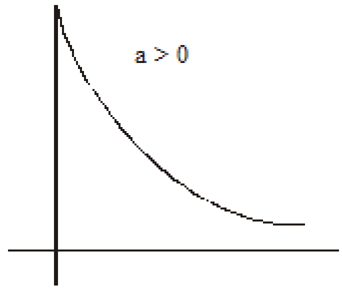
→ The physical existence of a signal is a sufficient condition for the existence of its transform.

3.5FOURIER TRANSFORM OF STANDARD SIGNAL

Example

$x(t) = e^{-at}\,u(t)\ \ a > 0$

Solution



$X(\omega)=\int_{-\infty}^{\infty}e^{-at}\,u(t)\,e^{-j\omega t}dt$

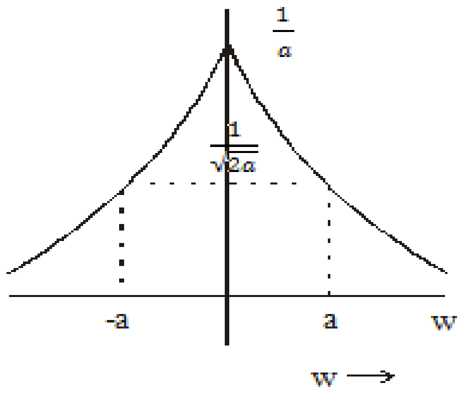
$X(\omega)=\int_{0}^{\infty}e^{at}e^{-j\omega t}dt=\int_{0}^{\infty}e^{-t(a+j\omega)}dt$

$=\frac{e^{-(a+j\omega)}}{-(a-j\omega)}$

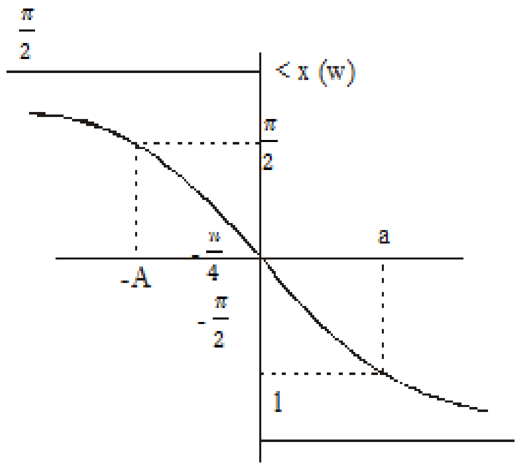
$X(\omega)=\frac{E}{2}$

$e^{-at}u(t)\leftrightarrow\frac{1}{a+j\omega}$ Fourier Transform

$|X(\omega)|=\frac{1}{\sqrt{a^2+\omega^2}},\ <X\,\omega=-\tan^{-1}\left(\frac{\omega}{a}\right)$



Even function of ω



Odd function of ω

Conjugate Symmetry Property

If g(t) is real function of t, then G(ω) and G(-ω) and complex conjugates, that is G(-ω) = G*(ω)

Therefore $|G(-\omega)| = |G^*(\omega)|$

$\theta_g(\omega) = -\theta_g(\omega)$

Thus for real g(t), the amplitude spectrum $|G(\omega)|$ is even function and $\theta_g(\omega)$ phase section is an odd function of ω

3.6 PROPERTIES OF THE CONTINUOUS - TIME FOURIER TRANSFORM

Basic properties of the Fourier transform are presented in the following. Many of these properties are similar to those the Laplace transform

A. Linearity

$$a_1x_1(t)+a_2x_2(t)\leftrightarrow a_1X_2(\omega)+a_2X_2(\omega)$$

B. Time Shifting

$$x(t-t_0)\leftrightarrow e^{-j\omega t_v}X(\omega)$$

Equation shows that the effect of a shift in the time domain is simply to add a linear term – ωt_0 to the original phase spectrum θ (ω). This is known as a linear phase shift of the Fourier transform X(ω).

C. Frequency Shifting

$$e^{j\omega_0 t}x(t)\leftrightarrow X(\omega-\omega_0)$$

The multiplication of x(t) by a complex exponential signal $e^{j\omega_0 t}$ is sometimes called complex modulation. Thus, Equation shows that complex modulation in the time domain corresponds to a shift of X(ω) in the frequency domain. Note that the frequency-shifting property Equation is the dual of the time-shifting property Equation.

D. Time Scaling:

$$x(at)\leftrightarrow\frac{1}{|a|}X\left(\frac{\omega}{a}\right)$$

Where a is a real constant. This property follows directly from the definition of the Fourier transform. Equation indicates that scaling the time variable t by the factor a causes and inverse scaling of the frequency variable ω by 1/a, as well as an amplitude scaling of X(ω/a) by 1/|a|. Thus, the scaling property implies that time compression of a signal (a>1) results in its spectral expansion and that time expansion of the signal (a<1) results in its spectral compression.

E. Time Reversal

$$x(-t)\leftrightarrow X(-\omega)$$

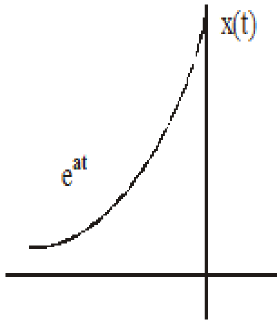
Thus, time reversal of x(t) produces a like reversal of the frequency axis for X(ω). Equation is readily obtained by setting a = -1 in Equation

Example : Find Fourier Transform of x(t)?,
$x(t) = e^{at}u(-t)$, a > 0

Solution: x(t) ↔ X(ω)

x(-t) ↔ X(-ω)

$$\therefore X(\omega)=\frac{1}{a-j\omega}$$



In Fourier Transform property only phase spectrum charge.

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega ft}dt$$

$$\int_{-\infty}^{\infty} x(-t)\, e^{j\omega t}\, dt$$

if x(t) real

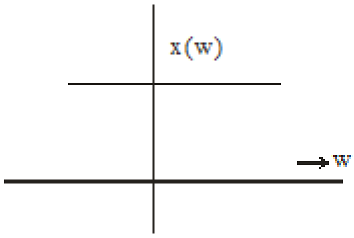
$X(-\omega) = x(-t)$

Example: Find Fourier Transform of x(t) ?, $x(t) = \delta(t)$

Solution:

$$X(\omega) = \int_{0}^{2T} \delta(t)e^{-j\omega t}dt = 1$$

$X(\omega) = 1$

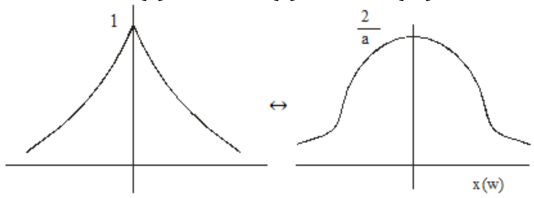


Impulse in time domain because constant in frequency Domain.

ANY TRANFORM OF IMPULSE IS ONE

Example : Find Fourier Transform of x(t) ?, $x(t) = e^{-a|t|}$, $a > 0$

Solution: $x(t) = e^{-at}u(t) + e^{at}u(-t)$



Even

$$X(\omega) = \frac{1}{a + j^{\omega}} + \frac{1}{a - j^{\omega}}$$

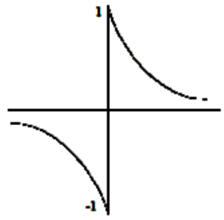
$$X(\omega) = \frac{2a}{a^2 + \omega^2} |X\ (\omega)|, <\ X\ (\omega) = 0$$

Example: Find Fourier Transform of x(t) ?, $x(t) = e^{-at}u(t) - e^{-at}u(-t)$

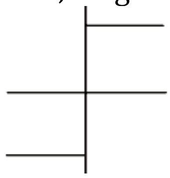
Solution:

$$x(\omega) = \frac{1}{a + j^{\omega}} - \frac{1}{a - j^{\omega}}$$

$$= \frac{-j\omega}{a^2 + \omega^2} = \frac{-2j\omega}{a^2 + \omega^2}$$



For signam function, magnitude spectrum =



$$\mathrm{Sgn}(t) = \mathrm{Lim}\ x(t) = \mathrm{Lim}\ e^{-at}u(t) - \mathrm{Lim}\ e^{-at}u(-t)$$

$a \to 0 \qquad a \to 0 \qquad a \to 0$

$= [u(t) - u(-t)] = \mathrm{Sgn}(t)$

$$\mathrm{Lim}\ X(\omega) = \mathrm{Lim} \frac{-j\omega}{a^2 + \omega^2}$$

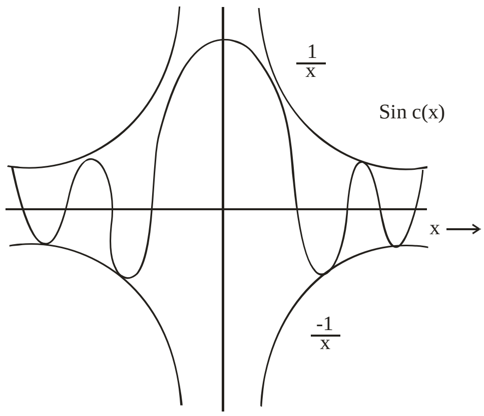
$a \to 0 \qquad a \to 0$

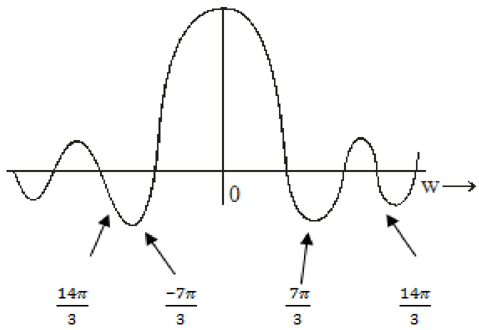
$$\mathrm{Sgn}(t) = \frac{-2j\omega}{\omega^2} = \frac{2}{j^{\omega}}$$

Important points to be remember :

1) sin c (x) is an even function of x
2) sin c (x)=0, for x = ± nπ, n = 1, 2, 3, 4, 5
3) Using L – hospital rule, Sin c (0) = 1
4) sin cx is product of or osc[r] Signal sin x (T=2π) and monotonically decreasing function $\frac{1}{x}$

Therefore sin c(x) exhibits sin. osc[r] or period 2π with amplitude decreasing continuously as $\frac{1}{x}$

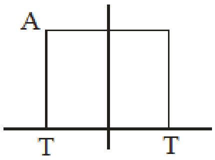




Example: Find Fourier Transform of x(t) ?,

$x(t) = A\text{rect}\left(\frac{t}{2T}\right)$

Solution:



$X(\omega) = \int_{-T}^{T} Ae^{-j\omega t}dt$

$= A\left(\frac{e^{-j\omega t}}{-j\omega}\right)$

$= A\left(\frac{e^{-j\omega T/2} - e^{j\omega T/2}}{j\omega}\right)$

$= \frac{2A}{\omega}\sin(\omega T) = 2TA\frac{\sin\omega T}{\omega T}$

$= 2TA\sin c(\omega T)$

I. Practical BW of sin c(t) is $\frac{2\pi}{T}$

II. Both positive and negative ω be together called as spectral width = $\frac{4\pi}{T}$

III. Null to Null BW is $\frac{2\pi}{T}$ (zero crossing BW)

IV. A signal can't be time limited and frequency band limited simultaneously.

Example: Inverse Fourier transform of $X(\omega) = \delta(\omega)$

Solution :

Inverse Fourier transform =

$\delta(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(\omega)\, e^{j\omega t}dt = \frac{1}{2\pi}$

$\frac{1}{2\pi} \leftrightarrow \delta(\omega)$

$1 \leftrightarrow 2\pi\delta(\omega)$

Spectrum of const signal g(t) = 1 is an impulse $2\pi\delta(\omega)$ i.e. g(t)=1, is a d.c. signal which has a single frequency ω = 0

Example: Find Fourier Transform of x(t) ?,

a) $X(t) = e^{j\omega ot}$

$1 \leftrightarrow 2\pi\delta(\omega)$

$e^{j\omega ot} \leftrightarrow 2\pi\delta(\omega - \omega_o)$

b) $e^{-j\omega ot}$

$e^{-j\omega ot} \leftrightarrow 2\pi\delta(\omega - \omega_o)$

c) $x(t) = \cos\omega_o t = \frac{e^{j\omega ot} - e^{-j\omega ot}}{2j}$

$\leftrightarrow \frac{1}{2j}\left[2\pi\delta(\omega - \omega_o) - 2\pi\delta(\omega + \omega_o)\right]$

$\leftrightarrow -j\left[\pi\delta(\omega - \omega_o) - \pi\delta(\omega + \omega_o)\right]$

$\leftrightarrow -j\left[\pi\,\delta(\omega - \omega_o) - \pi\delta(\omega + \omega_o)\right]$

$\sin\omega_o t \leftrightarrow -j\pi\left[\delta(\omega - \omega_o) - \delta(\omega + \omega_o)\right]$

Example Find Fourier Transform of following signal if f(t) ↔ F(ω) ?

1) $f\left(3(t) + \frac{7}{3}\right) = f(3t + 7)$

2) f(3t - 7)

3) f(-3t - 7)

4) f(-3t + 7)

Solution :

$f(t) \leftrightarrow F(\omega)$

$f(3t) \leftrightarrow \frac{1}{|3|}f\left(\frac{\omega}{3}\right)$

$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}dt$

1) $F(\omega) = \int_{-\infty}^{\infty} f(3t+7)e^{-j\omega}\left(\frac{P-7}{3}\right)dt$

$3t + 7 = P \rightarrow t = \frac{P-7}{3}$

$3dt = dp \Rightarrow dt = \frac{dp}{3}$

$$= \int_{-\infty}^{\infty} f(P)e^{-j\omega\left(\frac{P-7}{3}\right)}\frac{dp}{3}$$

$$= \int_{-\infty}^{\infty} f(P)e^{-j\frac{\omega P}{3}}e^{+j\frac{\omega 7}{3}}\frac{dp}{3}$$

$$= e^{+j\frac{\omega 7}{3}}\int_{-\infty}^{\infty} f(P)e^{-j\frac{\omega P}{3}}\frac{dp}{3}$$

$$= e^{+j\frac{\omega 7}{3}}\int_{-\infty}^{\infty} f(P)e^{-jP}\left(\frac{\omega}{3}\right)dp$$

$$= \frac{e^{+j\frac{\omega 7}{3}}}{3}F\left(\frac{\omega}{3}\right)$$

2) $F(\omega) = f(3t+7)e^{-j\omega t}dt$

$$3t-1 = P \rightarrow t = \frac{P+7}{3}$$

$$3dt = dp \Rightarrow dt = \frac{dp}{3}$$

$$= \int_{-\infty}^{\infty} f(P)e^{-j\omega\left(\frac{P+7}{3}\right)}\frac{dp}{3}$$

$$= \frac{e^{-j\frac{\omega 7}{3}}}{3}\int_{-\infty}^{\infty} f(P)e^{-j\left(\frac{\omega}{3}\right)}dp$$

$$= \frac{1}{3}F\left(\frac{\omega}{3}\right)e^{-j\left(\frac{7\omega}{3}\right)}$$

3) $F(\omega) = \int_{-\infty}^{\infty} f(-3t-7)e^{-j\omega t}dt$

$$-3t-7 = P \rightarrow t = \frac{-7-p}{3}$$

$$-3dt = dp \Rightarrow dt = -\frac{dp}{3}$$

$$= \int_{-\infty}^{\infty} f(p)e^{+j\omega\left(\frac{7+p}{3}\right)}\frac{dp}{3}$$

$$= \frac{-e^{-j\frac{7}{3}\omega}}{3}\int_{-\infty}^{\infty} f(p)e^{-j\left(\frac{\omega}{3}\right)p}dp$$

$$= \frac{e^{j\frac{7}{3}\omega}}{3}\int_{-\infty}^{\infty} f(p)e^{-j\left(-\frac{\omega}{3}\right)p}dp$$

$$= \frac{1}{3}e^{j\frac{7}{3}\omega}F\left(\frac{-\omega}{3}\right)$$

4) $F(\omega) = \int_{-\infty}^{\infty} f(-3t+7)e^{-j\omega t}dt$

$$-3t+7 = P \rightarrow t = \frac{7-p}{3}$$

$$-dt = \frac{dp}{3}$$

$$= -\int_{-\infty}^{\infty} f(p)e^{-j\omega\left(\frac{7-p}{3}\right)}\frac{dp}{3}$$

$$= e^{j\frac{7}{3}\omega}\int_{-\infty}^{\infty} f(p)e^{+j\left(\frac{\omega}{3}\right)p}\frac{dp}{3}$$

$$= \frac{e^{-j\frac{7}{3}\omega}}{3}\int_{-\infty}^{\infty} f(p)e^{-j\left(-\frac{\omega}{3}\right)p}dp$$

F. Duality (or Symmetry):

The duality property of the Fourier transform has significant implications. This property allows us to obtain both of these dual Fourier transform pairs from one evaluation of Equation.

$X(t) \leftrightarrow 2\pi\, x(-\omega)$

Example:

Find Fourier Transform of $x(t) = \frac{2}{\pi t}$?

Solution: $\text{Sgn}(t) \leftrightarrow = \frac{2}{j\omega}$

$$\frac{2}{j\omega} \leftrightarrow 2\pi\ \text{Sgn}(-\omega)$$

$$\frac{2}{\pi t} \leftrightarrow -j\ \text{Sgn}(\omega)$$

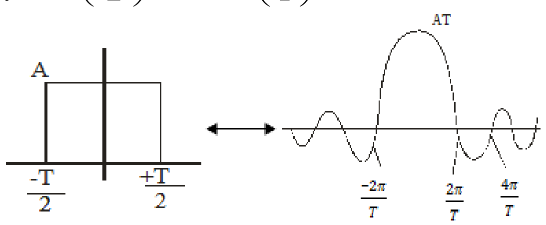
Example : Find Fourier Transform of

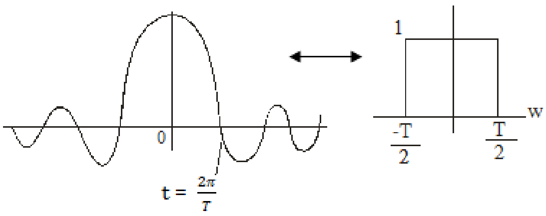
$$x(t) = \frac{T}{2\pi}\sin c\left(\frac{tT}{2}\right)?$$

Solution: $A\ \text{rect}\ \frac{T}{2\pi} \leftrightarrow AT\ \text{Sin c}\left(\frac{\omega T}{2}\right)$

$$AT\sin c\left(\frac{tT}{2}\right) \leftrightarrow 2\pi\ A\ \text{rect}\left(\frac{-\omega}{T}\right)$$

$$\frac{T}{2\pi}\sin c\left(\frac{tT}{2}\right) \leftrightarrow \text{rect}\left(\frac{\omega}{T}\right)$$





Example

Find Fourier Transform of $x(t)=\frac{\sin at}{\pi t}$?

Solution:

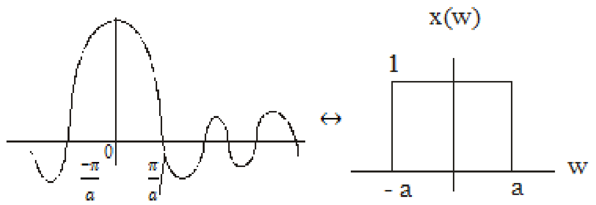
$$\frac{\sin at}{\pi t} \qquad A\,\text{rect}\left(\frac{t}{T}\right) \leftrightarrow \frac{AT\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

$$a=\frac{T}{2} \qquad A\,\text{rect}\left(\frac{t}{T}\right) \leftrightarrow \frac{2\sin\frac{\omega T}{2}}{\omega}$$

$$\frac{2.\sin ta}{t} \leftrightarrow 2\pi\ \text{rect}\left(\frac{-\omega}{2a}\right)$$

$$\frac{\sin at}{\pi t} \leftrightarrow \text{rect}\left(\frac{\omega}{2a}\right)$$

$$\frac{a}{\pi}\times\frac{\sin at}{at} \leftrightarrow \text{rect}\left(\frac{\omega}{2a}\right)$$

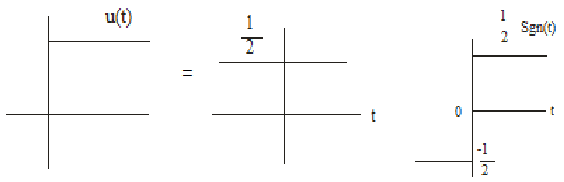


Example: Find Fourier Transform of $x(t)=u(t)$?

Solution: $u(t) = x_e(t) + x_o(t)=\frac{1}{2}+\frac{1}{2}\text{sgn}(t)$

Fourier Transform $u(t)=\frac{2\pi(\omega)}{2}+\frac{1}{2}\times\frac{2}{j\omega}$

$$=\pi\delta(\omega)+\frac{1}{j\omega}$$



Example

Find Fourier Transform of $x(t)=u(-t)$?

Solution: $u(-t)\to u(t)\leftrightarrow \pi\delta(\omega)+\frac{1}{j\omega}$

$$u(-t)\leftrightarrow \pi\delta(-\omega)-\frac{1}{j\omega}$$

$$u(-t)\ \leftrightarrow\ \pi\delta\pi(\omega)-\frac{1}{j\omega}$$

G. Differentiation in the Time Domain

$$\frac{dx(t)}{dt}\leftrightarrow j\omega X(\omega)$$

Equation shows that the effect of differentiation in the time domain is the multiplication of $X(\omega)$ by $j\omega$ in the frequency domain.

H. Differentiation in the Frequency Domain:

$$(-jt)x(t)\leftrightarrow\frac{dX(\omega)}{d\omega}$$

Equation is the dual property of Equation.

I. Integration in the Time Domain:

$$\int_{-\infty}^{t}x(\tau)d\tau\leftrightarrow\pi X(0)\delta(\omega)+\frac{1}{j\omega}X(\omega)$$

Since integration is the inverse of differentiation, Equation shows that the frequency-domainoperation corresponding to time domain integration is multiplication by $1/j\omega$, but an additional term is needed to account for a possible dc component in the integrator output. Hence, unless $X(0) = 0$, a dc component is produced by the integrator.

J. Convolutio

$$x_1(t)*x_2(t)\leftrightarrow X_1(\omega)X_2(\omega)$$

Equation is referred to as the time convolution theorem, and it states that convolution in the time domain becomes multiplication in the frequency domain. As in the case of the Laplace transform, this convolution property plays an important role in the study of continuous-time LTI systems and also forms the basis for our discussion of filtering.

K. Multiplication

$$x_1(t)x_2(t) \leftrightarrow \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$$

The multiplication property is the dual property of Equation and is often referred to as the frequency convolution theorem. Thus, multiplication in the time domain becomes convolution in the frequency domain.

L. Additional Properties:

If x(t) is real, let

$$x(t) = x_e(t) + x_0(t)$$

Where $x_e(t)$ and $x_o(t)$ are the even odd components of x(t), respectively. Let

$$x_1(t) \leftrightarrow X(\omega) \leftrightarrow A(\omega) + jB(\omega)$$

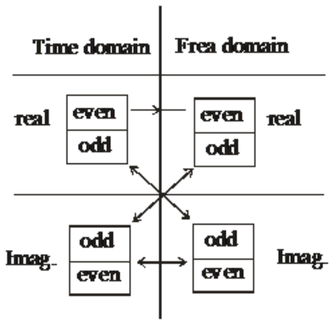
Then

$$X(-\omega) = X^*(\omega)$$

$$x_e(t) \leftrightarrow \text{Re}\{X(\omega)\} = A(\omega)$$

$$x_o(t) \leftrightarrow j\,\text{Im}\{X(\omega)\} = jB(\omega)$$

Equation is the necessary and sufficient condition for x(t) to be real Equations and show that the Fourier transform of an odd signal is a pure imaginary function of ω.



M. Parseval's Relations:

$$\int_{-\infty}^{\infty} x_1(\lambda)X_2(\lambda)d\lambda = \int_{-\infty}^{\infty} X_1(\lambda)x_2(\lambda)d\lambda$$

$$\int_{-\infty}^{\infty} x_1(t)x_2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\lambda)X_2(-\omega)d\omega$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega| X_1(\lambda)X_2(-\omega)d\omega$$

Equation is called Parseval's identity (or Parseval's theorem) for the Fourier transform. Note that the quantity on the left-hand side of Eq. is the normalized energy content E of x(t) Parseval's identity says that this energy content E can be computed by intergrating $|X(\omega)|^2$ over all frequencies ω. For this reason $|X(\omega)^2|$ is often referred to as the energy-density spectrum of x(t), and Eq. is also known as the energy theorem.

Parseval's Relations: - $x(t) \leftrightarrow X(\omega)$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |x(\omega)|^2 d\omega \, |X(\omega)|^2 =$$

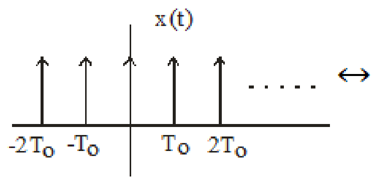
ESD (Energy Spectral Density)

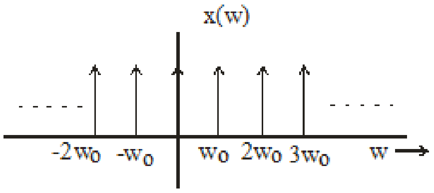
3.7 FOURIER TRANSFORM OF PERIODIC SIGNAL :

Periodic Signal is not absolute integrable its Fourier Transform cannot be determine directly. It can be approximated from Exponential. Fourier series representation.

Example : Find the Fourier Transform of a periodic signal x(t) with period To

Solution:





$$x(t) = \sum_{h=-\infty}^{\infty} c_n e^{-jn\omega o t}, \omega_o = \frac{2\pi}{T_0}$$

$$X(\omega) = 2\pi \sum_{h=-\infty}^{\infty} c\,\delta n(\omega - n\omega o)$$

The Fourier Transform of a periodic signal consists of a sequence of equidistant impulses located at the harmonic frequencies of the signal.

Example: Find the Fourier Transform of periodic impulse train

$$\delta_{To}(t) = \sum_{h=-\infty}^{\infty} \delta\ (t - nT_o)$$

$$\delta_{To}(t) = \frac{1}{T_0} \sum_{h=-\infty}^{\infty} e^{jn\omega ot}$$

$$C_k = \frac{1}{T_0} \int \delta(t)\ e^{-jn\omega ot}\ dt = \frac{1}{T_0}$$

$$F\left[\delta_{To}(t)\right] = \frac{2\pi}{T_0} \sum_{h=-\infty}^{\infty} \delta\ (\omega - n\omega_o)$$

$$\delta_{To}(t) \leftrightarrow \omega_o \sum_{h=-\infty}^{\infty} \delta\ (\omega - n\omega_o)$$

Thus, the Fourier Transform of a unit impulse pulse train is also similar impulse train.

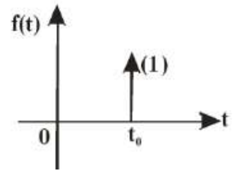
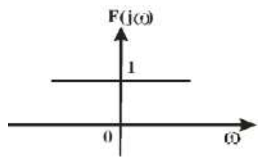
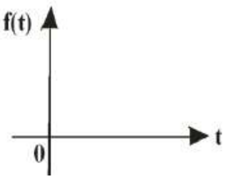
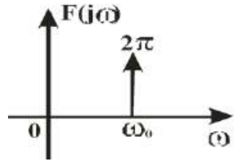
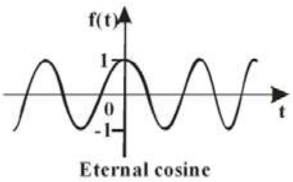
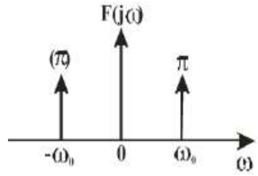
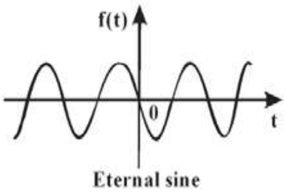
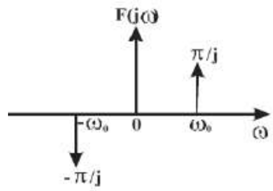
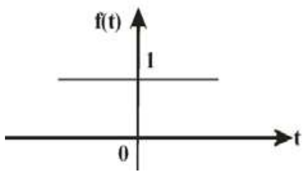
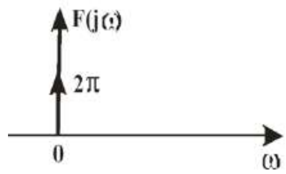
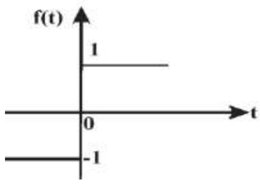
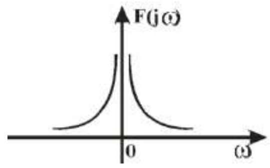
PROPERTIES OF FOURIER TRANSFORM

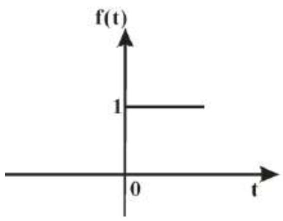
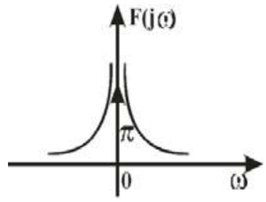
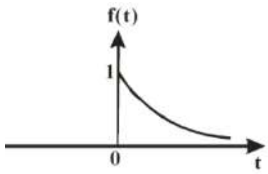
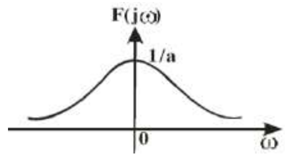
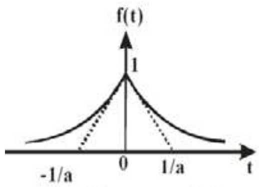
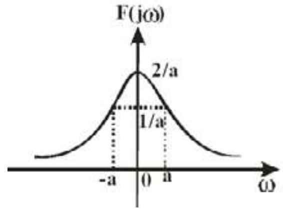
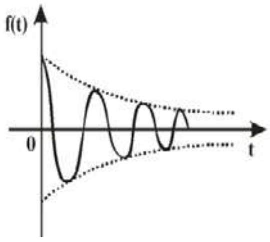
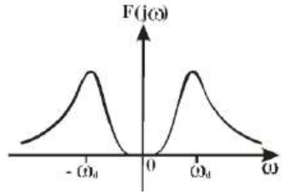
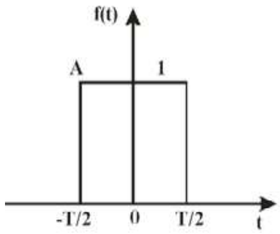
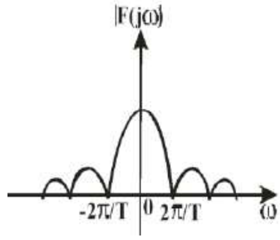
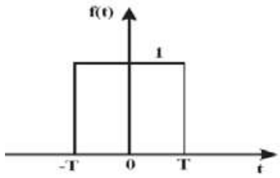
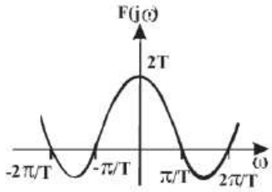
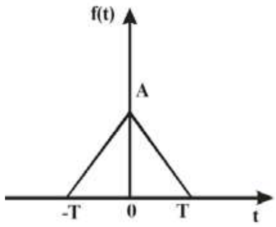
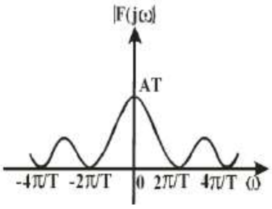
Table 3.1 Important properties of the Fourier Transform

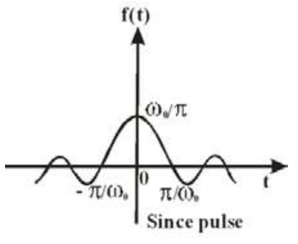
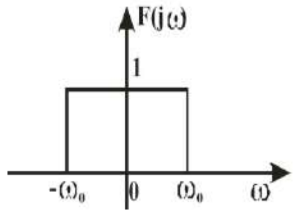
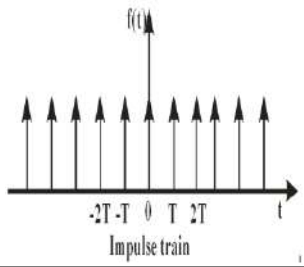
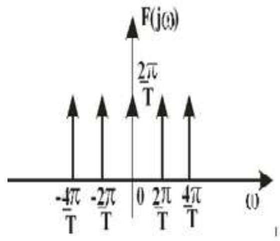
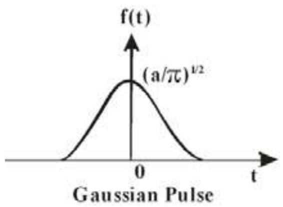
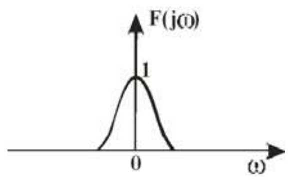
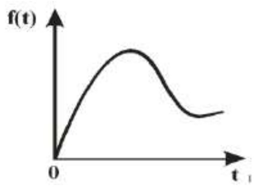
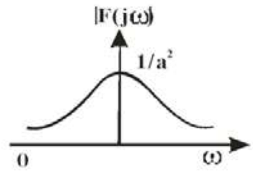
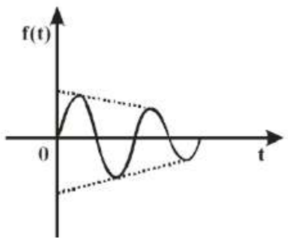
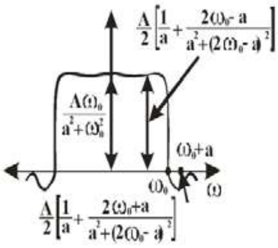
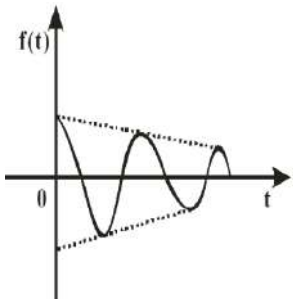
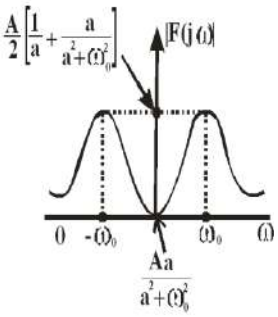
Operation	**f(t)**	**F(jω)**
Transform	$f(t)$	$\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$
Inverse transform	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{-j\omega t}d\omega$	$F(j\omega)$
Linearity	$af_1(t) + bf_2(t)$	$aF_1(j\omega) + bF_2(j\omega)$
Time-reversal	$f(-t)$	$F(-j\omega) = F^*(j\omega), f(t)\ \text{real}$
Time-shifting (Delay)	$f(t - t_0)$	$F(j\omega)e^{-j\omega t_0}$
Time-Sealing	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{j\omega}{a}\right)$
Time-differentiation	$\frac{d^n}{dt^n}f(t)$	$(j\omega)^n F(j\omega)$
Frequency-differentiation	$(-jt)f(t)$	$\frac{dF(j\omega)}{d\omega}$
Time-integration	$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{1}{j\omega}F(j\omega) + \pi F(0)\delta(\omega)$
Frequency-integration	$\frac{1}{(-jt)}f(t)$	$\int^{\omega} f(j\omega')d\omega'$
Time convolution	$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)d\tau$	$F_1(j\omega)F_2(j\omega)$
Frequency convolution (Modulation)	$f(t)e^{j\omega_0 t}$	$F(j\omega - j\omega_0)$
Symmetry	$F(jt)$	$2\pi f(-\omega)$

Real-time function	$f(t)$	$F(j\omega)=F^*(-j\omega)$ $\mathrm{Re}[F(j\omega)]=\mathrm{Re}[F(-j\omega)]$ $\mathrm{Im}[F(j\omega)]=-\mathrm{Im}[F(-j\omega)]$ $\lvert F(j\omega)\rvert=\lvert F(-j\omega)\rvert$ $\varphi f(j\omega)=-\varphi f(j\omega)$
Parseval\`s theorem	$E=\int_{-\infty}^{\infty}\lvert f(t)\rvert^2\,dt$	$E=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert F(j\omega)\rvert^2\,d\omega$
Duality	If $f(t)\Leftrightarrow g(j\omega)$, then $g(t)\Leftrightarrow 2\pi f(-j\omega)$	

Table 3.2 Fourier Transform of some important signals

Sl.	Time domain f(t)		Frequency domain F (jω)	
1.		$\delta(t-t_0)$	$e^{-j\omega t_0}$	
2.		$e^{-j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
3.		$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	
4.		$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	
5.		1	$2\pi\delta(\omega)$	
6.		$f(t)=\mathrm{sgn}(t)=\frac{t}{\lvert t\rvert}$	$\frac{2}{j\omega}$	

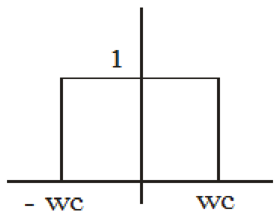
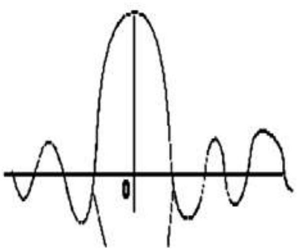
7.		$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
8.		$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	
9.	 Double exponential	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	
10		$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	
11		$A\left\{u\left(t+\dfrac{T}{2}\right)-u\left(t-\dfrac{T}{2}\right)\right\}$	$AT\,\text{Sinc}\left(\dfrac{\omega T}{2}\right)$ $T\dfrac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$	
12		$f(t)=1, \text{for} \|t\|\le T$ $=0, \text{otherwise}$ $f(t)=u(t+T)-u(t-T)$	$2T\dfrac{\sin(\omega T)}{\omega T}$ $=2T\sin c(\omega T)$	
13		$f(t)=A\left[1-\dfrac{\|t\|}{T}\right], \|t\|\le T$ $=\text{elsewhere}$	$AT\left[\dfrac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}\right]^2$ $=AT\sin c^2\left(\dfrac{\omega T}{2}\right)$	

14		$\frac{\omega_0}{\pi}\sin c\left(\frac{\omega_0 t}{\pi}\right)$	$u(\omega+\omega_0)-u(\omega-\omega_0)$	
15		$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi n}{T}\right)$	
16		$\left(\frac{a}{\pi}\right)^{\frac{1}{2}}\exp\left(-at^2\right)$	$\exp\left(\frac{-\pi f^2}{a}\right)$	
17		$t\exp(-at)u(t)$	$\frac{1}{(a+j\omega)^2}$	
18		$\frac{t^{n-1}}{(n-1)!}\exp(-at)u(t)$	$\frac{1}{(a+j\omega)^n}$	
19		$A\exp(-at)\sin(\omega_0 t)u(t)$	$\frac{A\omega_0}{(a+j\omega)^2+\omega_0^2}$	
20		$A\exp(-at)\cos(\omega_0 t)u(t)$	$A.\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	
21		$\cos\omega_0[u(t+T)-u(t-T)]$		

			$T\left[\frac{\sin(\omega-\omega_0)T}{(\omega-\omega_0)T}+\frac{\sin(\omega+\omega_0)T}{(\omega+\omega_0)T}\right]$	

Band Limited Signal:- A signal x(t) is called a band limited signal if $|x(\omega)| = 0, |\omega| > \omega m$

Ideal LPF :-

	$H(\omega)=h(t)\leftrightarrow H(\omega)$	$\begin{cases} 1 & \lvert\omega\rvert < \omega_c \\ 0 & \lvert\omega\rvert > \omega_c \end{cases} \leftrightarrow \text{rect}\left(\frac{\omega}{2\omega c}\right)$ $\text{rect}\left(\frac{t}{t}\right)\leftrightarrow T\sin c\left(\frac{\omega T}{2}\right)$ $\leftrightarrow \frac{T\sin\frac{\omega T}{2}}{\omega\frac{T}{2}}$
	$h(t)=\frac{\sin\omega t}{\pi t}=$	$\text{rect}\left(\frac{t}{t}\right)\leftrightarrow\frac{2\sin\frac{\omega T}{2}}{\omega}$ $\frac{2\sin\frac{\omega T}{2}}{\omega}\leftrightarrow 2\pi\ \text{rect}\left(\frac{-\omega}{T}\right)$ $\frac{\sin\omega ct}{\pi t}\leftrightarrow \text{rect}\left(\frac{\omega}{2\omega c}\right)$ Impulse response of ideal low pass filter

3.8 RAYLEIGH'S ENERGY THEOREM OR PARSEVAL'S POWER THEOREM

$x(t)\leftrightarrow X(\omega)$

$$\int_{-\infty}^{\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|x(\omega)|^2d\omega$$

Spectral Density

ESD or PSD

if x(t) is energy signal → ESD

if x(t) is power signal → PSD

Application :-

i) Frequency content in energy spectrum.

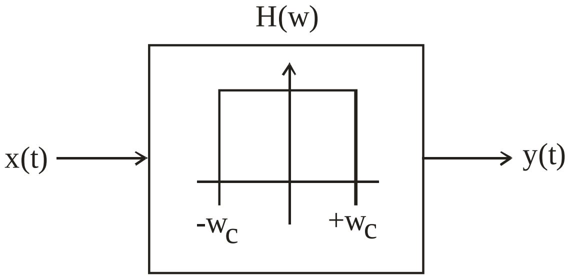
ii) Auto correlation & cross correlation.

Adaptive filter:-

Impulse response changes for every instant.

Example: For a given signal $x(t) = e^{-2t}u(t)$ then it pass through a low pass filter (LPF), h(t). Output signal energy $E_y = \frac{1}{8}$ then calculate cutoff frequency for low pass filter (LPF). $\omega_c = ?$

Solution : $x(t) = e^{-2t}u(t)$



$$x(\omega) = \frac{1}{2 + j\omega}$$

$$y(\omega) = X(\omega) . H(\omega)$$

$$y(\omega) = \frac{1}{j\omega + 2} 1$$

$$-\omega_c < \omega < \omega_c$$

$$E_x(t) = \int_{-\infty}^{\infty} e^{-ut}dt = \left[\frac{e^{-4t}}{-4}\right]_0^{\infty} = \frac{1}{4}$$

$$\therefore E_y(t) = \frac{1}{8}$$

Applying Rayleigh's theorem

$$E_y(t) = \int_{-\infty}^{\infty} |y(\omega)|^2 d\omega$$

$$\frac{1}{8} = \frac{1}{2\pi}\int_{-\omega c}^{\omega c} \frac{1}{(2 + j\omega)2} d\omega$$

$$\frac{2\pi}{8} = 2\int_0^{\omega c} \frac{1}{u + \omega^2} d\omega$$

$$\frac{2\pi}{8} = 2\int_0^{\omega} \frac{1}{u + \omega^2} d\omega$$

$$\frac{\pi}{4} = 2\left[\frac{1}{2}\tan-1(\frac{\omega}{2})\right]_0^{\omega c}$$

$$\frac{\pi}{4} = \tan^{-1}\left(\frac{\omega_c}{2}\right)$$

$$1 = \frac{\omega_c}{2}, \quad \omega_c = 2$$

3.9 CORRELATION

Comparing the original signal with its shifted version.

i) Cross Correlation → Both are different signal

ii) Auto Correlation → Both are same signal

Cross Correlation

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y(t + \tau)\, dt$$

Advanced by Z. t ↑ τ →p by change of variable

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t - \tau)\, y(t)\, dt$$

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} y(t)\, x(t + \tau)\, dt$$

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y(t + \tau)\, dt$$

Auto Correlation Function :- The correlation of a signal with itself is called auto correlation.

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t + \tau)\, dt$$

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t - \tau)\, x(t)\, dt$$

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t \pm \tau)\, dt$$

Auto Correlation Function of Energy Signal

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t - \tau)\, dt$$

$R_x(\tau) = R_x(-\tau)$ if x(t) is real

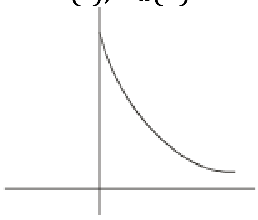
Auto Correlation Function is even function of τ

$F[R_x(\tau)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(t)\, x(t+\tau)\, e^{-j\omega\tau}\, d\tau$

$= x(t) \int_{-\infty}^{\infty} x(t+\tau)\, e^{-j\omega\tau}\, d\tau$

$= X(\omega) \int_{-\infty}^{\infty} x(t)\, e^{-j\omega\tau}\, dt = X(\omega)\, X\text{-}(-\omega)$

$F[R_x(\tau)] = |x(\omega)|^2$ = ESP = Energy per
unit band width $R_x(\tau) = x(\tau) * x(-\tau)$
$x(t) = e^{-j\omega\tau} d(t)$, $R_x(\tau) = ?$



$R_x(\tau) = x(\tau) * x(-\tau)$

$x(\tau) = e^{-a\tau} d(\tau), \quad x(-\tau) = e^{-a\tau} d(-\tau)$

$X(\omega) \leftrightarrow \frac{1}{a+j\omega}, \; X(-\omega) \leftrightarrow \frac{1}{a-j\omega}$

$R_x(\tau) = F^{-1}\left(\frac{1}{a+j\omega} \times \frac{1}{a-j\omega}\right)$

$F^{-1} \frac{1}{a^2+\omega^2} e^{-a|\tau|} \leftrightarrow \frac{2a}{a^2+\omega^2}$

$R_x(\tau) = \frac{e^{-a|\tau|}}{2a}$

$ESD = \frac{1}{a^2+\omega^2}$

GATE QUESTIONS(EC)

Q.1 Which of the following cannot be the Fourier series expansion of a periodic signal?

a) $x(t) = 2\cos t + 3\cos 3t$
b) $x(t) = 2\cos \pi t + 7\cos t$
c) $x(t) = \cos t + 0.5$
d) $x(t) = 2\cos 1.5\pi t + \sin 3.5\pi t$

[GATE -2002]

Q.2 The Fourier series expansion of a real periodic signal with fundamental frequency f_0 is given by $g_p(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_0 t}$ it is given that $c_3 = 3 + j5$ then c_{-3} is

a) $5 + j3$ b) $-3 - j5$
c) $-5 + j3$ d) $3 - j5$

[GATE -2003]

Q.3 Choose the function $f(t)\ -\infty < t < \infty$ for which a Fourier series cannot be defined.

a) $3\sin(25t)$
b) $4\cos(20t + 3) + 2\sin(710t)$
c) $\exp(-|t|)\sin(25t)$
d) 1

[GATE -2005]

Q.4 The Fourier series of a real periodic function has only

P. Cosine terms if it is even
Q Sine terms if it is even
R. Cosine terms if it is odd
S. Sine terms if it odd

Which of the above statement are correct?

a) P and S b) P and R
c) Q and S d) Q and R

[GATE -2009]

Q.5 The trigonometric Fourier series of an even function does not have the

a) dc term
b) Cosine terms
c) Sine terms
d) Odd harmonic terms

[GATE -2011]

Q.6 A periodic signal x(t) has a trigonometric Fourier expansion

$$x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

If $x(t) = -x(t - \pi/\omega_0)$, we can conclude that

a) A_n are zero for all n and b_n are zero for n even
b) A_n are zero for all n and b_n are zero for n odd
c) A_n are zero for n even and b_n are zero for n odd
d) A_n are zero for n odd and b_n are zero for n even

Q.7 Let $x(t)$ be a periodic function with period $T=10$. The Fourier series coefficient for this series are denoted by a_k , that is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

The same function $x(t)$ can also be considered as a periodic function with period $T'=40$. Let b_k be the Fourier series coefficient when period is taken as T'. If $\sum_{k=-\infty}^{\infty}|a_k| = 16$ then $\sum_{k=-\infty}^{\infty}|b_k|$ is equal to

a) 256 b) 64
c) 16 d) 4

[GATE -2018]

ANSWER KEY:

1	2	3	4	5	6	7
B	D	C	A	C	A	C

EXPLANATIONS

Q.1 **(b)**

$x(t) = 2\cos\pi t + 7\cos t$

$T_1 = \frac{2\pi}{\omega} = 2$

$T_2 = \frac{2\pi}{1} = 2\pi$

$\frac{T_1}{T_2} = \frac{1}{\pi} = irrational$

$\therefore x(t)$ is not periodic and does not satisfy Drichidt condition.

Q.2 **(d)**

$c_{-k} = C_k^*$

$c_3 = 3 + j5$

$c_{-3} = C_k^* = 3 - j5$

Q.3 **(c)**

All other functions are either periodic or constant function.

Q.4 **(a)**

The Fourier series of a real periodic function has only cosine terms if it is even and only sine sterms if it is odd.

Q.5 **(c)**

Trigonometric Fourier series of a no even function has dc and cosine terms only.

Q.6 **(a)**

Signal has odd and half wave symmetries. So, all a_n are zero and b_n are zero for n even

Q.7 **(c)**

$x(t)$ is periodic signal with time period $T = 10$ and its Fourier series coefficient is represented as a_k with

$$\sum_{k=-\infty}^{\infty} |a_k| = 16$$

Now, the same functions $x(t)$ can also be considered as periodic function with period $T' = 40$ and Fourier series coefficient is represented as b_k

We know that there is no change in Fourier series coefficient on applying time scaling on any signal $x(t)$.

If $x(t) \rightarrow C_n$

Then $x(\alpha t) \rightarrow C_n$

So, $\sum_{k=-\infty}^{\infty} |a_k| = \sum_{k=-\infty}^{\infty} |b_k| = 16$

GATE QUESTIONS(EC)

Q.1 A linear phase channel with phase delay T_p and group delay T_g must have

a) $T_p = T_g =$ constant

b) $T_p \propto f$ and $T_g \propto f$

c) $T_p =$ constant and $T_g \propto f$

d) $T_p \propto f$ and $T_g =$ constant (f denotes frequency)

[GATE-2002]

Q.2 The Fourier transform $F\{e^{-t}u(t)\}$ is equal to $\frac{1}{1+j2\pi f}$. Therefore $F\left\{\frac{1}{1+j2\pi t}\right\}$ is

a) $e^{f}u(f)$ b) $e^{-f}u(f)$

c) $e^{f}u(-f)$ d) $e^{-f}u(-f)$

[GATE-2002]

Q.3 Let $x(t)$ be the input to a linear, time-invariant system. The required output is $4x(t-2)$. The transfer function of the system should be

a) $4e^{j4\pi f}$ b) $2e^{-j8\pi f}$

c) $4e^{-j4\pi f}$ d) $2e^{j8\pi f}$

[GATE-2003]

Q.4 Let $H(f)$ denote the frequency response of the RC-LPF Let f_1 be the highest frequency such that $0 \le |f| \le f_1; \frac{|H(f_1)|}{H(0)} \ge 0.95$ Then f_1 (in Hz) is

a) 327.8 b) 163.9

c) 52.2 d) 104.4

[GATE-2003]

Q.5 Let $t_g(f)$ be the group delay function of the give RC-LPF and $f_2 = 100Hz$. $t_g(f_2)$ in ms is

a) 0.717 b) 7.17

c) 71.7 d) 4.505

[GATE-2003]

Q.6 The Fourier transform of a conjugate symmetric function is always

a) Imaginary

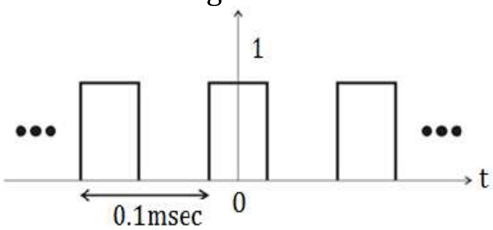
b) Conjugate anti symmetric

c) real

d) Conjugate symmetric

[GATE-2004]

Q.7 A rectangular pulse train s(t) as shown in the figure is convolved with the signal $\cos^2(4\pi \times 10^3 t)$. The convolved signal will be a



a) DC

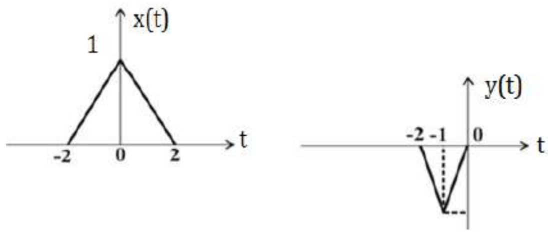
b) 12 KHz sinusoid

c) 8 KHz sinusoid

d) 14 KHz sinusoid

[GATE-2004]

Q.8 Let $x(t)$ and y(t) (with Fourier transforms X(f) and Y(f) respectively) be related as shown in the given figure.



Then Y (f) is

a) $-\frac{1}{2}X(f/2)e^{-j2\pi f}$

b) $-\frac{1}{2}X(f/2)e^{j2\pi f}$

c) $-X(f/2)e^{j2\pi f}$

d) $-X(f/2)e^{-j2\pi f}$

[GATE-2004]

Q.9 Match the following and choose the correct combination.

Group 1

E – Continuous and aperiodic signal

F – Continuous and periodic signal

G – Discrete and aperiodic signal

H – Discrete and periodic signal

Group 2

1. Fourier representation is continuous & aperiodic
2. Fourier representation is discrete and aperiodic
3. Fourier representation is continuous and periodic
4. Fourier representation is discrete and periodic

a) E-3, F-2, G-4, H-1
b) E-1, F-3, G-2, H-4
c) E-1, F-2, G-3, H-4
d) E-2, F-1, G-4, H-3

[GATE-2005]

Q.10 For a signal $x(t)$ the Fourier transform is X(f) . Then the inverse Fourier transform of X(3f + 2) is given by

a) $\frac{1}{2}\times\left(\frac{t}{2}\right)e^{j3\pi t}$ b) $\frac{1}{3}\times\left(\frac{t}{3}\right)e^{-j4\pi t/3}$

c) $3\times(3t)e^{-j4\pi t}$ d) $x(3t+2)$

[GATE-2005]

Q.11 The output y(t) of a linear time invariant system is related to its input $x(t)$ by the following equation: $y(t)=0.5x(t-t_d+T)+x(t-t_d)+0.5x(t-t_d-T)$ The filter transfer function H(ω) of such a system is given by

a) $(1 + \cos\omega T)e^{-j\omega t_d}$

b) $(1 + 0.5\cos\omega T)e^{-j\omega t_d}$

c) $(1 + \cos\omega T)e^{-j\omega t_d}$

d) $(1 - 0.5\cos\omega T)\ e^{-j\omega t_d}$

[GATE-2005]

Q.12 Let $\leftrightarrow X(j\omega)$ be Fourier Transform pair. The Fourier Transform of the signal x(5t – 3) in terms of X (jω) is given as

a) $\frac{1}{5}e^{-\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$

b) $\frac{1}{5}e^{\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$

c) $\frac{1}{5}e^{-j3\omega}X\left(\frac{j\omega}{5}\right)$

d) $\frac{1}{5}e^{j3\omega}X\left(\frac{j\omega}{5}\right)$

[GATE-2006]

Q.13 The 3–dB bandwidth of the low – pass signal $e^{-t}u(t)$ Where u(t) is the unit step function , is given by

a) $\frac{1}{2\pi}Hz$ b) $\frac{1}{2\pi}\sqrt{\sqrt{2}-1}Hz$

c) ∞ d) 1Hz

[GATE-2007]

Q.14 The signal x(t) is described by

$$x(t) = \begin{cases} 1 & \text{for } -1 \le t \le +1 \\ 0 & \text{otherwise} \end{cases}$$

Two of the angular frequencies at which its Fourier transform becomes zero are

a) $\pi, 2\pi$ b) $0.5\pi, 1.5\pi$

c) $0, \pi$ d) $2\pi, 2.5\pi$

[GATE-2008]

Statement for Linked Answer Questions 15 & 16

The impulse response h(t) of a linear time – invariant continuous time system is given by h(t)=exp(-2t)u(t), where u(t) denotes the unit step function.,

Q.15 The frequency response $H(\omega)$ of this system in terms of angular frequency ω is given by. $H(\omega)$ =

a) $\frac{1}{1+j2\omega}$ b) $\frac{\sin(\omega)}{\omega}$

c) $\frac{1}{2+j\omega}$ d) $\frac{j\omega}{2+j\omega}$

[GATE-2008]

Q.16 The output of this system, to the sinusoidal input $x(t) = 2\cos(2t)$ for all time t, is

a) 0

b) $2^{-0.25}\cos(2t - 0.125\pi)$

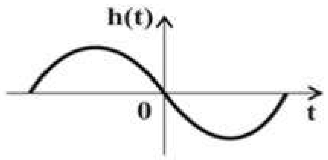
c) $2^{-0.5}\cos(2t - 0.125\pi)$

d) $2^{-0.5}\cos(2t - 0.25\pi)$

[GATE-2008]

Q.17 Consider a system whose input x and output y are related by the equation

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(2\tau)d\tau.$$



Where h(t) is shown in the graph. Which of the following four properties are possessed by the system?

BIBO: Bounded input gives a bounded output.

Causal: The system is causal

LP: The system is low pass

LTI: The system is linear and time – invariant.

a) Causal, LP b) BIBO, LTI

c) BIBO, Causal, LTI d) LP, LTI

[GATE-2009]

Q.18 A function is given by $f(t) = \sin^2 t + \cos 2t$. which of the following is true?

a) f has frequency components at 0 and $1/2\pi$ Hz.

b) f has frequency components at 0 and $1/\pi$ Hz.

c) f has frequency components at $1/2\pi$ and $1/\pi$ Hz

d) f has frequency components at 0, $1/2\pi$ and $1/\pi$ Hz

[GATE-2009]

Q.19 The Fourier transform of a signal h(t) is $H(j\omega) = (2\cos\omega)(\sin 2\omega)/\omega$. The value of h(0) is

a) 1/4 b) 1/2

c) 1 d) 2

[GATE-2012]

Q.20 Let g(t) $= e^{-\pi t^2}$ and h(t) is a filter matched to g(t). If g(t) is applied as input to h(t), then the Fourier transform of the output is

a) $e^{-\pi f^2}$ b) $e^{-\pi f^2/2}$

c) $e^{-\pi|f|}$ d) $e^{-2\pi f^2}$

[GATE-2013]

Q.21 The value of the integral

$$\int_{-\infty}^{\infty} \sin c^2(5t)dt$$ is........

[GATE2014, Set 2]

Q.22 Consider the function $g(t) = e^{-t}\sin(2\pi t)u(t)$ where u(t) is the unit step function. The area under g(t) is.....

[GATE 2015, SET-3]

Q.23 The energy of the signal $x(t) = \frac{\sin(4\pi t)}{4\pi t}$ is.........

[GATE 2016, SET 2]

Q.24 A continuous time-signal $x(t) = 4\cos(200\pi t) + 8\cos(400\pi t)$, where t is in seconds, is the input to a linear time invariant (LTI) filter with the impulse response

$$h(t) = \begin{cases} \frac{2\sin(300\pi t)}{\pi t}, & t \neq 0 \\ 600, & t = 0 \end{cases}$$

Let y(t) be the output of this filter. This maximum value of |y(t)| is.......

[GATE 2017, SET-1]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
a	c	c	c	a	c	a	b	c	b	a	a	a	a
15	16	17	18	19	20	21	22	23	24				
c	d	b	b	c	d	0.2	0.155	0.25	8				

EXPLANATIONS

Q.1 (a)

$\theta(\omega) = -\omega t_o$

$T_p = \frac{-\theta(\omega)}{\omega} = t_o$

$T_g = -\frac{d\theta(\omega)}{d\omega} t_o$

$T_p = T_g = t_o = constant$

Q.2 (c)

$f(t) = e^{-t}u(t)$

$F = \frac{1}{1+j2\pi f}$

$f(-f) = e^{f}u(-f)$

$F(t) = f(-f)$

Q.3 (c)

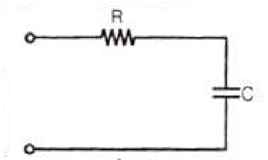
$y(t) = 4x(t-2)$

$Y(s) = 4e^{-2s}X(s)$

$\frac{Y(s)}{X(s)} = 4e^{-2s}$

$H(f) = 4e^{-2\times j2\pi j} = 4e^{-j4\pi f}$

Q.4 (c)



$H(f) = \frac{1}{1+j2\pi fRC}$

$H(0) = 1$

$\frac{|H(f_1)|}{H(0)} = \frac{1}{\sqrt{1+4\pi^2 f_1^2 R^2 C^2}} \geq 0.95$

$\Rightarrow \frac{1}{1+4\pi^2 f_1^2 (RC)^2} \leq 0.9025$

$\Rightarrow 1.108 \leq 1 + 4\pi^2 f_1^2 (RC)^2$

$\Rightarrow 0.108 \geq 1 + 4\pi^2 f_1^2 (RC)^2$

$f_1^2 \leq \frac{0.108}{4\pi^2 (RC)^2}$

$f_1 \leq \frac{0.329}{2\pi \times 10^{-3}}$

$\Rightarrow f_{1max} = 52.2Hz$

Q.5 (a)

$H(w) = \frac{1}{1+j\omega RC}$

$\theta(\omega) = -tan^{-1}RC\omega$

$H(\omega) = \frac{1}{1+j\omega RC}$

$\theta(\omega) = -tan^{-1}RC\omega$

$t_g = \frac{10^{-3}}{1+10^{-6}\times 4\pi^2 \times 10^{14}}$

$t_g = 0.717ms$

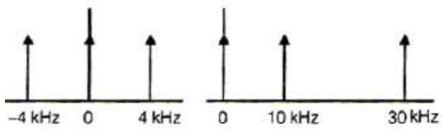
Q.6 (c)

Q.7 (a)

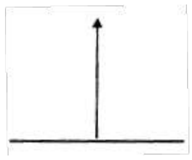
$T_0 = 0.1 \times 10^{-3} = 10^{-4}$

$f_0 = \frac{1}{T_0} = 10^4 = 10kHz$

$cos^2(4\pi \times 10^3 t)$ has frequency =4kHz



So only at '0', we get output after convolution (only odd harmonics are present.



Constant in time domain.

Q.8 (b)

$y(t) = -x[2(t+1)]$

$x(t-t_0) \leftrightarrow X(f)e^{-j2\pi f t_0}$

$t_o = -1$

$x(at) = \frac{1}{|a|}X\left(\frac{f}{a}\right)$

$a = -2$

$\therefore y(f) = -\frac{1}{2}X\left(\frac{f}{2}\right)e^{-j2\pi f}$

Q.9 (c)

Q.10 (b)

$X\left[3\left(f+\frac{2}{3}\right)\right] = \frac{1}{3} \times \left(\frac{t}{3}\right)e^{-j4\pi t/3}$

Applying scaling Shifting property.

Q.11 (a)

$y(t) = 0.5x(t - t_d + T) +$

$0.5x(t - t_d - T) + x(t - t_d)$

Taking Fourier transform

$Y(\omega) = [0.5e^{j\omega(-t_d+T)} +$

$0.5e^{j\omega(-t_d-T)} + e^{-j\omega t_d}]X(\omega)$

$\frac{Y(\omega)}{X(\omega)} e^{-j\omega t_d}[0.5e^{j\omega T} + 0.5e^{-j\omega T} + 1]$

$\frac{Y(\omega)}{X(\omega)} e^{-j\omega t_d}[\cos\omega T + 1]$

$H(\omega) = (1 + \cos\omega T)e^{-j\omega t_d}$

Q.12 (a)

$x(t) \leftrightarrow X(j\omega)$

$x\left[5\left(t - \frac{3}{5}\right)\right] = \frac{1}{5}X\left(\frac{j\omega}{5}\right).e^{-\frac{j3\omega}{5}}$

Using scaling and shifting property

Q.13 (a)

Laplace transform of

$e^{-t}u(t) = \frac{1}{s+1}$

∵magnitude at 3dB frequency

$= \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{s+1} = \frac{1}{\sqrt{1+\omega^2}}$

$\omega = 1\text{rad}$

$\therefore f = \frac{1}{2\pi}\text{Hz}$

Q.14 (a)

$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$

$= \int_{-1}^{1} e^{-j\omega t}\,dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^{1}$

$= \frac{e^{-j\omega}}{-j\omega} - \frac{e^{j\omega}}{-j\omega}$

$= \frac{1}{j\omega}(e^{j\omega} - e^{-j\omega})$

$X(\omega) = 0$

$\Rightarrow e^{j\omega} - e^{-j\omega} = 0$

$\Rightarrow e^{j\omega} - \frac{1}{e^{j\omega}} = 0$

$\Rightarrow e^{2j\omega} - 1 = 0$

$\Rightarrow e^{2j\omega} = 1$

$\Rightarrow e^{j\omega} = \pm 1$

$\Rightarrow \omega = \pi, 2\pi$

Q.15 (c)

$h(t) = \exp(-2t)u(t)$

$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}\,dt$

$= \int_0^{\infty} e^{-2t}e^{-j\omega t}\,dt$

$= \int_0^{\infty} e^{-(2+j\omega)t}\,dt$

$= -\frac{1}{2+j\omega}.e^{-(2+j\omega)t}\Big|_0^{\infty}$

$= \frac{1}{2+j\omega}$

Q.16 (d)

Input $x(t) = 2\cos(2t)$

Frequency response

$X(\omega) = 2\pi[\delta(\omega - 2) + \delta(\omega + 2)]$

$H(\omega) = \frac{1}{2+j\omega}$

Output $Y(\omega) = H(\omega)X(\omega)$

$= \frac{1}{2+j\omega}.2\pi[\delta(\omega - 2) + \delta(\omega + 2)]$

$= \frac{2\pi}{2+j2}.\delta(\omega - 2) + \frac{2\pi}{2+j\omega}\delta(\omega + 2)$

$= \frac{2\pi}{8}[(2 - j2)\delta(\omega - 2) +$

$(2 + j2)\delta(\omega + 2)$

$= \frac{\pi}{4}[2\{\delta(\omega - 2) + \delta(\omega + 2)\} +$

$j2\{\delta(\omega + 2) + \delta(\omega - 2)\}]$

$= \frac{\pi}{2}[\delta(\omega - 2) + \delta(\omega + 2)]$

$-j\frac{\pi}{2}[\delta(\omega - 2) + \delta(\omega + 2)]$

$= \frac{\cos 2t}{2} + \frac{\sin 2t}{2}$

$= \frac{\sqrt{2}}{2}\left[\frac{1}{\sqrt{2}}\cos 2t + \frac{1}{\sqrt{2}}\sin 2t\right]$

$= \frac{1}{\sqrt{2}}\cos(2t - 0.25\pi)$

$= 2^{-0.5}\cos(2t - 0.25\pi)$

Q.17 (b)

$f(t) = \frac{1}{2}(1 - \cos 2t) + \cos 2t.$

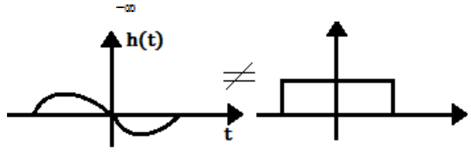
Frequency components are

$f_1 = 0$

$f_2 = \frac{\omega_2}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi}Hz$

Q.18 (b)

$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(2\tau)d\tau.$



It is not low pass filter .

But the system is LTI and BIBO

Q.19 (c)

$H(j\omega) = 2\cos\omega.\frac{2\sin 2\omega}{\omega}$

$= \frac{e^{j\omega}+e^{-j\omega}}{2}.\frac{2\sin 2\omega}{\omega}$

$= \frac{1}{2}\left[e^{j\omega}.\left(\frac{2\sin 2\omega}{\omega}\right) + e^{-j\omega}\left(\frac{2\sin 2\omega}{\omega}\right)\right]$

Let $X(\omega) = \frac{2\sin 2\omega}{\omega}$

Then $x(t) = \begin{cases} 1 & ; \quad -2 < t < 2 \\ 0 & ; \quad \text{otherwise} \end{cases}$

By time shifting property

$$h(t) = \frac{1}{2}\begin{cases} x(t+1) & ; \quad -3 < t < -1 \\ x(t+1) + x(t-1) & ; \quad -1 < t < 1 \\ x(t-1) & ; \quad 1 < t < 3 \end{cases}$$

$\therefore \quad h(0) = \frac{1}{2}[x(1) + x(-1)]$

$= \frac{1}{2}[1+1] = 1$

Q.20 (d)

$g(t) = e^{-\pi t^2}$

$g(f) = e^{-\pi f^2}$

$h(f) = e^{-\pi f^2}$

$Y(f) = g(f)h(f)$

$Y(f) = e^{-\pi f^2}.e^{\pi f^2}$

$Y(f) = e^{-2\pi f^2}$

Q.21 0.2

$$\int_{-\infty}^{\infty} \sin c^2(5t)dt$$

$$f(t) = \sin c(5t)$$

$$= Sa(5\pi t)$$

$$TSa\left(t\frac{T}{2}\right) \leftrightarrow 2\pi G_T \omega$$

$$\frac{T}{2} = 5\pi$$

$$T = 10\pi$$

$$10\pi Sa(5\pi t) \leftrightarrow 2\pi G_{10\pi}(\omega)$$

$$ESD = |F(\omega)|^2$$

$$E_f = \int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} ESD_f d\omega$$

$$\int_{-\infty}^{\infty} \sin c^2(5t)dt = \frac{1}{2\pi}\int_{-5\pi}^{5\pi}\left(\frac{1}{5}\right)^2 d\omega$$

$$= \frac{1}{2\pi}\times\frac{1}{25}\times 10\pi = \frac{1}{5}$$

Q.22 0.155

$$g(t) = e^{-t}\sin(2\pi t)$$

$$\text{Area} = \int_{-\infty}^{\infty} g(t)dt$$

$$\int_{-\infty}^{\infty} \sin 2\pi u(t)e^{-t}dt$$

$$\left.\frac{2\pi}{s^2 + 4\pi^2}\right|_{s=1} = 0.155$$

Q.23 0.25

Q.24 8

Q.25 8

$$y(t) = \frac{1}{2}\times 8\cos(20\pi t + \phi)$$

$$= 4\cos(20\pi t + \phi)$$

$$P_Y = \frac{4^2}{2} = 8W$$

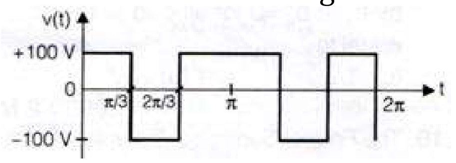
GATE QUESTIONS(EE)

Q.1 If an a.c. voltage wave is corrupted with an arbitrary number or harmonics, then the overall voltage waveform differs from its fundamental frequency component in terms of

a) only the peak values
b) only the rms values
c) only the average values
d) all the three measures (peak, rms and average values)

[GATE-2000]

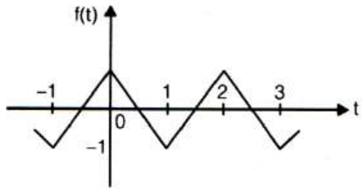
Q.2 What is the rms value of the voltage waveform shown in figure?



a) $200/\pi V$ b) $100/\pi V$
c) 200V   d) 100 V

[GATE-2002]

Q.3 Fourier Series for the waveform, f(t) shown in figure is



a) $\frac{8}{\pi^2}\left[\sin(\pi t)+\frac{1}{9}\sin(3\pi t)+\frac{1}{25}\sin(5\pi t)+\cdots\right]$

b) $\frac{8}{\pi^2}\left[\sin(\pi t)-\frac{1}{9}\cos(3\pi t)+\frac{1}{25}\sin(5\pi t)+\cdots\right]$

c) $\frac{8}{\pi^2}\left[\cos(\pi t)+\frac{1}{9}\cos(3\pi t)+\frac{1}{25}\cos(5\pi t)+\cdots\right]$

d) $\frac{8}{\pi^2}\left[\cos(\pi t)-\frac{1}{9}\sin(3\pi t)+\frac{1}{25}\sin(5\pi t)+\cdots\right]$

[GATE-2002]

Q.4 x(t) is a real valued function of a real variable with period T. It's trigonometric Fourier series expansion contains no terms of frequency $\omega = 2\pi(2k)/T; k = 1,2,\ldots$ Also, no sine terms are present. Then x(t) satisfies the equation

a) $x(t) = -x(t-T)$
b) $x(t) = x(T-t) = -x(-t)$
c) $x(t) = x(T-t) = -x(t-T/2)$
d) $x(t) = x(t-T) = x(t-T/2)$

[GATE-2006]

Q.5 A signal x(t) is given by

$$x(t) = \begin{cases} 1, -T/4 < t \le 3T/4 \\ -1, 3T/4 < t \le 7T/4 \\ -x(t+T) \end{cases}$$

Which among the following gives the fundamental fourier term of x(t)?

a) $\frac{4}{\pi}\cos\left(\frac{\pi t}{T}-\frac{\pi}{4}\right)$ b) $\frac{\pi}{4}\cos\left(\frac{\pi t}{2T}+\frac{\pi}{4}\right)$
c) $\frac{4}{\pi}\sin\left(\frac{\pi t}{T}-\frac{\pi}{4}\right)$ d) $\frac{\pi}{4}\sin\left(\frac{\pi t}{2T}+\frac{\pi}{4}\right)$

[GATE-2007]

Q.6 Let x(t) be a periodic signal with time period T, Let $y(t) = x(t-t_0) + x(t+t_0)$ for some t_0. The Fourier Series coefficients of y(t) are denoted by b_k. If $b_k = 0$ for all odd K. then t_0 can be equal to

a) T/8   b) T/4
c) T/2   d) 2T

[GATE-2008]

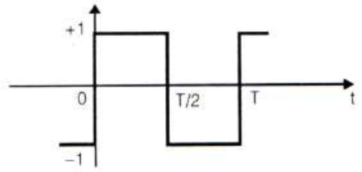
Q.7 The Fourier series coefficients, of a periodic signal x(t) expressed as $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$ a re given by $a_{-2} = 2-j1;\ a_{-1} = 0.5+j0.2;\ a_0 = j2;$
$a_1 = 0.5-j0.2;\ a_2 = 2+j1;$
and $a_k = 0$ for $|k| > 2$

Which of the following is true?

a) x(t) has finite energy because only finitely many coefficients are non-zero
b) x(t) has zero average value because it is periodic
c) The imaginary part of x(t) is constant
d) The real part of x(t) is even

[GATE-2009]

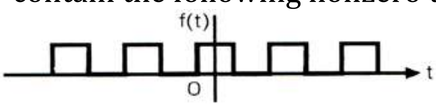
Q.8 The second harmonic component of the periodic waveform given in the figure has amplitude of



a) 0 b) 1
c) $2/\pi$ d) $\sqrt{5}$

[GATE-2010]

Q.9 The Fourier expansion $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$ Of the periodic signal shown below will contain the following nonzero terms



a) a_0 and $b_n, n = 1, 3, 5, \ldots \infty$
b) a_0 and $a_n, n = 1, 2, 3, \ldots \infty$
c) a_0, a_n and $b_n, n = 1, 3, 5, \ldots \infty$
d) a_0, a_n and $n = 1, 3, 5, \ldots \infty$

[GATE-2011]

Q.10 For a periodic signal $v(t)=30\sin 100t+100\cos 300t+6\sin(500t+\pi/4)$ the fundamental frequency in rad/s is

a) 100 b) 300
c) 500 d) 1500

[GATE-2013]

Q.11 A band-limited signal with a maximum frequency of 5 kHz is to be sampled. According to the sampling theorem, the sampling frequency in which is not valid is

a) 5kHz b) 12kHz
c) 15 kHz d) 20 kHz

[GATE-2013]

Q.12 Let g: $[0,\infty) \rightarrow [0,\infty)$ be a function defined by g(x)=x-[x], where [x] represents the integer part of x. (That is the largest integer which is less than or equal to x). The value of the constant term in the Fourier series expansion of g(x) is________

[GATE-2014]

Q.13 The signum function is given by

$$\text{Sign}(x)=\begin{cases} \frac{x}{|x|}; & x \neq 0 \\ 0; & x = 0 \end{cases}$$

The Fourier series expansion of the sgn(cos(t)) has

a) Only sine terms with all harmonics
b) Only cosine terms with all harmonics
c) Only sine terms with even numbered harmonics
d) Only cosine terms with odd numbered harmonics

[GATE-2015]

Q.14 Let f(x) be a real, periodic function satisfying f(-x) = -f(x). The general form of its Fourier series representation would be

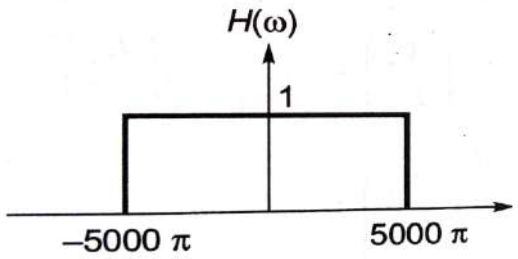
a) $F(x) = a_0 + \sum_{k=1}^{\infty} a_k \cos(kx)$
b) $F(x) = \sum_{k=1}^{\infty} b_k \sin(kx)$
c) $F(x) = a_0 + \sum_{k=1}^{\infty} a_{2k} \cos(kx)$
d) $F(x) = \sum_{k=1}^{\infty} a_{2k+1} \sin(2k+1)x$

[GATE-2016]

Q.15 Let the signal

$x(t)= \sum_{k=-\infty}^{+\infty} (-1)^k \delta\left(t - \frac{k}{2000}\right)$

be passed through an LTI system with frequency response $H(\omega)$, as given in the figure below.



The Fourier Series representation of the output is given as

a) $4000 + 4000\cos(2000\pi t) + 4000 \cos(4000\pi t)$
b) $2000 + 2000\cos(2000\pi t) + 2000 \cos(4000\pi t)$
c) $4000 \cos(2000\pi t)$
d) $2000 \cos(2000\pi t)$

[GATE-2017]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11
(d)	(d)	(c)	(c)	(a)	(b)	(c)	(a)	(d)	(a)	(a)
12	13	14	15							
0.5	(d)	(b)	(c)							

EXPLANATIONS

Q.1 (d)

Q.2 (d)

RMS value

$= \left[\frac{1}{\pi}\left\{\int_0^{\pi/3}(100)^2 dt + \int_{\pi/3}^{2\pi/3}(-100)^2 dt + \int_{2\pi/3}^{\pi}(100)^2 dt\right\}\right]^{12}$

$= \left[\frac{1}{\pi}.100^2\pi\right]^{1/2} = 100V$

Q.3 (c)

∵ f(t) is an even function with half wave symmetry,

∵ dc term as well as sine terms=0

Only the cosine terms with odd harmonics will be present.

∵ Option (c) is correct.

Q.4 (c)

Since trigonometric fourier series has no sine terms and has only cosine terms therefore this will be an even signal i.e. it will satisfy

$x(t) = x(-t)$

Or we can write

$x(t-T) = x(-t+T)$

but signal is periodic with period T.

therefore $x(t-T) = x(t)$

therefore

$x(t) = x(T-t)$ ...(i)

Now since signal contains only odd harmonics i.e. no terms of frequency

$\omega = \frac{2\pi \times 2k}{T}, k = 1, 2, 3, 4$

i.e. no even harmonics.

This means signal contains half wave symmetry this implies that

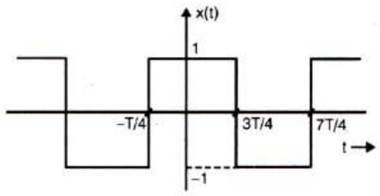
$x(t) = -x(t-T/2)$ ...(ii)

form (i) and (ii)

$x(t) = x(T-t) = -x(t-T/2)$

Q.5 (a)

According to definition of signal given in question the x(t) will be as



So it is periodic with period

$T_0 = \frac{7T}{4} - \left(-\frac{T}{4}\right)$

$T_0 = 2T$

∴ Fundamental angular frequency

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T}$

Now,

$C_1 =$ (Exponential series coefficient for k=1)

$C_1 = \frac{1}{T_0}\int_{T_0} X(t)e^{-j\omega_0 t}dt$

$C_1 = \frac{1}{T_0}\left[\int_{-T/4}^{3T/4} e^{-j\omega_0 t}dt + \int_{3T/4}^{7T/4} -1e^{-j\omega_0 t}dt\right]$

$C_1 = \frac{1}{j\omega_0 T_0}\left[\{-e^{-j\omega_0 t}\}_{-T/4}^{3T/4} + \{e^{-j\omega_0 t}\}_{3T/4}^{7T/4}\right]$

$C_1 = \frac{1}{j2\pi}\left[2e^{j\pi/4} + 2e^{j\pi/4}\right]$

$C_1 = \frac{2}{j\pi}e^{\frac{j\pi}{4}} = \frac{2}{\pi}e^{-\frac{j\pi}{4}}$

$= \frac{2}{\pi}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right]$

Comparing it with

$C_1 = \frac{a_1}{2} = \frac{jb_1}{2}$

$a_1 = \frac{4}{\sqrt{2}\pi}, b_1 = \frac{4}{\sqrt{2}\pi}$

∴ Fundamental Fourier term will be

$= a_1 \cos\omega_0 t + b_1 \sin\omega_0 t$

$= \frac{4}{\sqrt{2}\pi}\cos\left(\frac{\pi}{T}t\right) + \frac{4}{\sqrt{2}\pi}\sin\left(\frac{\pi}{T}t\right)$

$= \sqrt{\left(\frac{4}{\sqrt{2}\pi}\right)^2 + \left(\frac{4}{\sqrt{2}\pi}\right)^2}\cos\left(\frac{\pi}{T}t - \frac{\pi}{4}\right)$

$= \frac{4}{\pi}\cos\left[\frac{\pi}{T}t - \frac{\pi}{4}\right]$

Q.6 (b)

$y(t) = x(t - t_0) + x(t + t_0)$

Since y(t) is periodic with period T.

Therefore $x(t - t_0)$ and $x(t + t_0)$ will also be periodic with period T.

$\therefore b_k = a_k e^{-jk\omega_0 t_0} + a_k e^{jk\omega_0 t_0}$

a_k is fourier series coefficient of signal x(t) therefore

$b_k = a_k\left[e^{-jk\omega_0 t_0} + e^{jk\omega_0 t_0}\right]$

$b_k = 2a_k \cos k\omega_0 t_0$

Since $b_k = 0$ for odd k

i.e. $k\omega_0 t_0 = k\frac{\pi}{2}$ where

$k = \pm 1, \pm 3, \pm 5, \ldots$

$\Rightarrow k\omega_0 t_0 = \pi/2$

$t_0 = \frac{\pi}{2} \times \frac{T}{2\pi} = \frac{T}{4}$

Q.7 (c)

$a_{-2} = 2 - j1,$

$a_{-1} = 0.5 + j0.2$

$a_0 = j2$

$a_1 = 0.5 - j0.2$

$a_2 = 2 + j1$

These are exponential series coefficient.

From this we will find the values of $b_1, b_2, c_1, c_2,$ the coefficient of trigonometric series.

Since x(t) is summation, $a_0 +$ sin and cosine terms (real).

But $a_0 = j2$ which is imaginary.

So imaginary part of x(t) is constant.

x(t) can't have finite energy because it is a periodic signal and periodic signal is always a power signal having infinite energy and finite power.

Q.8 (a)

The given signal is odd as well as having half wave symmetry.

So it has only sine terms with odd harmonics.

So for second harmonic term amplitude = 0.

Q.9 (d)

The pulse train is even and half wave symmetry.

So, $b_n = 0$

$a_n = 0$ for $n = 2, 4, 6, \ldots \infty$

Only a_0, a_n for $n = 1, 3, 5, 7 \ldots \infty$ present.

Q.10 (a)

$\omega_1 = 100$

$\omega_2 = 300$

$\omega_3 = 500$

H.C.F. of ω_1, ω_2 and $\omega_3 =$

H.C.F (100, 300, 500)

$\omega = 100\ rad/sec$

Q.11 (a)

$(f_s)_{min} = 2f_m$

$(f_s)_{min} = 2 \times 5 = 1 = kHz$

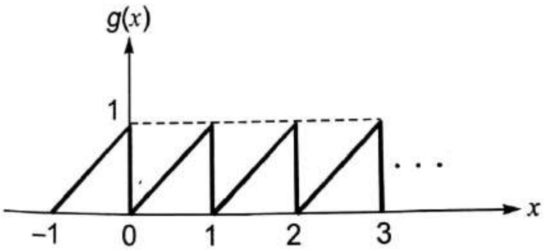
So, $f_s \geq 10kHz$

Q.12 0.5

Given function g[x] = x – [x]

Where [x] is a integer part of x

Then function g[x] will be



The value of the constant term (or) dc term in the Fourier series expansion of g[x] is

$$a_0 = \frac{1}{T}\int_0^T f(x)dx = \frac{\text{Area in one period}}{\text{one period}}$$

$$= \frac{1}{2} \times 1 \times 1 = 0.5$$

Q.13 (d)

So, cos(t) is

So, sgn(cost) is a rectangular signal which is even and has half wave symmetry.
So, Fourier series will have only cosine terms with odd harmonics only.

Q.14 (b)

Given that, f(-x) = - f(x)
So function in an odd function.
So the Fourier series will have sine term only so

$$f(x) = \sum_{k=-\infty}^{\infty} b_x \sin(kx)$$

Q.15 (c)

GATE QUESTIONS(EE)

Q.1 Let $x(t) = \text{rect}\left(t - \frac{1}{2}\right)$ (where rect (t)=1 for $-\frac{1}{2} \le x \le \frac{1}{2}$ and zero otherwise). Then $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, the Fourier Transformer of $x(t) + x(-t)$ will be given by

a) $\text{sinc}\left(\frac{\omega}{2\pi}\right)$

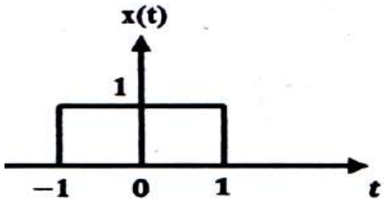
b) $2\text{sinc}\left(\frac{\omega}{2\pi}\right)$

c) $2\text{sinc}\left(\frac{\omega}{2\pi}\right)\cos\left(\frac{\omega}{2}\right)$

d) $\text{sinc}\left(\frac{\omega}{2\pi}\right)\sin\left(\frac{\omega}{2}\right)$

[GATE-2008]

Q.2 x(t) is a positive rectangular pulse from $t = -1$ to $t = +1$ with unit height as shown in the figure. The value of $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ {where $X(\omega)$ is the Fourier transform of $x(t)$} is



a) 2
b) 2π
c) 4
d) 4π

[GATE-2010]

Q.3 The Fourier transform of a signal h(t) is $H(j\omega) = (2\cos\omega)(\sin 2\omega)/\omega$. The value of h(0) is

a) 1/4
b) 1/2
c) 1
d) 2

[GATE-2012]

Q.4 Let f(t) be a continuous time signal and let $F(\omega)$ be its Fourier Transform defined by

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

And g(t) is defined by

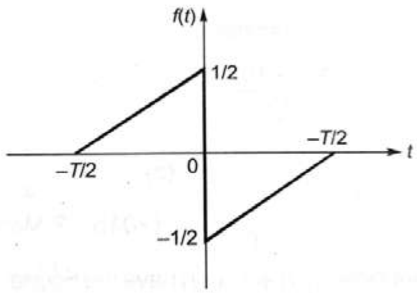
$$g(t) = \int_{-\infty}^{\infty} F(u) e^{-jut} du$$

What is the relation between the f(t) and g(t)?

a) g(t) would always be proportional to the f(t).
b) g(t) would be proportional to the f(t) if f(t) is an even function.
c) g(t) would be proportional to the f(t) only if f(t) is a sinusoidal function.
d) g(t) would never be proportion to the f(t).

[GATE-2014]

Q.5 A function f(t) is shown in the figure.



The Fourier transform $F(\omega)$ of f(t) is

a) real and even function of ω
b) real and odd function of ω
c) imaginary and odd function of ω
d) imginary and even function of ω

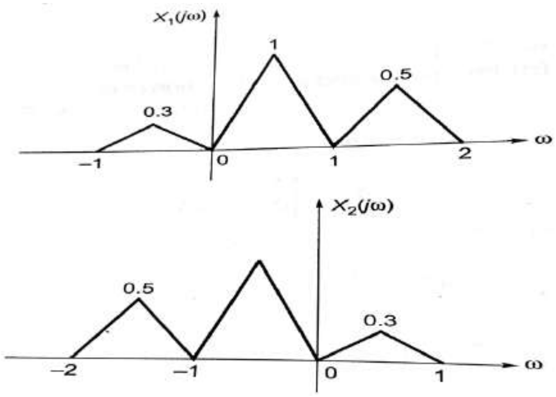
[GATE-2014]

Q.6 A differentiable non constant even function x(t) has a derivative y(t), and their respective Fourier Transforms are $X(\omega)$ and $Y(\omega)$. Which of the following statement is TRUE?

a) $X(\omega)$ and $Y(\omega)$ are both real
b) $X(\omega)$ is real and $Y(\omega)$ is imaginary
c) $X(\omega)$ and $Y(\omega)$ are both imaginary
d) $X(\omega)$ is imaginary and $Y(\omega)$ is real

[GATE-2014]

Q.7 Suppose $x_1(t)$ and x_2t have the Fourier transforms as shown below.



Which of the following statement is TRUE?

a) $x_1(t)$ and $x_2(t)$ are complex and $x_1(t)\ x_2(t)$ is also complex with nonzero imaginary part

b) $x_1(t)$ and $x_2(t)$ are complex but $x_1(t)\ x_2(t)$ is also complex with nonzero real part

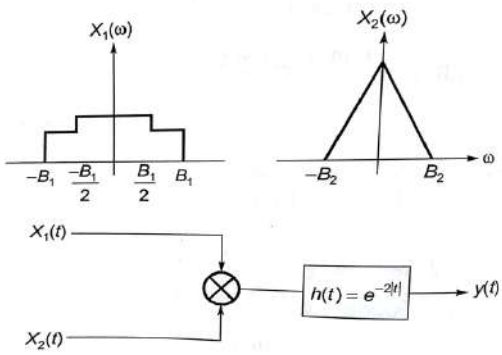
c) $x_1(t)$ and $x_2(t)$ are complex but $x_1(t)\ x_2(t)$ is real

d) $x_1(t)$ and $x_2(t)$ are imaginary but $x_1(t)\ x_2(t)$ is real

[GATE-2016]

Q.8 Suppose the maximum frequency in a band limited signal x(t) is 5kHz. Then, the maximum frequency in $x(t)\cos(2000\pi t)$, in kHz, is____________ **[GATE-2016]**

Q.9 Let $x_1(t) \leftrightarrow X_1(\omega)$ and $x_2(t) \leftrightarrow X_2(\omega)$ be two signals whose Fourier Transforms are as shown in the figure below. In the figure, h (t) = $e^{-2|t|}$ denotes the impulse response.



For the system shown above, the minimum sampling rate required to sample y(t), so that y(t) can be uniquely reconstructed from its samples, is

a) $2B_1$ b) $2(B_1 + B_2)$

c) $4(B_1 + B_2)$ d) ∞

[GATE-2016]

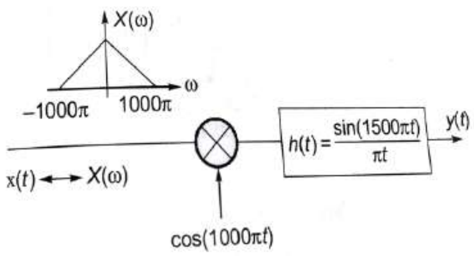
Q.10 The value of the integral $2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)dt$ is equal to

a) 0 b) 0.5

c) 1 d) 2

[GATE-2016]

Q.11 The output y(t) of the following system is to be sampled, so as to reconstruct it from its samples uniquely. The required minimum sampling rate is



a) 1000 samples/s

b) 1500 samples/s

c) 2000 samples/s

d) 3000 samples/s

[GATE-2017]

Q.12 The Fourier transform of a continuous-time signal $x(t)$ is given by $X(\omega) = \frac{1}{(10 + j\omega)^2}$, $-\infty < \omega < \infty$,

where $j = \sqrt{-1}$ and ω denotes frequency. Then the value of $|\ln x(t)|$ at $t = 1$ is ________ (up to 1 decimal place). (ln denotes the logarithm to base *e*).

ANSWER KEY:

1	2	3	4	5	6	7
(c)	(d)	(c)	(b)	(c)	(b)	(c)
8	9	10	11	12		
6	(b)	(d)	(b)	10		

EXPLANATIONS

Q.1 (c)

$x(t) = rect[t - 1/2]$

$x(t) = 1 \qquad 0 \le t \le 1$

$= 0 \qquad$ otherwise

$F[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$

$= \int_0^1 1e^{-j\omega t} dt = \frac{1}{j\omega}(1 - e^{-j\omega})$

$= \frac{2}{\omega}\left[\frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}\right] x e^{-j\omega/2}$

Where $x(-t) = 1, -1 \le t \le 0$

0, otherwise

$F[x(t)] = \int_{-\infty}^{\infty} x(-t) e^{-j\omega t} dt$

$= \int_{-1}^{0} 1. e^{-j\omega t} dt = \frac{1}{-j\omega}(e^{-j\omega t})_{-1}^{0}$

$= \frac{1}{j\omega}(e^{j\omega} - 1)$

$= \frac{1}{j\omega}[e^{j\omega/2} - e^{-j\omega/2}] e^{j\omega/2}$

$= \frac{2}{\omega}\left[\frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}\right] e^{j\omega/2}$

$F[x(-t)] = \frac{\sin\omega/2}{\omega/2} e^{j\omega/2}$

$F[x(t) + x(-t)] = \frac{\sin\omega/2}{\omega/2}$

$[e^{-j\omega/2} + e^{j\omega/2}]$

$= \frac{\sin\omega/2}{\omega/2} 2\cos\omega/2$

$= 2\text{sinc}\left(\frac{\omega}{2\pi}\right) \cos\left(\frac{\omega}{2}\right)$

Q.2 (d)

$\int_{-\infty}^{\infty} |X(\omega)^2| d\omega = \int_{-\infty}^{\infty} X(\omega). X^*(\omega) d\omega$

$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt\right]. X^*(\omega) d\omega$

$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} X^*(\omega) e^{-j\omega t} d\omega\right] x(t) dt$

$= 2\pi \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \frac{1}{2\pi}\{X(\omega) e^{-j\omega t}\}^* d\omega\right] x(t) dt$

$= 2\pi \int_{-\infty}^{\infty} X^*(t) x(t)\, dt$

$= 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= 2\pi \int_{-1}^{1} 1 dt$

$= 2\pi \times 2$

$= \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 4\pi$

Q.3 (c)

$H(j\omega) = 2\cos\omega. \frac{2\sin 2\omega}{\omega}$

$= \frac{e^{j\omega} + e^{-j\omega}}{2}. \frac{2\sin 2\omega}{\omega}$

$= \frac{1}{2}\left[e^{j\omega}.\left(\frac{2\sin 2\omega}{\omega}\right) + e^{-j\omega}\left(\frac{2\sin 2\omega}{\omega}\right)\right]$

Let $X(\omega) = \frac{2\sin 2\omega}{\omega}$

Then $x(t) = \begin{cases} 1 & ; \quad -2 < t < 2 \\ 0 & ; \quad \text{otherwise} \end{cases}$

By time shifting property

$h(t) = \frac{1}{2}\begin{cases} x(t+1) & ; \quad -3 < t < -1 \\ x(t+1) + x(t-1) & ; \quad -1 < t < 1 \\ x(t-1) & ; \quad 1 < t < 3 \end{cases}$

$\therefore \quad h(0) = \frac{1}{2}[x(1) + x(-1)]$

$= \frac{1}{2}[1 + 1] = 1$

Q.4 (b)

Given that,

$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}$

and $g(t) = \int_{-\infty}^{\infty} F(u) e^{-jut} du$

$\left.\begin{matrix} p(t) \leftrightarrow p(f) \\ p(t) \leftrightarrow p(-f) \end{matrix}\right\} \ldots\ldots(i)$

$g(t) = F.T.[F.T.f(t)] \ldots.(ii)$

Put, equation (i) in (ii) so that the Fourier transform gives the additional negation.

Hence g(t) would be proportional to f(t) if f(t) is an even function.

Q.5 (c)

Given signal f(t) is an odd signal. Hence $F(\omega)$ is imaginary and odd function of ω.

Q.6 (b)

For even function x(t) the Fourier transform $X(\omega)$ is always real. Y(t) is a derivative of x(t) which is an odd function and hence, Fourier transform $Y(\omega)$ is imaginary.

Q.7 (c)

By observing $X_1(j\omega)$ and $X_2(j\omega)$we can say that they are not conjugate symmetric.

Since the fourier transform is not conjugate symmetric the signal will not be real.

So $x_1(t)$, $x_2(t)$ are not real.

Now, the fourier transform of $x_1(t)$, $x_2(t)$ are not real.

Now the fourier transform of $x_1(t)$ $x_2(t)$ will be $\frac{1}{2\pi}X_1(j\omega)*X_2(j\omega)$

And by looking at $X_1(j\omega)$ and $X_2(j\omega)$ we can say that $X_1(j\omega)*X_2(j\omega)$ will be conjugate symmetric and thus $x_1(t)\times x_2(t)$ will be real.

Q.8 6kHz

Maximum possible frequency of

x(t) $(2000\pi t) = (f_1 + f_2)$

= 5kHz + 1kHz = 6kHz

Q.9 (b)

Given that,

Bandwidth of $X_1(\omega)=B_1$

Bandwidth of $X_2(\omega)=B_2$

System has $h(t)=e^{-2|t|}$ and input to the system is $x_1(t)\cdot x_2(t)$

The bandwidth of $x_1(t)\cdot x_2(t)$ is B_1+B_2.

The bandwidth of output will be B_1+B_2

So sampling rate will be $2(B_1+B_2)$.

Q.10 (d)

The Fourier transform of

$$\frac{2\sin(t\tau/2)}{t}\rightarrow 2\pi\,\text{rect}\left(\frac{\omega}{\tau}\right)$$

$$\frac{\sin(2\pi t)}{\pi t}\rightarrow \text{rect}\left(\frac{\omega}{4\pi}\right)$$

$$\text{So, }\int_{-\infty}^{\infty}\frac{\sin(2\pi t)}{\pi t}e^{-j\omega t}dt=\text{rect}\left(\frac{\omega}{4\pi}\right)$$

Putting $\omega=0$ in above equation

$$\int_{-\infty}^{\infty}\frac{\sin(2\pi t)}{\pi t}dt=1$$

$$2\int_{-\infty}^{\infty}\frac{\sin(2\pi t)}{\pi t}dt=2$$

Q.11 (b)

Q.12 10

We know that

$$e^{-at}u(t)\xleftrightarrow{FT}\frac{1}{a+j\omega}$$

Put $a=10$

$$e^{-10t}u(t)\xleftrightarrow{FT}\frac{1}{10+j\omega}$$

Apply multiplication by t property
[frequency differentiation property]

i.e. $tx(t) \leftrightarrow j\frac{d}{d\omega}X(\omega)$

$$te^{-10t}u(t) \leftrightarrow j\frac{d}{d\omega}\left[\frac{1}{10+j\omega}\right]$$

$$te^{-10t}u(t) \leftrightarrow \left[\frac{1}{(10+j\omega)^2}\right]$$

$$\left|\ln x(t)\right|_{t=1} = \left|\ln\left\{te^{-10t}u(t)\right\}\right|_{t=1}$$

$$= \left|\ln(t) + \ln\left[e^{-10t}\right] + \ln\left[u(t)\right]\right|_{t=1}$$

$$= \left|\ln(1) + (-10t) + \ln\left[u(t)\right]\right|$$

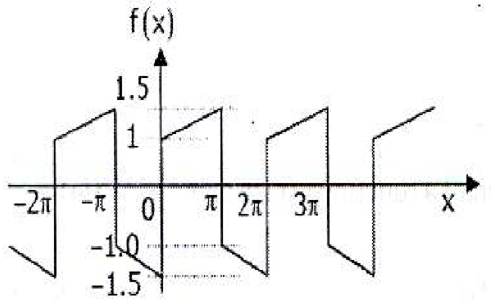
$$= \left|0 + (-10\times 1) + \ln(1)\right|$$

$$= \left|0 - 10 + 0\right|$$

$$= \left|-10\right| = 10$$

GATE QUESTIONS(IN)

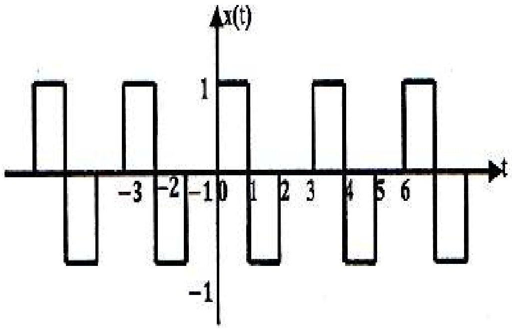
Q.1 f(x) , shown in the adjoining figure is represented by $f(x) = a_0 + \sum_{n=1}^{\infty} \{(a_n\cos(nx) + b_n\sin(nx))\}$
the value of a_0 is



a) 0 b) $\frac{\pi}{2}$
c) π d) 2π

[GATE-2010]

Q.2 Consider a periodic signal x(t) as shown below



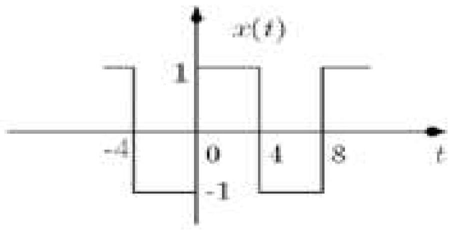
It has a Fourier series representation $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T)kt}$

Which one of the following statements is TRUE?

a) $a_k = 0$, for k odd integer and T =3
b) $a_k = 0$, for k even integer and T =3
c) $a_k = 0$, for k even integer and T =6
d) $a_k = 0$, for k odd integer and T =6

[GATE-2011]

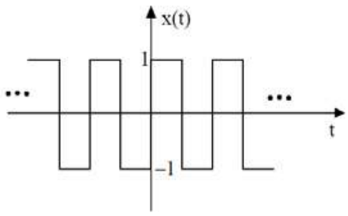
Q.3 For the periodic signal x(t) shown below with period T = 8 s, the power in the 10th harmonic is



a) 0 b) $\frac{1}{2}\left(\frac{2}{10\pi}\right)^2$
c) $\frac{1}{2}\left(\frac{4}{10\pi}\right)^2$ d) $\frac{1}{2}\left(\frac{4}{5\pi}\right)^2$

[GATE-2010]

Q.4 A ideal square wave with period of 20 ms shown in the figure, is passed through an ideal low pass filter with cut-off frequency 120 Hz. Which of the following is an accurate description of the output?



a) Output is zero

b) Output consists of both 50 Hz and 100 frequency components

c) Output is a pure sinusoid of frequency 50 Hz.

d) Output is a square wave of fundamental frequency of 50 Hz.

[GATE-2018]

ANSWER KEY:

1	2	3	4
a	b	a	(c)

EXPLANATIONS

Q.1 (a)

The given function $f(x)$ is periodic function with period, $T = 2\pi$ F(x) is shown over one period from 0 to 2π in fig.

The d.c value of

$f(x) = a_0 = \frac{1}{T}\int_0^T f(x)\,dx$

$a_0 = \frac{\text{Area under } f(x)}{T} = 0$

As can be seen from Fig.

Q.2 (b)

Clearly , period of the signal x(t) is 3

So, T =3

And $a_0 = \frac{1}{T}\int_0^T x(t)\,dt$

$= \frac{1}{3}\int_0^3 x(t)dt$

$= \frac{1}{3}(1-1)$

$= 0$

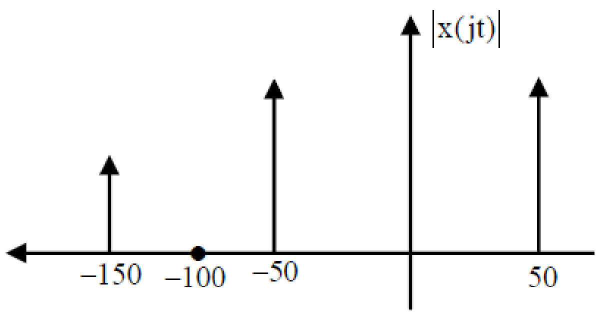
So $a_k = 0$ for k even integer.

Q.3 (a)

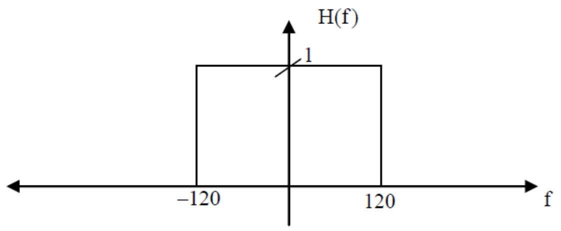
The given square wave satisfy odd and half wave symmetry so it does not have any eigen harmonic. Since 10th harmonic amplitude is 0. So Re also 0

Q.4 (c)

Given square wave passed half wave symmetry. Thus only odd frequency compound exist.



The frequency response of filter is



At the output of filter only one frequency component exist. Thus output will have pure sinusoidal of 50 Hz.

GATE QUESTIONS(IN)

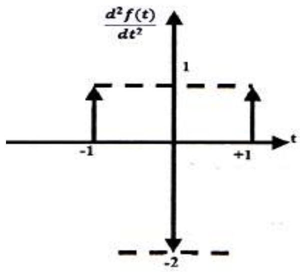
Q.1 A real function f(t) has a fourier transform F(ω). The fourier transform of [f(t) – f(-t)] is

a) zero b) real

c) real and odd d) imaginary

[GATE-2003]

Q.2 If the waveform, shown in the following figure , corresponds to the second derivative of a given function f(t), then the Fourier transform of f(t) is



a) $1 + sin\omega$ b) $1 + cos\omega$

c) $\frac{2(1-cos\omega)}{\omega^2}$ d) $\frac{2(1+cos\omega)}{\omega^2}$

[GATE-2006]

Q.3 The Fourier transform of a function g(t) is given as $G(\omega) = \frac{\omega^2+21}{\omega^2+9}$. Then the function g(t) is given as,

a) $\delta(t) + 2\exp(-3|t|)$

b) $\cos 3\omega t + 21\exp(-3t)$

c) $\sin 3\omega t + 7\cos\omega t$

d) $\sin 3\omega t + 21\exp(3t)$

[GATE-2006]

Q.4 Consider the signal $x(t) = \begin{cases} e^{-t}, t \geq 0 \\ 0, t < 0 \end{cases}$

Let $x(\omega)$ denote the Fourier transform of this signal. The integral $\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) d\omega$ is

a) 0 b) $\frac{1}{2}$

c) 1 d) ∞

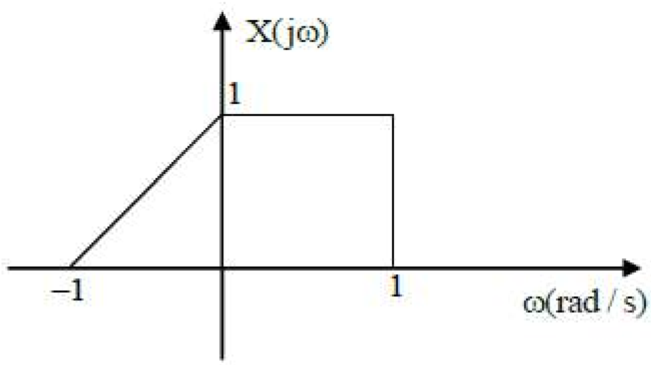
[GATE-2011]

Q.5 The Fourier transform of a signal h(t) is H(jω)=(2cosω)(sin2ω)/ ω. The vlue of h(0) is

a) 1/4 b) 1/2

c) 1 d) 2

Q.6 The Fourier transform of a signal $x(t)$, denoted by $X(j\omega)$ is shown in the figure.



Let $y(t) = x(t) + e^{jt}x(t)$. The value of Fourier transform of $y(t)$ evaluated at the angular frequency $\omega = 0.5\ rad/s$ is

a) 0.5 b) 1

c) 1.5 d) 2.5

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6
d	c	a	c	c	c

EXPLANATIONS

Q.1 (d)

Q.2 (c)

Rom the given plot,

$\frac{d^2f(t)}{dt^2} = 1\delta(t+1) - 2\delta(t) + 1\delta(t) + 1\delta(t-1)$

Take F.T on both sides

Use the following pairs and properties :

Let $f(t) \rightarrow F(\omega)$, then $\frac{df(t)}{dt} \rightarrow j\omega F(\omega)$

$\frac{d^2f(t)}{dt^2} \rightarrow (j\omega)^2F(\omega) = -\omega^2F(\omega)$

$\delta(t) \rightarrow 1\delta(t \mp 1) = e^{\mp j\omega}$

$-\omega^2F(\omega) = e^{j\omega} - 2 + e^{-j\omega} = 2\cos(\omega) - 2$

$\omega^2F(\omega) = 2(1-\cos\omega)$,

$F(\omega) = \frac{2}{\omega^2}(1-\cos(\omega))$

Q.3 (a)

$g(t) \rightarrow G(\omega)$

Given , $G(\omega) = \frac{\omega^2+21}{\omega^2+9}$

$G(\omega) = \frac{\omega^2+9+12}{\omega^2+9} = 1 + \frac{12}{\omega^2+9}$

Use the F.T pairs:

$\delta(t) \rightarrow 1$

$e^{-3t}u(t) \rightarrow \frac{1}{3+j\omega}$

If t replaced by $(-t)$, ω is replaced by $(-\omega)$

$e^{+3t}u(t) \rightarrow \frac{1}{3-j\omega}$

$e^{-3t}u(t) + e^{3t}u(-t) \rightarrow \frac{1}{3+j\omega} + \frac{1}{3-j\omega}$

$= \frac{6}{9+\omega^2}$

$e^{-3|t|} \rightarrow \frac{6}{9+\omega^2}$, $2e^{-3|t|} \rightarrow \frac{12}{\omega+9}$

In general, $Ke^{-\alpha|t|} \rightarrow K\frac{2\alpha}{\alpha^2+\omega^2}$

$\therefore g(t) = 1\delta(t) + 2e^{-3|t|}$

Q.4 (c)

Given $x(t) = e^{-t}, t \geq 0$

$= 0, t < 0$

$x(t) \rightarrow X(f), X(\omega)$

Area property:

$\int_{-\infty}^{\infty} X(f)\,df = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\,d\omega = x(t)|_{t=0} = 1$

Q.5 (c)

$$H(j\omega) = \cos\omega \cdot \frac{2\sin 2\omega}{\omega}$$

$$= \frac{e^{j\omega}+e^{-j\omega}}{2} \cdot \frac{2\sin 2\omega}{\omega}$$

$$= \frac{1}{2}\left[e^{j\omega} \cdot \left(\frac{2\sin 2\omega}{\omega}\right) + e^{-j\omega}\left(\frac{2\sin 2\omega}{\omega}\right)\right]$$

Let, $X(\omega) = \frac{2\sin 2\omega}{\omega}$

Then,

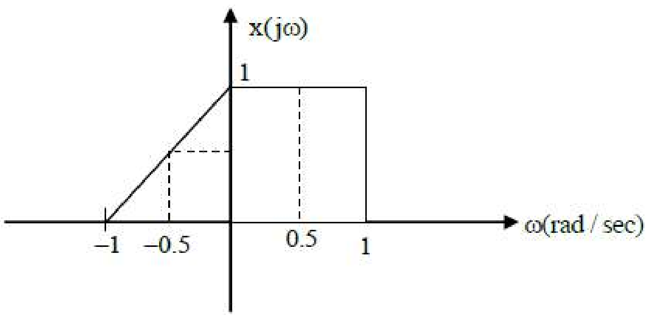
$$x(t) = \begin{cases} 1 ; -2 < t < 2 \\ 0 ; \text{otherwise} \end{cases}$$

$$h(t) = \frac{1}{2}\begin{cases} x(t+1) & ; \quad -3 < t < -1 \\ x(t+1) + x(t-1) & ; \quad -1 < t < 1 \\ x(t-1) & ; \quad 1 < t < 3 \end{cases}$$

$$\therefore \quad h(0) = \frac{1}{2}\left[x(1) + x(-1)\right]$$

$$= \frac{1}{2}[1+1] = 1$$

Q.6 (c)



$$y(t) = x(t) + e^{jt}x(t)$$

$$Y(j\omega) = X(j\omega) + X(j[\omega - 1])$$

$$Y(j0.5) = X(j0.5) + X(j[-0.5])$$

$$= 1 + 0.5 = 1.5$$

4 LAPLACE TRANSFORM

4.1 INTRODUCTION TO THE LAPLACE TRANSFORM

For a general continuous-time signal x(t), the Laplace transform X(s) is defined as

$$X(s)=\int_{-\infty}^{\infty} x(t)e^{-st}dt$$

The variable s is generally complex-valued and is expressed as

$$s=\sigma+j\omega$$

The Laplace transform defined in Equation is often called the bilateral (or two-sided) Laplace transform in contrast to the unilateral (or one-sided) Laplace transform, which is defined as

$$X_1(s)=\int_{0^-}^{\infty} x(t)e^{-st}dt$$

Where $0^-=\lim_{\varepsilon\to 0}(0-\varepsilon)$. clearly the bilateral and unilateral transforms are equivalent only if x (t) =0 for t<0. We will omit the word "bilateral" except where it is needed to avoid ambiguity. Equation is sometimes considered an operator that transforms a signal x (t) into a function X(s) symbolically represented by

$$X(s)=L\{x(t)\}$$

and the signal x(t) and its Laplace transform X(s) are said to form a Laplace transform pair denoted as $x(t)\leftrightarrow X(s)$

4.2 THE REGION OFCONVERGENCE

The range of values of the complex variables s for which the Laplace transform converges is called the region of convergence (ROC). To illustrate the Laplace transform and the associated ROC let us consider some examples.

Example:
Consider the signal x(t) =e^{-at}u(t), where a is real then the Laplace transform of x(t) is = X(s) ?
Solution:

$$X(s)=\int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}dt$$

$$X(s)=\int_{0^+}^{\infty} e^{-(s+a)t}dt$$

$$X(s)=-\frac{1}{s+a}e^{-(s+a)t}\Big|_{0^+}^{\infty}$$

$$X(s)=\frac{1}{s+a}\quad Re(s)>-a$$

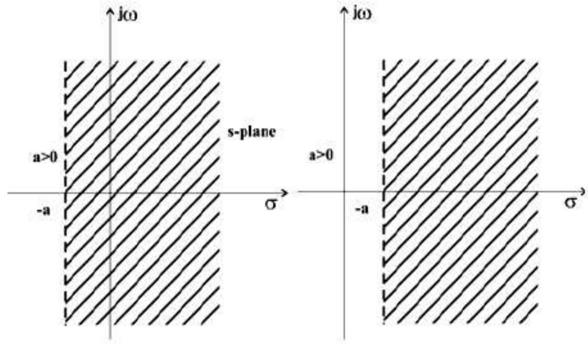


Fig. ROC for Example

Because

$\lim t\to\infty e^{-(s+a)t}=0$ only if $Re(s+a)>0$

or $Re(s)>-a$.

Thus, the ROC for this example is specified in Eq. as Re(s) > -a and is displayed in the complex plane as shown in Fig. by the shaded area to the right of the line Re(s) = -a. In Laplace transform applications, the complex plane is commonly referred to as the s-plane. The horizontal and vertical axes are sometimes referred to as the a-axis and the jω-axis, respectively.

Example:
Consider the signal $x(t)-e^{-at}u(-t)$ a real
Its Laplace transform X(s) is =?
Solution:

$$X(s)=\frac{1}{s+a}\qquad Re(s)<-a$$

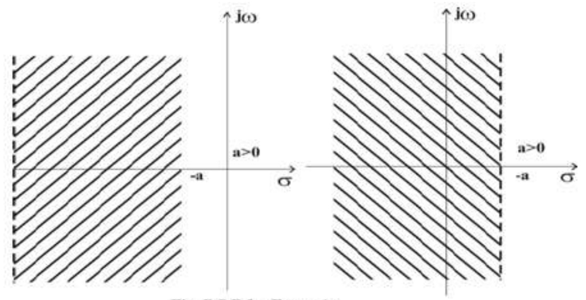


Fig. ROC for Example

Thus, the ROC for this example is specified in Eq. as Re(s) < -a and is displayed in the complex plane as shown in Fig. by the shaded area to the left of the line Re(s) = -a. Comparing Eqs. and we see that the algebraic expressions for X(s) for these two different signals are identical except for the ROCs. Therefore, in order for the Laplace transform to be unique for each signal x (t), the ROC must be specified as part of the transform.

4.3 PROPERTIES OF THE ROC

As we saw in Examples, the ROC of X(s) depends on the nature of the signal.
The properties of the ROC are summarized below. We assume that X(s) is a rational function of s.

Property 1
The ROC does not contain any poles.

Property 2
If **x (t)** is a finite-duration signal, that is, **x (t)** =0 except in a finite interval $t_1 \le t \le t_2 (-\infty < t_1 \text{ and } t_2 < \infty)$, then the ROC is the entire s-plane except possibly s=0 or s=∞.

Property 3
If x (t) is a **right-sided** signal, that is, x (t) = **0** for t< t< ∞, then the ROC is of the form $\text{Re}(s) > \sigma_{maz}$

Where σ_{maz} equals the maximum real part of any of the poles of X(s). Thus, the ROC is a half-plane to the right of the vertical line Reb(s) = σ_{maz} in the s-plane and thus to the right of all of the poles of X(s).

Property 4
If x (t) is a left-sided signal, that is, x (t) = 0 for t > t_2> -∞, then the ROC is of the form $\text{Re}(s) < \sigma_{min}$

Where σ_{min} equals the minimum real part of any of the poles of X(s). Thus, the ROC is a half-plane to the left of the vertical line $\text{Re}(s) < \sigma_{min}$ the s-plane and thus to the left of all of the poles of X(s).

Property 5
If x (t) is a two-sided signal, that is, x (t) is an infinite-duration signal that is neither right-sided nor left-sided, then the ROC is of the form $\sigma_1 < \text{Re}(s) < \sigma_2$

Where σ_1 and σ_2 are the real parts of the two poles of X(s). Thus, the ROC is a vertical strip in the s-plane between the vertical lines Re(s) = σ_1 and Re(s) = σ_2.

Note:
That Property 1 follows immediately from the definition of poles; that is, **X(s)** is infinite at a pole.

LAPLACE TRANSFORMS OF SOME COMMON SIGNALS

A. Unit Impulse Function

$\delta(t)$:

Using Eqs., we obtain

$$L[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)e^{-st}dt = 1 \text{ all } s$$

B. Unit Step Function u(t)

$$L[u(t)] = \int_{-\infty}^{\infty} u(t)e^{-st}dt = \int_{0^+}^{\infty} e^{-st}dt$$

$$= -\frac{1}{s}e^{-st}\Big|_{0^+}^{\infty} = \frac{1}{s} \qquad \text{Re}(s) > 0$$

Where o⁺ = $\lim_{\varepsilon \to 0}(0+\varepsilon)$.

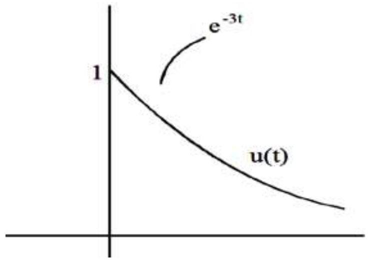
Example:
Find the Laplace Transform of following signals?

1) $e^{-3t}u(t)$ 2) $e^{3t}u(+t)$

3) $e^{3t}u(-t)$ 4) $e^{-3t}u(-t)$

Solution:

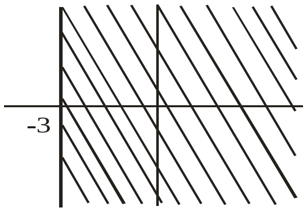
1) $e^{-3t}u(t)$



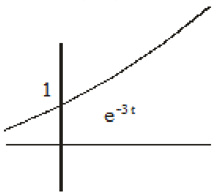
$$X(s)=\int_{-\infty}^{\infty} e^{-3t}u(t)^{-st}$$

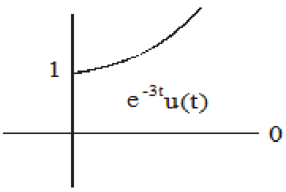
$$\int_{-\infty}^{\infty} e^{-3t}e^{-st}dt=\left(\frac{e^{-(s+3)}}{s+3}\right)$$

$$+\frac{1}{s+3}\text{ROC, Re (s) > - 3}$$



2) $e^{3t}u(+t)$

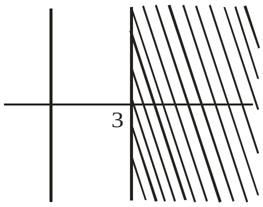




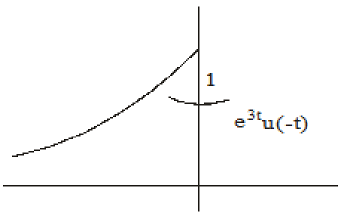
$$x(s)=\int_{\infty}^{\infty} e^{-3t}u(t)e^{-st}dt$$

$$=\int_{\infty}^{\infty} e^{-(s-3)t}u(t)-\left|\frac{e^{-(s-3)}}{s-3}\right|$$

$$x(s)=\frac{1}{s-3}\quad \text{ROC, Re (s) > 3}$$



3) $e^{3t}u(-t)$



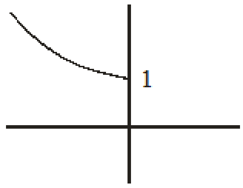
$$=\int_{\infty}^{\infty} e^{-3t}e^{-st}dt=\left|\frac{e^{-(s-3)}}{s-3}\right|$$

$$X(s)=\int_{\infty}^{\infty} e^{-3t}u(-t)e^{-st}dt$$

$$x(s)=-\frac{1}{s-3}\quad \text{ROC, Re (s) < 3}$$

$$-e^{-3t}u(-t)\leftrightarrow\frac{1}{s-3},e^{3t}u(t)\leftrightarrow\frac{1}{s-3}$$

4) $e^{-3t}u(-t)$



$$X(s)=\int_{\infty}^{\infty} e^{-3t}u(-t)e^{-st}dt$$

$$X(s)=\int_{\infty}^{\infty} e^{-3t}e^{-st}dt=\left|\frac{e^{-(s-3)t}}{s+3}\right|$$

$$X(s)=-\frac{1}{s+3}\quad \text{ROC Re (s) < -3}$$

$$e^{-at}u(t)\leftrightarrow\frac{1}{s+3}\text{ROC, } R_e(s)<-3,$$

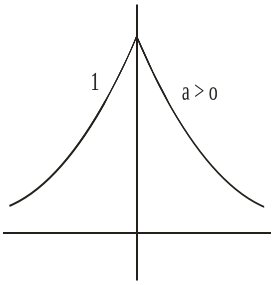
$$-e^{-at}u(-t)\leftrightarrow\frac{1}{s+3}\quad \text{ROC, Re (s) < -3}$$

Example:

Find the Laplace transform X(s) of x (t) $=e^{-a|t|}$?

Solution:

$x(t)=e^{-a|t|}$, X(s) = ?



$$x(t)=e^{-a|t|}$$

$$x(t)=e^{-at}u(t)+e^{at}u(-t)$$

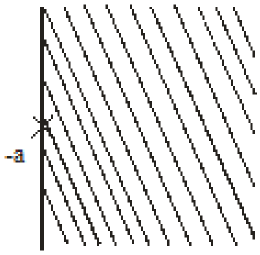
$$\downarrow \qquad\qquad \downarrow$$

$$\frac{1}{s+a}\rightarrow R_e(s)>-a\quad \frac{1}{s+a}\rightarrow R_e(s)>-a$$

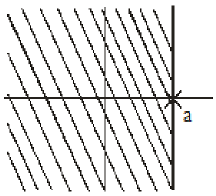
$$X(s)=\frac{-2a}{(s+a)(s-a)}$$

$$X(s)=\frac{-2a}{s^2-a^2}\; -a<R_e(s)<a,\; a>o$$

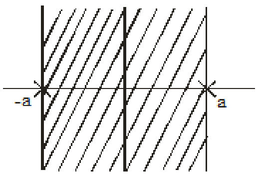
$\frac{1}{s+a} \leftarrow$ if $a>o$



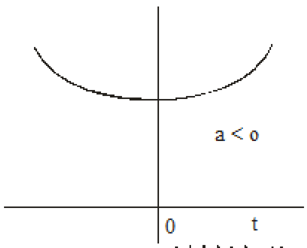
$-\frac{1}{s\text{-}a} \leftarrow$ if $a>o$



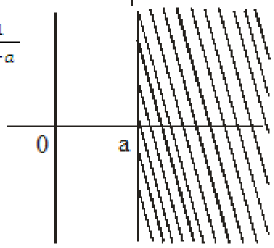
Common ROC



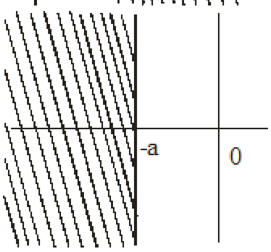
If $a<0 \quad x(t)=e^{-a|t|}$



$\frac{1}{s+a}$



$\frac{1}{s-a}$



There is no common ROC, thus x(t) has no transform X(s)

4.4 PROPERTIES OF THE LAPLACE TRANSFORM

Basic properties of the Laplace transform are presented in the following.

A. Linearity:

If

$x_1(t) \leftrightarrow X_1(s) \qquad ROC = R_1$

$x_2(t) \leftrightarrow X_2(s) \qquad ROC = R_2$

Then

$a_1x_1(t)+a_2x_2(t) \leftrightarrow a_1X_1(s)+a_2X_2(s)$

$R' \supset R_1 \cap R_2$

The set notation $A \supset B$ means that set A contains set B, while $A \cap B$ denotes the intersection of sets A and B, that is, the set containing all elements in both A and B. Thus, Eq. indicates that the **ROC** of the resultant Laplace transform is at least as large as the region in common between R1and R2. Usually we have simply $R' = R_1 \cap R_2$

1) Unit impulse function:-

$$L[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)\, e^{-st}\, dt = 1 \text{ all s}$$

2) Unit step function:-

$$L[u(t)] = \int_{-\infty}^{\infty} u(t)\, e^{-st}\, dt = \int_{-\infty}^{\infty} e^{-st}\, dt$$

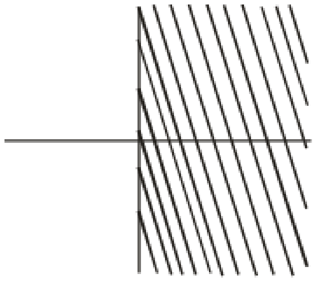
$$-\frac{1}{s}e^{-st}\left(=\frac{1}{s}\text{Re}(s)\right)>0$$

$$L[u(-t)] = \int_{-\infty}^{\infty} u(-t)\, e^{-st}\, dt = \int_{-\infty}^{\infty} e^{-st}\, dt$$

$$-\frac{1}{s}e^{-st}\left(=-\frac{1}{s}\text{Re}(s)\right)>0$$

$[u(-t)] \quad \leftrightarrow \quad \frac{1}{s}R_e(s)<0$

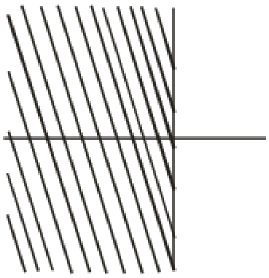
$L[u(t)+u(-t)] \quad \leftrightarrow \quad \text{ROCu(t)}$



$\frac{1}{s} - \frac{1}{s}$

No common ROC then no Laplace transform.

L[u(t) - u(-t)] = L[Sgn(t)]



$\frac{1}{s} - \frac{1}{s}$

No common ROC, the no Laplace transform

B. Time Shifting:

If $x(t) \leftrightarrow X(s) \qquad ROC = R$

Then, $x(t-t_0) \leftrightarrow e^{-st_0}X(s) \qquad R` = R$

Equation indicates that the ROCs before and after the time-shift operation are the same.

C. Shifting in the s-Domain:

If $x(t) \leftrightarrow X(s) \qquad ROC = R$ then

$e^{-s_0 t}x(t) \leftrightarrow X(s-s_0) \qquad R` = R + Re(s_0)$

Equation indicates that the ROC associated with X(s – s0) is that of X(s) shifted by Re (s0).This is illustrated in Fig

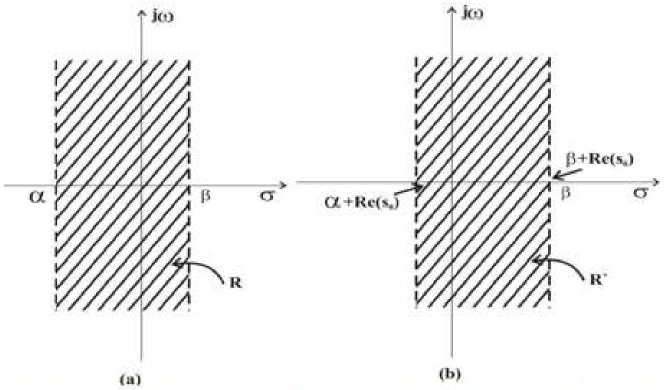


Fig. Effect on the ROC of shifting in the s-domain. (a) ROC of X(S); (b) ROC of X(S-S0)

D. Time Scaling:

If $x(t) \leftrightarrow X(s) \qquad ROC = R$

Then $x(at) \leftrightarrow \frac{1}{|a|}X\left(\frac{s}{a}\right) \qquad R` = aR$

Equation indicates that scaling the time variable t by the factor a causes an inverse scaling of the variable s by l / a as well as an amplitude scaling of X(s/a) by 1/|a|. The corresponding effect on the ROC is illustrated in Fig.

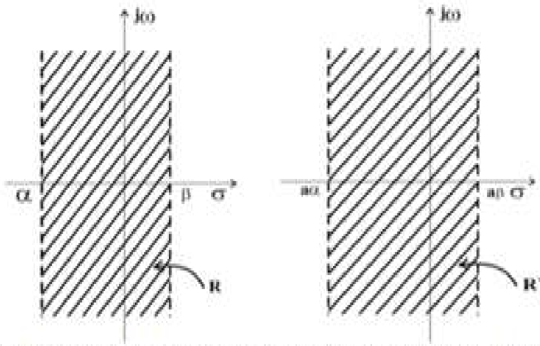


Fig: Effect on the ROC of Time scaling (a) ROC of X(s); (b) ROC of X(s/a)

E. Time Reversal:

If $x(t) \leftrightarrow X(s) \qquad ROC = R$

Then $x(-t) \leftrightarrow X(-s) \qquad R` = -R$

'Thus, time reversal of x (t) produces a reversal of both the σ – and jω-axes in the s-plane.

F. Differentiation in the Time Domain:

If $x(t) \leftrightarrow X(s) \qquad ROC = R$

Then, $-tx(t) \leftrightarrow \frac{dX(s)}{ds} \qquad R' = R$

Equation shows that the effect of differentiation in the time domain is multiplication of the corresponding Laplace transform by s. The associated ROC is unchanged unless there is a pole-zero cancellation at s = 0.

4.5 THE INVERSE LAPLACE TRANSFORM

Inversion of the Laplace transform to find the signal x (t) from its Laplace transform X(s) is called the inverse Laplace transform, symbolically denoted as

$$x(t) = L^{-1}\{X(s)\}$$

A. Inversion Formula

There is a procedure that is applicable to all classes of transform functions that involves the evaluation of a line integral in complex s-plane; that is,

$x(t) = \frac{1}{2\pi j}\int_{c+j\infty}^{c+j\infty} X(s)e^{st}ds$ In this integral, the real c is to be selected such that if the ROC of X(s) is $\sigma_1 < Re(s) < \sigma_2$, then $\sigma_1 < c < \sigma_2$. The evaluation of this inverse Laplace transform integral requires an understanding of complex variable theory.

B. Use of Tables of Laplace Transform Pairs:

In the second method for the inversion of X(s), we attempt to express X(s) as a sum $X(s) = X_1(s) + \ldots + X_n(s)$
Where X1(s), . . . , $X_n(s)$ are functions with known inverse transforms $x_1(t)$, . . . , $x_n(t)$. From the linearity property it follows that $x(t) = x_1(t) + \text{- -} + x_n(t)$

C. Partial-Fraction Expansion:

If X(s) is a rational function, that is, of the form

$$X(s) = \frac{N(s)}{D(s)} = k\frac{(s-z_1)........(s-z_m)}{(s-p_1).......(s-p_n)}$$

a simple technique based on partial-fraction expansion can be used for the inversion of X(s).

4.6 THE SYSTEM FUNCTION

A. THE SYSTEM FUNCTION

We showed that the output y (t) of a continuous-time LTI system equals the convolution of the input x(t) with the impulse response h(t)that is,
$y(t) = x(t) * h(t)$
Applying the convolution property we obtain where Y(s), X(s), and H(s) are the Laplace transforms of y (t), x (t) and h (t), respectively. Equation can be expressed as

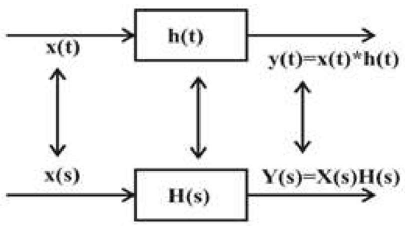


Figure: Impulse response and system Function

The Laplace transform H(s) of h (t) is referred to as the system function (or the transfer Function) of the system. By Equation, the system function H(s) can also be defined as the ratio of the Laplace transforms of the output y (t) and the input x (t). The system function H (t) completely characterizes the system because the impulse response h (t) completely characterizes the system. Figure illustrates the relationship of Equation.

4.7 CHARACTERIZATION OF LTI SYSTEMS

Many properties of continuous-time LTI systems can be closely associated with the characteristics of H(s) in the s-plane and in particular with the pole locations and the ROC.

1. Causality

For a causal continuous-time LTI system, we have h (t) = 0   t < 0

Since h (t) is a right-sided signal, the corresponding requirement on H(s) is that the ROC of H(s) must be of the form

$Re(s) > \sigma_{max}$

That is, the ROC is the region in the s-plane to the right of all of the system poles. Similarly, if the system is anti causal, then h (t) = 0 t > 0

And h (t) is left-sided. Thus, the ROC of H (s) must be of the form $Re(s) < \sigma_{min}$

That is, the ROC is the region in the s-plane to the left of all of the system poles.

2. Stability

A continuous-time LTI system is BIBO stable if and only if $\int_{-\infty}^{\infty} |h(t)| dt < \infty$

The corresponding requirement on H(s) is that the ROC of H(s) contains the jω-axis

3. Causal and Stable Systems

If the system is both causal and stable, then all the poles of H(s) must lie in the left half of the s-plane; that is, they all have negative real parts because the ROC is of the form $Re(s) > \sigma_{max}$, , and since the jω axis is included in the ROC, we must have

$\sigma_{max} < 0$.

4.8 THE UNILATERAL LAPLACE TRANSFORM

A. Definitions

The unilateral (or one-sided) Laplace transform $X_1(s)$ of a signal x(t) is defined as

$$X_1(s) = \int_{0^-}^{\infty} x(t) e^{-st} dt$$

The lower limit of integration is chosen to be 0^- (rather than 0 or 0^+) to permit x (t) to Include δ (t) or its derivatives. Thus, we note immediately that the integration from 0- to $0^+$is zero except when there is an impulse function or its derivative at the origin. The unilateral Laplace transform ignores x (t) for t < 0. Since x (t) in Eq. is a right-sided signal, the ROC of $X_1(s)$ is always of the form $Re(s) > \sigma_{max}$, that is, a right half-plane in the s-plane.

B. Basic Properties

Most of the properties of the unilateral Laplace transform are the same as for the bilateral transform. The unilateral Laplace transform is useful for calculating the response of a causal system to a causal input when the system is described by a linear constant coefficient differential equation with nonzero initial conditions. The basic properties of the unilateral Laplace transform that are useful in this application are the time-differentiation and time-integration properties which are different from those of the bilateral transform.

They are presented in the following:-

1. Differentiation in the Time Domain

$$\frac{dx(t)}{dt} \leftrightarrow sX_1(s) - x(0^-)$$

Provided that $\lim \frac{dx(t)}{dt} \leftrightarrow sX_1(s) - x(0^-)$ Repeated application of this property yields

$$\frac{d^2x(t)}{dt^2} \leftrightarrow s^2X_1(s) - x`(0^-)$$

$$\frac{d^nx(t)}{dt^n} \leftrightarrow s^nX_1(s) - s^{n-1}x(0^-)$$

$-s^{n-2}x`(0^-) - \ldots.. - x^{(n-1)}(0^-)$ where

$$x^r(0^-) = \left.\frac{d^rx(t)}{dt^r}\right|_{t=0^-}$$

2. Integration in the Time Domain

$$\int_{0^-}^{t} x(\tau)d\tau \leftrightarrow \frac{1}{s}X_1$$

$$\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \frac{1}{s}X_1(s) + \frac{1}{s}\int_{-\infty}^{0^-} x(\tau)d\tau$$

C. System Function:

Note that with the unilateral Laplace transform, the system function **H(s)** = Y (s)/ X (s)is defined under the condition that the LTI system is relaxed, that is, all initial conditions are zero.

Table 4.1 Important Laplace Transform pair

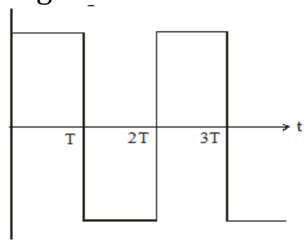
S. No.	f(t)	F(s)	S. No.	f(t)	F(s)
1.	δ (t)	1	14.	$\cos h\, \omega_0 t$	$\frac{s}{s^2-\omega_0^2}$
2.	δ (t-a)	e^{-as}	15.	$e^{-at}\sin h\, \omega_0 t$	$\frac{\omega_0}{(s+a)^2-\omega_0^2}$
3.	u(t)	$\frac{1}{s}$	16	$e^{-at}\cos h\, \omega_0 t$	$\frac{s+a}{(s+a)^2-\omega_0^2}$
4.	u(t – a)	$\frac{e^{-as}}{s}$	17.	$\sin(\omega_0 t + \theta)$	$\frac{s\sin\theta-\omega_0\cos\theta}{s^2+\omega_0^2}$
5.	$\frac{t^n}{n!}$u(t), n positive integer	$(-1)^n\frac{1}{s^{n+1}}$	18.	$\cos(\omega_0 t + \theta)$	$\frac{s\cos\theta-\omega_0\sin\theta}{s^2+\omega_0^2}$
6.	e^{-at}u(t)	$\frac{1}{s+a}$			
7.	$\frac{t^n e^{-at}}{n!}$u(t)	$\frac{1}{(s+a)^{n+1}}$			
8.	$\sin(\omega_0 t)$ u(t)	$\frac{\omega_0}{s^2+\omega_0^2}$			
9.	$\cos(\omega_0 t)$ u(t)	$\frac{s}{s^2+\omega_0^2}$			
10.	$t\cos(\omega_0 t)$ u(t)	$\frac{s^2-\omega_0^2}{(s^2+\omega_0^2)^2}$			
11.	$t\sin(\omega_0 t)$ u(t)	$\frac{2\omega_0 s}{(s^2+\omega_0^2)^2}$			
12.	$e^{-at}\sin(\omega_0 t)$ u(t)	$\frac{\omega_0}{(s^2+a)^2+\omega_0^2}$			
13.	$e^{-at}\cos(\omega_0 t)$ u(t)	$\frac{s+a}{(s+a)^2+\omega_0^2}$			

Table 4.2 Properties of Laplace transform

S.No.	Property	Time domain	Frequency domain
1.	Linearity	$a f_1(t) \pm b f_2(t)$ a and b are constants	$a F_1(s) \pm b F_2(s)$
2.	Scalar multiplication	$kf(t)$	$kF(s)$
3.	Scale change	$f(at), a \geq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
4.	Time delay	$f(t-a), a > 0$	$F(s)e^{-as}$
5.	s-shift	$e^{-at} f(t)$	$F(s+a)$
6.	Multiplication by t^n	$t^n f(t), n = 1, 2, \ldots\ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$
7.	Time differentiation	$f'(t)$ $f''(t)$ $f^n(t')$	$s F(s) - f(0)$ $s^2 F(s) - sf(0) - f'(0)$ $s^n F(s) - s^{n-1} f(0) - s^{n-2}$ $f'(0) - \ldots\ldots - f^{n-1}(0)$
8.	Time integration	$\int_0^t \frac{(t-u)^{n-1}}{(n-1)!} f(u)du$	$\frac{F(s)}{s^n}$
9.	Frequency differentiation	$(-t)^n f(t)$ $-t f(t)$ $t^2 f(t)$	$F^n(s)$ $F'(s) = \frac{dF(s)}{ds}$ $F''(s)$
10.	Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)ds$
11.	Convolution	$f_1(t) * f_2(t)$ $= \int_0^t f_1(\tau) f_2(t-\tau)d\tau$	$F_1(s) F_2(s)$
12.	Final value	$f(\infty) = \lim_{t\to\infty} f(t)$	$\lim_{s\to 0} s F(s)$
13.	Initial value	$f(0^+) = \lim_{t\to 0^=} f(t)$	$\lim_{s\to\infty} s F(s)$
14.	Time periodicity	$f(t) = f(t + nT)$ $n = 1, 2, \ldots\ldots$	$\frac{1}{1-e^{-sT}} F_1(s)$ where $F_1(s) = \int_0^t f(t)e^{-st}dt$

Example

Find Laplace Transform of periodic rectangular waveform.



Period 2T

Solution :

$$F(s) = \frac{1}{1-e^{-st}} \int_0^T f(t) e^{-st} dt = \frac{F_1(s)}{1-e^{-st}}$$

$$L[f(t)] \frac{1}{1-e^{-st}} \int_0^{2T} f(t) e^{-st} dt$$

$$= \frac{1}{1-e^{-2Ts}} \left(\int_0^{2T} Ae^{-st} dt + \int_0^{2T} (A) e^{-st} dt \right)$$

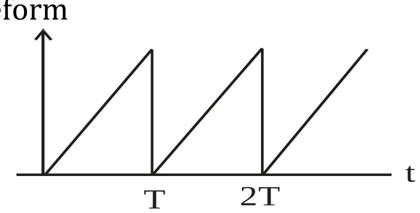
$$= \frac{A}{S}(e^{-st}) + \frac{A}{S} e^{-st} - \frac{A}{S} e^{-st}$$

$$= \frac{1}{1-e^{-2Ts}} \frac{A}{S}(1-e^{-ST})2 = \frac{A}{S}\left[\frac{1-e^{-ST}}{1+e^{-ST}}\right]$$

$$L[f(t)] = \frac{A}{s} \tan h\left(\frac{ST}{2}\right)$$

Example

Laplace Transform of periodic saw tooth waveform



Solution

$$f(t) = \begin{pmatrix} \frac{A}{T} t & 0 < t < T \\ 0 & \text{otherwise} \end{pmatrix}$$

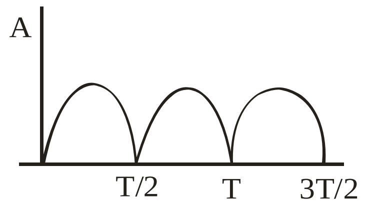
$$L[F(t)] = \frac{1}{1-e^{-sT}} \frac{A}{T} \int_0^T te^{-sT} dt$$

$$= \frac{A}{TS^2(1-e^{sT})}(1-e^{-sT} - STe^{-sT})$$

Example

Laplace Transform of full wave rectified output

f (t)



Solution:

$f(t) = A \sin \omega_0 t \quad 0 < t < T/2$

$$L[f(t)] = \frac{A}{1-e^{-sT/2}} \int_0^{T/2} \sin \omega_0 t \, e^{-sT} dt$$

$$= \frac{A}{1-e^{-sT/2}} \frac{w_0}{w^2+s^2}(1+e^{-sT/2})$$

$$= \frac{Aw_0}{w^2+s^2} \cdot \frac{e^{\frac{ST}{4}} + e^{\frac{-ST}{4}}}{e^{\frac{ST}{4}} - e^{\frac{-ST4}{4}}}$$

$$= \frac{Aw_0}{s^2+w_0^2} \cot \ h\left(\frac{ST}{4}\right)$$

GATE QUESTIONS(EC)

Q.1 The transfer function of a system is given by H(s) = $\frac{1}{s^2(s-2)}$. The impulse response of the system is
a) $(t^2 * e^{-2t})U(t)$
b) (t * e^{2t}) U(t
c) ($te^{-2}t$) U (t)
d) (te^{-2t})U(t)
(*denotes convolution, and u(t) is a unit step function)
[GATE-2001]

Q.2 The Laplace transform of a continuous –time signal $x(t)$ is $X(s) = \frac{5-s}{s^2-s-2}$ If the Fourier transform of this signal exists, then x(t) is
a) $e^{2t}u(t) - 2e^{-t}u(t)$
b) $-e^{2t}u(-t) + 2e^{-t}u(t)$
c) $-e^{2t}u(-t) - 2e^{-t}u(t)$
d) $e^{2t}u(-t) - 2e^{-t}u(t)$
[GATE-2002]

Q.3 The Laplace transform of i(t) is given by $I(s) = \frac{2}{s(1+s)}$ As $t \to \infty$ the value of i(t) tends to
a) 0
b) 1
c) 2
d) ∞
[GATE-2003]

Q.4 Consider the function f(t) having Laplace transform
$F(s) = \frac{\omega_0}{s^2+\omega_0^2}$ $Re[s] > 0$
The final value of f(t) would be :
a) 0
b) 1
c) $-1 \le f(\infty) \le 1$
d) ∞
[GATE-2006]

Q.5 If the Laplace transform of a signal y(t) is $Y(s) = \frac{1}{s(s-1)}$, then its final value is
a) -1
b) 0
c) 1
d) Unbounded
[GATE-2007]

Q.6 Given that F(s) is the one-sided Laplace transform of f(t), the Laplace transform of $\int_0^t f(\tau)d\tau$ is
a) $sF(s) - f(0)$
b) $\frac{1}{s}F(s)$
c) $\int_0^s F(\tau)d\tau$
d) $\frac{1}{s}[F(s) - f(0)]$
[GATE-2009]

Q.7 A continuous time LTI system is described by
$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$
Assuming zero initial conditions the response y(t) of the above system for the input $x(t) = e^{-2t}u(t)$ is given by
a) $(e^t - e^{3t})u(t)$
b) $(e^t - e^{-3t})u(t)$
c) $(e^{-t} + e^{-3t})u(t)$
d) $(e^t + e^{3t})u(t)$
[GATE-2010]

Q.8 If the unit step response of a network is $(1 - e^{\alpha t})$ then its unit impulse response is
a) $\alpha e^{-\alpha t}$
b) $\alpha^{-1}e^{-\alpha t}$
c) $(1 - \alpha^{-1})e^{-\alpha t}$
d) $(1 - \alpha)e^{-\alpha t}$
[GATE-2011]

Q.9 An input $x(t) = \exp(-2t)\,u(t) + \delta(t-6)$ is applied to an LTI system with impulse response $h(t) = u(t)$. The output is
a) $[1 - \exp(-2t)]u(t) + u(t+6)$
b) $[1 - \exp(-2t)]u(t) + u(t-6)$
c) $0.5[1 - \exp(-2t)]u(t) + u(t+6)$
d) $0.5\,[1 - \exp(-2t)]u(t) + u(t-6)$
[GATE-2011]

Q.10 If $F(s) = L[f(t)] = \frac{2(s+1)}{s^2+4s+7}$ then the initial and final values of f(t) are respectively
a) 0, 2
b) 2, 0
c) 0, 2/7
d) 2/7, 0

[GATE-2011]

Q.11 The unilateral Laplace transform of f(t) is $\frac{1}{s^2+s+1}$ The unilateral Laplace transform of t f(t) is

a) $-\frac{1}{(s^2+s+1)^2}$ b) $-\frac{1}{(s^2+s+1)^2}$

c) $\frac{s}{(s^2+s+1)^2}$ d) $\frac{2s+1}{(s^2+s+1)^2}$

[GATE-2012]

Q.12 The impulse response of a system is h(t) = t u(t). For an input u(t – 1), the output is

a) $\frac{t^2}{2}u(t)$

b) $\frac{t(t-1)}{2}u(t-1)$

c) $\frac{(t-1)^2}{2}u(t-1)$

d) $\frac{t^2-1}{2}u(t-1)$

[GATE-2013]

Q.13 A system is described by the following differential equation, where u(t) is the input to the system and y(t) is the output of the system.

$\dot{y}(t)+5y(t)=u(t)$

When y(0)=1 and u(t0 is a unit step function, y(t) is

a) $0.2+0.8e^{-5t}$ b) $0.2-0.2e^{-5t}$

c) $0.8+0.2e^{-5t}$ d) $0.8-0.8e^{-5t}$

[GATE 2014, SET-1]

Q.14 Let the signal f(t)=0 outside the interval $[T_1,T_2]$, where T_1 and T_2 are finite. Furthermore, $|f(t)|<\infty$. The region of convergence (RoC) of the signal's bilateral Laplace transform F(s) is

a) a parallel strip containing $j\Omega$ axis

b) a parallel strip not containing $j\Omega$ axis

c) the entire s-plane

d) a half- plane containing the $j\Omega$ axis

[GATE 2015, SET-2]

Q.15 Consider the following statements for continuous-time linear time invariant(LTI) systems.

I. There is no bounded input bounded output (BIBO) stable system with a pole in the right half of the complex plane.

II. There is no casual and BIBO stable system with a pole in the right half of the complex plane.

Which of the following is correct?

a) Both I and II are true

b) Both I and II are not true

c) Only I is true

d) Only II is true

[GATE 2017, SET-1]

ANSWER KEY:

1	2	3	4	5	6	7	8
b	d	c	c	d	b	b	a
9	10	11	12	13	14	15	
d	b	d	c	A	C	d	

EXPLANATIONS

Q.1 **(b)**

Impulse response of system is

$L^{-1}[H(s)]$

$\frac{1}{s^2(s-2)} = \frac{1}{s^2} \times \frac{1}{s-2} = (t^* e^{+2T})u(t)$

Q.2 **(d)**

$X(s) = \frac{5-s}{s^2-s-2} = \frac{5-s}{(s+1)(s-2)}$

$\frac{5-s}{(s+1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-2}$

$5 - s = A(s-2) + B(s+1)$

$s = 2{,}3 = 3B$

$\Rightarrow\ B = 1$

$s = -1{,}6 = -3A$

$\Rightarrow\ A = -2$

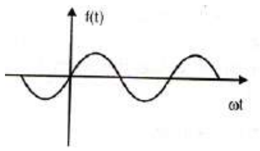
$\therefore x(t) = -2e^{t}u(t) + e^{2t}u(-t)$

Q.3 **(c)**

$\lim i_{t\to\infty}(t) = \lim_{s\to 0} sI(s)$

$= \lim_{s\to 0} s\frac{2}{s(1+s)} = 2$

Q.4 **(c)**



$L^{-1}[F(s)] = \sin\omega_0 t$

$F(t) = \sin\omega_0 t$

$-1 \le f(\infty) \le 1$

Q.5 **(d)**

Final value theorem is applicable only when all the poles of system lies in left half of s-plane.

$\because s = 1$ is right s-plane pole

$\therefore$ Unbounded.

Q.6 **(b)**

$\lambda\left[\int_0^t f(\tau)\,d\tau\right] = \frac{F(s)}{s} + \frac{f^{-1}(0^+)}{S} = \frac{F(s)}{s}$

With zero initial condition .

Q.7 **(b)**

$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t)$

$= 2\frac{dx(t)}{dt} + 4x(t)$

Taking Laplace transform on both sides

(Assuming zero initial conditions),

$s^2Y(s) + 4sY(s) + 3Y(s)$

$= 2sX(s) + 4X(s)$

Or $\frac{Y(s)}{X(s)} = \frac{2s+4}{s^2+4s+3}$

$= \frac{2(s+2)}{(s+1)(s+3)}$

Given that,

$x(t) = e^{-2t}u(t)$

$X(s) = \frac{1}{s+2}$

$Y(s) = \frac{2}{(s+1)(s+3)(s+2)}$

$= \frac{2}{(s+1)(s+3)}$

$= \frac{1}{s+1} - \frac{1}{s+3}$

Taking inverse Laplace transform on both sides,

$Y(t) = (e^{-t} - e^{-3t})\,u(t)$

Q.8 **(a)**

Unit step response

$s(t) = (1 - e^{-\alpha t})$

So, unit impulse response is

$h(t) = \frac{ds(t)}{dt} = \frac{d}{dt}(1 - e^{-\alpha t})$

$= \alpha e^{-\alpha t}$

Q.9 **(d)**

$x(t) = e^{-2t}u(t) + \delta(t-6)$

$X(s) = \frac{1}{s+2} + e^{-6s}$

$H(s) = \frac{1}{s}$

$\therefore Y(s) = X(s).H(s)$

$\Rightarrow\ Y(s) = \frac{1}{s(s+2)} + \frac{1}{s}e^{-6s}$

$\Rightarrow\ Y(s) = \frac{1}{2}\left(\frac{1}{s} - \frac{1}{s(s+2)}\right) + \frac{1}{s}e^{-6s}$

Taking inverse Laplace transform ,we have

$y(t)0.5\,(1 - e^{-2t})u(t) + u(t-6)$

Q.10 **(b)**

$$F(s) = L[f(t)] = \frac{2(s+1)}{s^2+4s+7}$$

Initial value ,

$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$

$= s.\lim_{s\to\infty} \frac{2(s+1)}{s^2+4s+7}$

$\lim_{s\to\infty} \frac{2s^2(1+\frac{1}{s})}{s^2(1+\frac{4}{s}+\frac{7}{s^2})}$

$\frac{2(1+0)}{(1+0+0)} = 2$

Final value ,

$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$

$= s.\lim_{s\to\infty} \frac{2(s+1)}{s^2+4s+7} = 0$

Q.11 (d)

If $L[f(t)] = \bar{f}(s)$then

$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\bar{f}(s)$

In this problem.

Given,

$L[f(t)] = \frac{1}{s^2+s+1} = \bar{f}(s)$

We need

$L[t\,f(t)] = (-1)^1 \frac{d^1}{ds^1}\bar{f}(s)$

$= -\frac{d}{ds}\bar{f}(s)$

$= -\frac{d}{ds}\left[\frac{1}{s^2+s+1}\right]$

$= -\left[\frac{1}{(s^2+s+1)^2}\right] \times (2s+1)$

$= \left[\frac{2S+1}{(s^2+s+1)^2}\right]$

Q.12 (c)

$h(t) = tu(t)$

Taking Laplace transform

$H(s) = \frac{1}{s^2}$

$x(t) = u(t-1)$

Taking Laplace transform

$X(s) = \frac{e^{-s}}{s}$

$\frac{Y(s)}{X(s)} = H(s)$

$Y(s) = H(s)X(s)$

$Y(s) = \frac{1}{s^2}.\frac{e^{-s}}{s} = \frac{e^{-s}}{s^3}$

Taking the inverse Laplace transform

$Y(t) = \frac{(t-1)^2}{2}u(t-1)$

Q.13 (a)

$y(t)+5y(t) = u(t)$

$sY(s)-y(0)+5Y(s) = \frac{1}{s}$

$(s+5)Y(s)-1 = \frac{1}{s}$

$(s+5)Y(s) = \frac{s+1}{s}$

$Y(s) = \frac{(s+1)}{s(s+5)}$

$\frac{A}{s}+\frac{B}{(s+5)} = \frac{(s+1)}{s(s+5)}$

$A = 0.2$

$B = \frac{-4}{-5} = 0.8$

$Y(s) = \frac{0.2}{s}+\frac{0.8}{(s+5)}$

$y(t) = 0.2+0.8e^{-5t}$

Q.14 (c)

ROC of a finite duration signal is entire s-plane

Q.15 (d)

A BIBO stable system can have poles in right half of complex plane, if it is a non causal system. So, statement-I is wrong.

A causal and BIBO stable system should have all poles in the left half of complex plane. So, statement-II is correct.

GATE QUESTIONS(EE)

Q.1 Consider the function. $F(s)=\frac{5}{s(s^2+3s+2)}$ where F(s) is the Laplace transform of the function f(t). The initial value of f(t) is equal to

a) 5 b) $\frac{5}{2}$

c) $\frac{5}{3}$ d) 0

[GATE-2004]

Q.2 The Laplace transform of a function f(t) is $F(s)=\frac{5s^2+23s+6}{s(s^2+2s+2)}$. As $t\to\infty, f(t)$ approaches

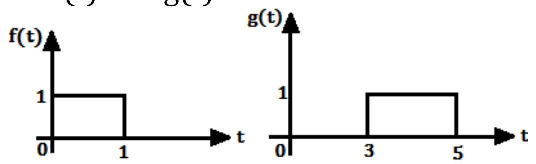
a) 3 b) 5

c) $\frac{17}{2}$ d) ∞

[GATE-2005]

Common Data Questions 3 & 4

Given f(t) and g(t) as shown below:



Q.3 g(t) can be expressed as

a) $g(t)=f(2t-3)$ b) $g(t)=f\left(\frac{t}{2}-3\right)$

c) $g(t)=f\left(2t-\frac{3}{2}\right)$ d) $g(t)=f\left(\frac{t}{2}-\frac{3}{2}\right)$

[GATE-2010]

Q.4 The Laplace transform of g(t) is

a) $\frac{1}{s}(e^{3s}-e^{5s})$ b) $\frac{1}{s}(e^{-5s}-e^{-3s})$

c) $\frac{e^{-3s}}{s}(1-e^{-2s})$ d) $\frac{1}{s}(e^{5s}-e^{3s})$

[GATE-2010]

Q.5 Let the Laplace transform of a function f(t) which exists for $t>0$ be $F_1(s)$ and the Laplace transform of its delayed version $f(t-\tau)$ be $F_2(s)$. $F_1^*(s)$ be the complex conjugate of $F_1(s)$ with the Laplace variable set as $s=\sigma+j\omega$. If $G(s)=\frac{F_2(s)F_1^*(s)}{|F_1(s)|^2}$, then the inverse Laplace transform of G(s) is

a) an ideal impulse $\delta(t)$

b) an ideal delayed impulse $\delta(t-\tau)$

c) an ideal step function $u(t)$

d) an ideal delayed step function $u(t-\tau)$

[GATE-2011]

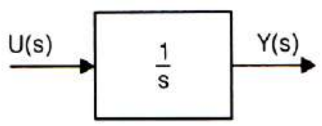
Q.6 The unilateral Laplace transform of f(t) is $\frac{1}{s^2+s+1}$ The unilateral Laplace transform of t f(t) is

a) $-\frac{1}{(s^2+s+1)^2}$ b) $-\frac{1}{(s^2+s+1)^2}$

c) $\frac{s}{(s^2+s+1)^2}$ d) $\frac{2s+1}{(s^2+s+1)^2}$

[GATE-2012]

Q.7 Assuming zero initial condition, the response y(t) of the system given below to a unit step input u(t) is



a) $u(t)$ b) $tu(t)$

c) $\frac{t^2}{2}u(t)$ d) $e^{-t}u(t)$

[GATE-2013]

Q.8 Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI system?

a) All the poles of the system must lie on the left side of the $j\omega$ axis.

b) Zeros of the system can lie anywhere in the s-plane.

c) All the poles must be within $|s|=1$

d) All the roots of the characteristic equation must be located on the left side of the $j\omega$ axis.

[GATE-2013]

Q.9 The Laplace Transform of $f(t)=2\sqrt{t/\pi}$ is $s^{-3/2}$. The Laplace Transform of $g(t)=\sqrt{1/t\pi}$ is

a) $\frac{3s^{-5/2}}{2}$ b) $s^{-1/2}$
b) $s^{1/2}$ d) $s^{3/2}$

[GATE-2015]

Q.10 The Laplace Transform of $f(t) = e^{2t}\sin(5t)u(t)$ is

a) $\frac{5}{s^2-4s+29}$ b) $\frac{5}{s^2+5}$
c) $\frac{s-2}{s^2-4s+29}$ d) $\frac{5}{s^2+5}$

[GATE-2016]

Q.11 The solution of the differential equation, for t>0, y''(t) + 2y'(t) + y(t)=0 with initial conditions y(0)=0 and y'(0)=1, is (u(t) denotes the unit step function),

a) $te^{-t}u(t)$ b) $(e^{-t}-te^{-t})u(t)$
c) $(e^{-t}+te^{-t})u(t)$ d) $e^{-t}u(t)$

[GATE-2016]

Q.12 Consider a Linear Time Invariant system with transfer function

$$H(s) = \frac{1}{(s+1)}$$

If the input is cos(t) and the steady state output is A cos(t+α), then the value of A is______

[GATE-2016]

ANSWER KEY:

1	2	3	4	5	6
d	a	d	c	b	d
7	8	9	10	11	12
b	c	b	a	a	0.707

EXPLANATIONS

Q.1 (d)

$F(s) = \frac{5}{s(s^2+3s+2)}$

By initial value theorem

$\text{Lim}_{t\to 0} f(t) = \text{Lt}_{s\to\infty}\ sF(s)$

Q.2 (a)

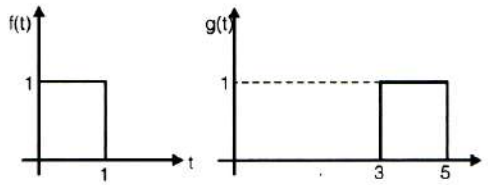
$F(s) = \frac{5s^2+23s+6}{s(s^2+2s+2)}$

By final value theorem,

$\text{Lim}_{t\to\infty} F(t) = \text{Lim}_{s\to 0} sF(s)$

$= \frac{6}{2} = 3$

Q.3 (d)



Since g(t) has width of 2-unit and f(t) has 1 unit therefore we have to first expand f(t) by 2 unit and for this we have to a scale f(t) by ½

i.e. $f(t) = f\left(\frac{1}{2}t\right)$

Now shift it by three unit to get g(t)

$g(t) = f_1(t-3)$

$g(t) = f\left(\frac{t-3}{2}\right)$

$= f\left(\frac{t}{2} - \frac{3}{2}\right)$

Q.4 (c)

$g(t) = u(t) \qquad 3 < t < 5$

$= 0 \qquad t < 3, t > 5$

$\Rightarrow g(t) = u(t-3) - u(t-5)$

$L\{g(t)\} = L\{u(t-3)\}$

$= \frac{1}{s}e^{-3s} - \frac{1}{s}e^{-5s}$

$-L\{u(t-5)\}$

$= \frac{1}{s}(e^{-3s} - e^{-5s})$

$= \frac{e^{-3s}}{s}(1 - e^{-2s})$

Q.5 (b)

$F_2(t) = L\{f(t-\tau)\} = e^{-s\tau}F_1(s)$

$G(s) = \frac{e^{-s\tau}F_1(s)F_1^*(s)}{|F_1(s)|^2} = e^{-s\tau}$

$G(t) = \delta(t-\tau)$

Q.6 (d)

If $L[f(t)] = \bar{f}(s)$ then

$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\bar{f}(s)$

In this problem.

Given,

$L[f(t)] = \frac{1}{s^2+s+1} = \bar{f}(s)$

We need

$L[t\,f(t)] = (-1)^1 \frac{d^1}{ds^1}\bar{f}(s)$

$= -\frac{d}{ds}\bar{f}(s)$

$= -\frac{d}{ds}\left[\frac{1}{s^2+s+1}\right]$

$= -\left[\frac{1}{(s^2+s+1)^2}\right] \times (2s+1)$

$= \left[\frac{2S+1}{(s^2+s+1)^2}\right]$

Q.7 (b)

$x(t) = u(t)$

Apply Laplace transform

$X(s) = \frac{1}{s}$

$H(s) = \frac{1}{s}$ (given)

$Y(s) = H(s)X(s)$

$= \frac{1}{s} \times \frac{1}{s} = \frac{1}{s^2}$

Taking inverse Laplace transform

$y(t) = tu(t)$

Q.8 (c)

For a causal and stable LTI system the ROC must be right sided and it must include the $s = j\omega$ line $(\sigma = 0)$

For all the poles must lie within $|s| = 1$ then all poles will lie between -1 to $+1$ We know that ROC does not include any pole, so in this case ROC will not be able to include $s = j\omega$ line $(\sigma = 0)$. Thus the system will not be stable .Hence this statement is NOT TRUE.

Q.9 (b)

Given that,

$$f(t) \xrightarrow{L.T.} s^{-3/2}$$

$$2\sqrt{\frac{t}{2\pi}} \xrightarrow{L.T.} s^{-3/2}$$

To find Laplace transform of $\sqrt{\frac{1}{\pi t}}$.

Using property that :

$$\text{if } f(t) \xrightarrow{L.T.} F(s)$$

$$\text{then, } \frac{1}{t} f(t) \xrightarrow{L.T.} -\int F(s)ds$$

$$\text{so, } 2 \cdot \sqrt{\frac{1}{\pi t}} \xrightarrow{L.T.} 2s^{-1/2}$$

$$\text{Thus, } \sqrt{\frac{1}{\pi t}} \xrightarrow{L.T.} s^{-1/2}$$

Q.10 (a)

$$\text{Laplace transform of } \sin 5t\, u(t) \longrightarrow \frac{5}{s^2+25}$$

$$e^{2t} \sin 5t\, u(t) \longrightarrow \frac{5}{(s-2)^2+25} = \frac{5}{s^2-4s+29}$$

Q.11 (a)

The differential equation is

Y''(t)+2y'(t)+y(t)=0

$$\text{So, } (s^2Y(s) - sy(0) - y'(0)) + 2[sY(s) - y(0)] + Y(s) = 0$$

$$\text{So, } Y(s) = \frac{sy(0) + y'(0) + 2y(0)}{(s^2+2s+1)}$$

Given that $y'(0) = 1, y(0) = 0$

$$\text{So, } Y(s) = \frac{1}{(s+1)^2}$$

$$\text{So, } y(t) = te^{-t}u(t)$$

Q.12 0.707

$$H(s) = \frac{1}{s+1}$$

put $s = j\omega$,

$$H(j\omega) = \frac{1}{j\omega+1}$$

$$|H(j\omega)| = \frac{1}{\sqrt{\omega^2+1}}$$

$\because$ input $x(t) = \cos(t)$

Here $x = 1$ rad / sec

and $|x(t)| = 1$

Hence, steady state output

$$y(t) = |x(t)| \times |H(j\omega)|_{\omega=1} \cos[t + \angle H(j\omega)]$$

$$A = |x(t)| \times |H(j\omega)|_{\omega=1}$$

$$A = \frac{1}{\sqrt{2}} = 0.707$$

GATE QUESTIONS(IN)

Q.1 Identify the transfer function corresponding to an all-pass filter from the following:

a) $\frac{1-s\tau}{1+s\tau}$ b) $\frac{1-s\tau 1}{1+s\tau 2}$

c) $\frac{1}{1+s\tau}$ d) $\frac{s\tau}{1+s\tau}$

[GATE-2005]

Q.2 Let the signal x(t) have the Fourier transform $X(\omega)$. Consider the signals $y(t) = \frac{d}{dt}[x(t - t_d)]$ where t_d is an arbitrary delay .The magnitude of the Fourier transform of y(t) is given by the expression

a) $|X(\omega)|.|\omega|$ b) $|X(\omega)|.\omega$

c) $\omega^2.|X(\omega)|$ d) $|\omega||X(\omega)|.e^{-j\omega td}$

[GATE-2007]

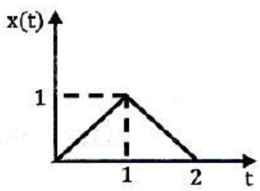
Q.3 $u(t)$represents the unit step function .The Laplace transform of $u(t-\tau)$ is

a)$\frac{1}{s\tau}$ b) $\frac{1}{s-\tau}$

c)$\frac{e^{-s\tau}}{s}$ d) $e^{-s\tau}$

[GATE-2010]

Q.4 The Laplace Transform representation of the triangular pulse shown below is



a)$\frac{1}{s^2}[1+e^{-2s}]$

b)$\frac{1}{s^2}[1-e^{-s}+e^{-2s}]$

c)$\frac{1}{s^2}[1-e^{-s}+2e^{-2s}]$

d)$\frac{1}{s^2}[1-2e^{-s}+e^{-2s}]$

[GATE-2013]

Q.5 If X(s), the Laplace transform of signal $x(t)$ is given by $\frac{(S+2)}{(S+1)(S+3)^2}$,then the value of x(t) as $t\rightarrow\infty$ is _____.

[GATE-2016]

Q.6 A system is described by the following differential equation:

$\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + x(t)$, $x(0)=y(0)=0$

Where x(t) and y(t) are the input and output variables respectively. The transfer function of the inverse system is

a) $\frac{s+1}{s-2}$ b) $\frac{s+2}{s+1}$

c) $\frac{s+1}{s+2}$ d) $\frac{s-1}{s-2}$

[GATE-2017]

Q.7 The Laplace transform of a causal signal y (t) is Y(s) $=\frac{s+2}{s+6}$. The value of the signal y (t) at t=0.1s is__________units.

[GATE-2017]

Q.8 Unit step resonance of a linear time invariant (LTI) system is given by $y(t)=(1-e^{-2t})u(t)$. Assuming zero initial condition, the transfer function of the system is

a) $\frac{1}{s+1}$ b) $\frac{2}{(s+1)(s+2)}$

c) $\frac{1}{s+2}$ d) $\frac{2}{s+2}$

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8
a	a	c	d	0	b	-2.195	d

EXPLANATIONS

Q.1 (a)

Q.2 (a)

$F\{x(t-t_d)\} = e^{-j\omega t d}.X(J\omega)$

$F\{\frac{d}{dt}(x(t-t_d))\} = J\omega.e^{-j\omega t d}.X(J\omega)$

$\therefore |Y(J\omega)| = .|\omega||X(\omega)|$

Q.3 (c)

Laplace transform of u(t)=1/s

Use Time shifting property:

If the L.T of f(t) is F(s),

Then L.T of $x(t)= f(t-\tau)$ is

$X(s) = e^{-s\tau}F(s)$

$\therefore$ L.T of $u(t-\tau)$ is $\frac{e^{-s\tau}}{s}$

Q.4 (d)

$$x(t)=r(t)-2r(t-1)+r(t-2)$$

$$X(s)=\frac{1}{s^2}\left[1-2e^{-s}+e^{-2s}\right]$$

Q.5 (0)

$$t\xrightarrow{\text{l t f}}\infty(t)=s\frac{s+2}{(s+1)(s+2)}=0$$

Q.6 (C)

$$\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + x(t)$$

$$\text{So, } H(s) = \frac{Y(s)}{X(s)} = \frac{s+1}{s+2}$$

Q.7 -2.195

$y(s) = \frac{s+2}{s+6}$ is proper converting

to strictly proper

$$y(s)=1-\frac{4}{s+6}$$

$$y(t) = \delta(t)-4e^{-6t}U(t)$$

$$y(0.1) = 0-4e^{-6t}U(t) =-2.195$$

Q.8 (d)

Given unit step response

$$y(t)=\left(1-e^{-2t}\right)u(t)$$

impulse response

$$h(t)=\frac{d}{dt}y(t)$$

$$h(t)=\delta(t)-e^{-2t}\delta(t)+2e^{-2t}u(t)$$

$$=2e^{-2t}u(t)$$

$$L\{h(t)\}=\text{Transfer function}$$

$$\text{T.F}=L\{2e^{-2t}u(t)\}=\frac{2}{s+2}$$

5 Z-TRANSFORM

5.1 Introduction to Z – Transform

For a general discrete-time signal x[n], the z-transform X (z) is defined as

$$X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

The variable z is generally complex-valued and is expressed in polar form as $Z = re^{j\Omega}$
Where r is the magnitude of ***z*** and Ω is the angle of ***z.*** The z-transform defined in equation is often called the bilateral (or two-sided) z-transform in contrast to the unilateral (or one-sided) z-transform, which is defined as

$$X_1(z)=\sum_{n=-0}^{\infty} x[n]z^{-n}$$

Clearly the bilateral and unilateral z-transforms are equivalent only if x[n] = 0 for n< 0. We will omit the word "bilateral" except where it is needed to avoid ambiguity.
As in the case of the Laplace transform, equation is sometimes considered an operator that transforms a sequence x[n] into a function X (z), symbolically represented by

$$X(z)=3\{x[n]\}$$

The x[n] and X (z) are said to form a z-transform pair denoted as

$$x[n]\leftrightarrow X(z)$$

5.2 THE REGION OF CONVERGENCE

As in the case of the Laplace transform, the range of values of the complex variable z for which the z-transform converges is called the region of convergence. To

illustrate the z-transform and the associated ROC, let us consider some examples.

Example:
Consider the sequence x[n] = $a^n u[n]$, a is real then by equation the z-transform of x[n]is?
Solution:

$$X(z)=\sum_{n=-\infty}^{\infty} a^n u[n]z^{-n}=\sum_{n=-0}^{\infty}\left(az^{-1}\right)^n$$

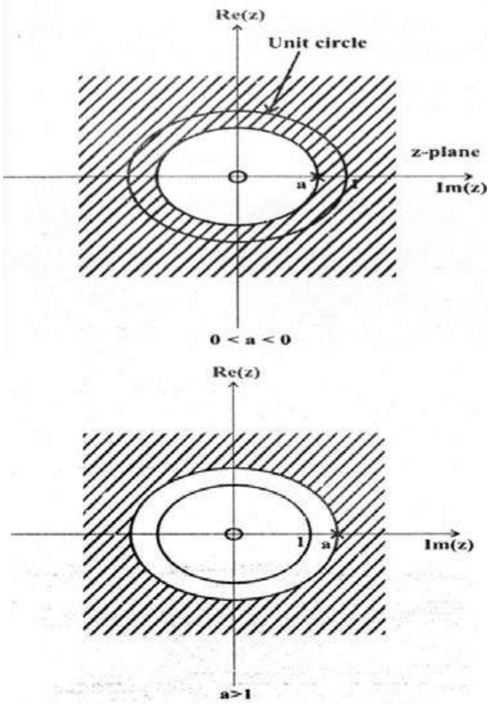
For the convergence of X (z) we require that

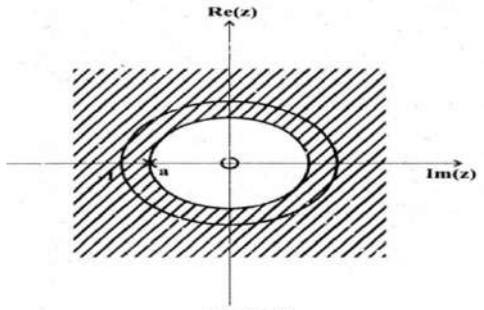
$$\sum_{n=-0}^{\infty}\left|az^{-1}\right|^n<\infty$$

Thus, the ROC is the range of values of z for which $\left|az^{-1}\right|^n<1$ or, equivalently, $|z|>|a|$
Then

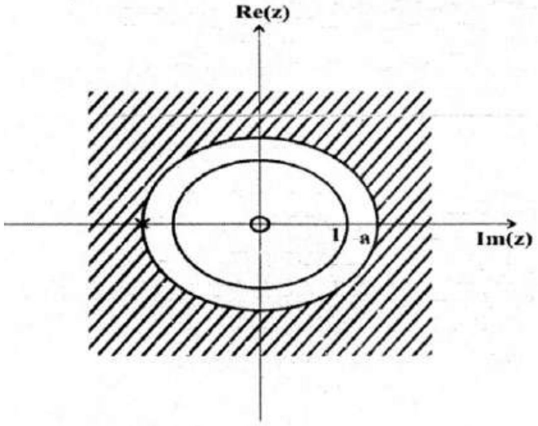
$$X(z)=\sum_{n=-0}^{\infty}\left(az^{-1}\right)^n=\frac{1}{1-az^{-1}}\quad |z|>|a|$$

Fig ROC of the Form |z|>|a|





$-1 < a < 0$



$a < -1$

Alternatively, by multiplying the numerator and denominator of equation by *z,* we may write X (z). As from equation, we see that X(z) is a rational function of z. Consequently, just as with rational Laplace transforms, it can be characterized by its zeros (the roots of the numerator polynomial) and its poles (the roots of the denominator polynomial). From equation we see that there is one zero at z = 0 and one pole at z ***=a.*** The ROC and the pole-zero plots for this example are shown in Fig. In z-transform applications, the complex plane is commonly referred to as the z-plane.

Example: Consider the sequence $x[n] = -a^n u[-n-1]$, its z-transform X (z) is given by

Solution:

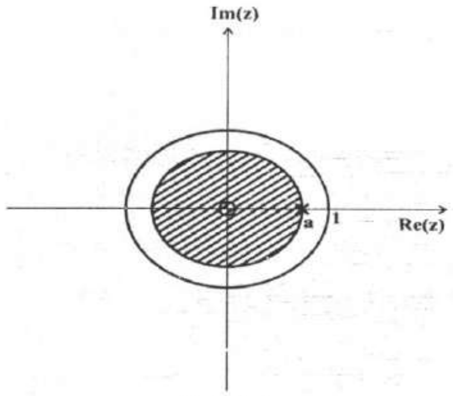
$$X(z)=\frac{1}{1-az^{-1}}\quad |z|<|a|$$

Again, as before, X (z) may be written as

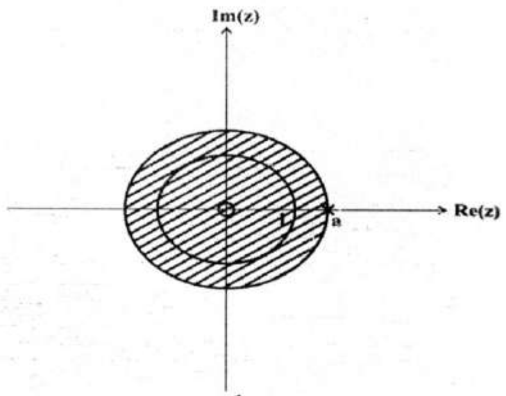
$$X(z)=\frac{z}{z-a}\quad |z|<|a|$$

Thus, the ROC and the pole-zero plots for this example are shown in Fig. Comparing equation, we see that the algebraic expressions of X (z)for two different sequences are identical except for the ROCs. Thus, as in the Laplace transform, specification of the z-transform requires both the algebraic expression and the ROC.

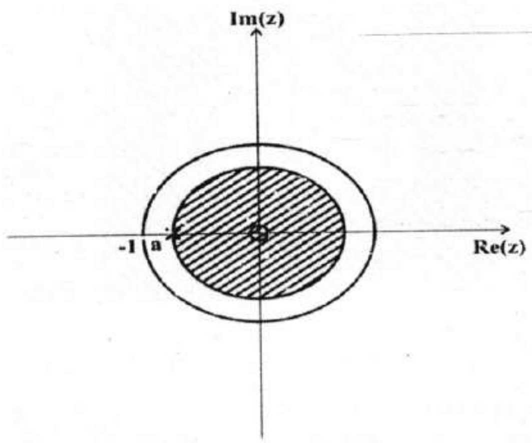
Fig. ROC of the Form |z|<|a|



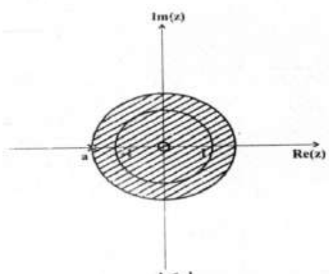
$0 < a < -1$



$a>1$



$-1 < a < 0$



$A < -1$

5.3 PROPERTIES OF THE REGION OF CONVERGENCE (ROC)

As we saw in the Examples and the ROC of X (z) depends on the nature of x[n]. The properties of the ROC are summarized below. We assume that X (z) is a rational function of z.

Property 1
The ROC does not contain any poles.

Property 2
If x[n] is a finite sequence (that is, x[n] =0 except in a finite interval $N_1 \le n \le N_2$ where N_1 and N_2 are finite) and X (z) converges for some value of z, then the ROC is the entire z-plane except possibly z = 0 or z = ∞.

Property 3
If x[n]is a right-sided sequence then, the ROC is entire z plane except z=0

Property 4
If x[n]is a left-sided sequence, then the ROC is entire z plane except z=∞

Property 5
If x (n) is a two-sided sequence, then the ROC is entire z plane except at z=0 and z=∞

Z-TRANSFORMS OF SOME COMMON SEQUENCES

A. Unit Impulse Sequence δ[n]:
From definition

$$X(z) = \sum_{n=-\infty}^{\infty} \delta[n] z^{-n} = z^{-0} = 1 \text{ For all z}$$

B. Unit Step Sequence u[n]
Setting a = 1 in equation, we obtain

$$u[n] \leftrightarrow \frac{1}{1-z^{-1}} = \frac{z}{z-1} |z| > 1$$

Example
Find X (z) & its ROC for given discrete sequence x[n]?

a) $x(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{3}\right)^n u(n)$

b) $x(n) = \left(\frac{1}{3}\right)^n u(n) + \left(\frac{1}{2}\right)^n u(-n-1)$

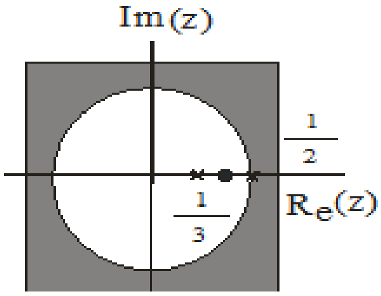
c) $x(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{3}\right)^n u(-n-1)$

Solution

a) $\left(\frac{1}{2}\right)^n u(n) \leftrightarrow \frac{z}{z-\frac{1}{z}}$ $\quad |z| > \frac{1}{z}$

$\left(\frac{1}{3}\right)^n u(n) \leftrightarrow \frac{z}{z-\frac{1}{3}}$ $\quad |z| > \frac{1}{3}$

$$X(z) = \frac{z}{z-\frac{1}{2}} + \frac{z}{z-\frac{1}{3}}$$

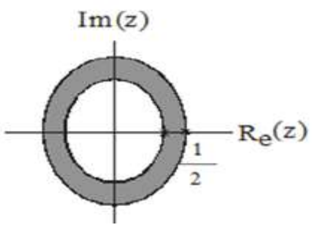


$$X(z) = \frac{2z\left(z-\frac{5}{12}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} \quad |z| > \frac{1}{2}$$

b) $\left(\frac{1}{3}\right)^n u(n) \leftrightarrow \frac{z}{z-\frac{1}{3}}$ $\quad |z| > \frac{1}{3}$

$\left(\frac{1}{2}\right)^n u(n) \leftrightarrow \frac{z}{z-\frac{1}{2}}$ $\quad |z| > \frac{1}{2}$

$$X(z) = \frac{z}{z-\frac{1}{3}} + \frac{z}{z-\frac{1}{2}}$$



$$X(z) = -\frac{1}{6}\frac{z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$

$$= \frac{1}{3} < |z| < \frac{1}{2}$$

c) $\left(\frac{1}{2}\right)n\ u(n) \quad |z| > \frac{1}{2}$

$$\leftrightarrow \frac{z}{z-\frac{1}{2}}$$

$\left(\frac{1}{3}\right)n\ u(n) \quad |z| > \frac{1}{3}$

$$\leftrightarrow \frac{z}{z-\frac{1}{3}}$$

There is no common ROC, X[z] is not valid for given x (n).

Example

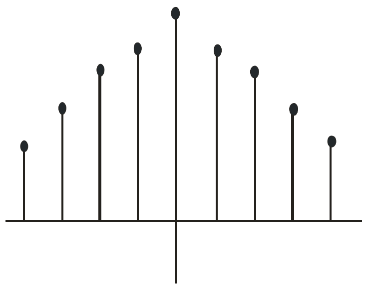
Find X (z) & its ROC for given discrete sequence x (n) = $a^{|n|}$?

Solution:

x (n) = $a^{|n|}$

a) x (n) for a <1 and a > 1, X(z) & ROC for both cases

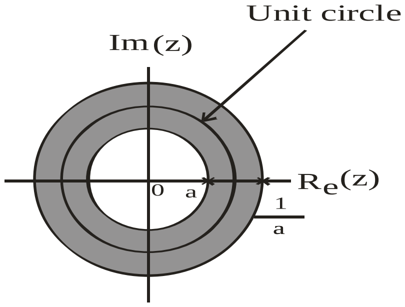
x (n) = $a^{|n|}$ 0 < a < 1



x(n) = $a^n u(n) + a^{-n} u(-n-1)$

$$a^n u(n) \leftrightarrow \frac{z}{z-a}\left(|z| > a\right)$$

$$a^n u(\text{-n-1}) \leftrightarrow \frac{z}{z-\frac{1}{a}} |z| < \frac{1}{a}$$

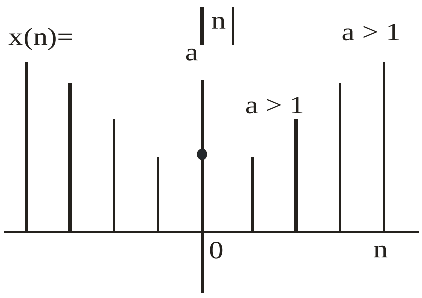


$$X(z) = \frac{z}{z-a} - \frac{z}{z-\frac{1}{a}}$$

$$X(z) = \frac{a^2-1}{a} \cdot \frac{z}{(z-a)\left(z-\frac{1}{a}\right)}$$

$a < |z| < \frac{1}{a}, 0 < a < 1$, overlapped ROC

d) a > 1



$$x(z) = \frac{z}{z-a} - \frac{z}{z-\frac{1}{a}}$$

Pole at z . a, $z = \frac{1}{a}$

$a < |z| < \frac{1}{a}, \quad a > 1$

Do not overlap. There is no common ROC. Thus x (n) will have no X (z)

5.4 THE INVERSE Z-TRANSFORM

Inversion of the z-transform to find the sequence x[n]from its z-transform X (z) is called the inverse z-transform, symbolically denoted as

$$x[n] = Z^{-1}[X(z)]$$

A. Inversion Formula:

As in the case of the Laplace transform, there is a formal expression for the inverse z-transform in terms of an integration in the z-plane; that is,

$$x[n]=\frac{1}{2\pi}\oint_{c}X(z)z^{n-1}dz$$

Where C is a counter clockwise contour of integration enclosing the origin. Formal evaluation of Eq. requires an understanding of complex variable theory.

B. Use of Tables of z-Transform Pairs:

In the second method for the inversion of X (z), we attempt to express X (z) as a sum $X(z)=X_1(z)+\ldots+X_n(z)$

Where $X_1(z),\ldots, X_n(z)$ are functions with known inverse transforms $x_1[n],\ldots, x_n[n]$. From the linearity property it follows that

$$x[n]=x_1[n]+\ldots\ldots x_n[n]$$

C. Power Series Expansion:

The defining expression for the z-transform equals is a power series where the sequence values x[n] are the coefficients of z-". Thus, if X (z) is given as a power series in the form

$$X[z]=\sum_{n=-\infty}^{\infty}x[n]z^{-n}$$

$$=\ldots+x[-2]z^{2}+x[-1]z+x[1]z^{-1}+x[2]z^{-2}+\ldots\ldots$$

We can determine any particular value of the sequence by finding the coefficient of the appropriate power of z^{-1}. This approach may not provide a closed-form solution but is very useful for a finite-length sequence where X (z) may have no simpler form than a polynomial in z^{-1}.

D. Partial-Fraction Expansion:

As in the case of the inverse Laplace transform, the partial-fraction expansion method provides the most generally useful inverse z-transform, especially when X (z) is a rational function of z. Let

$$X(z)=\frac{N(z)}{D(z)}=k\frac{(z-z_1)\ldots\ldots(z-z_m)}{(z-p_1)\ldots\ldots(z-p_n)}$$

Assuming n ≥ m and all poles are simple, then

$$\frac{X(z)}{z}=\frac{c_0}{z}+\frac{c_1}{z-p_1}+\frac{c_2}{z-p_2}+\ldots+\frac{c_n}{z-p_n}$$

$$=\frac{c_0}{z}+\sum_{k=1}^{n}\frac{c_k}{z-p_k}$$

Where

$$c_0=X(z)\big|_{z=0}$$

$$c_k=(z-p_k)\frac{X(z)}{z}\bigg|_{z=p_k}$$

Hence, we obtain

$$X(z)=c_0+c_1\frac{z}{z-p_1}+\ldots+c_n\frac{z}{z-p_n}$$

$$=c_0+\sum_{k=1}^{n}c_k\frac{z}{z-p_k}$$

Inferring the ROC for each term in equals from the overall ROC of X(z) and using we can then invert each term, producing thereby the overall inverse z-transform If m > n in equals, then a polynomial of z must be added to the right-hand side of equals the order of which is (m - n). Thus for m> n, the complete partial-fraction expansion would have the form

$$X(z)=\sum_{q=0}^{m-n}b_q z^{q}+\sum_{k=1}^{n}c_k\frac{z}{z-P_k}$$ If X (z)

has multiple-order poles, say **pi** is the multiple pole with multiplicity r, then the expansion of X(z)/z will consist of terms of the form

$$\frac{\lambda_1}{z-P_1}+\frac{\lambda_2}{(z-P_1)^2}+\ldots+\frac{\lambda_r}{(z-P_1)^r}$$

Where

$$\lambda_{r-k}=\frac{1}{k!}\frac{d^k}{dz^k}\left[(z-p_i)^r\frac{X(z)}{z}\right]\Bigg|_{r=p_i}$$

5.5 FUNCTION OF DISCRETE-TIME LTI SYSTEMS

THE SYSTEM FUNCTION

The output y[n] of a discrete-time LTI system equals the convolution of the input x[n] with the impulse response h[n]; that is

y[n] = x[n] * h[n]

Applying the convolution property of the z-transform, we obtain

Y[z] = X[z] H[z]

Where Y (z), X (z), and H (z) are the z-transforms of y[n], x[n], and h[n], respectively.

Equation can be expressed as

$$H(z)=\frac{Y(z)}{X(z)}$$

The z-transform H (z) of h[n] is referred to as the system function (or the transfer function) of the system. By equals the system function H (z) can also be defined as the ratio of the z-transforms of the output y[n] and the input x[n]. The system function H (z) completely characterizes the system.

Characterization of Discrete-Time LTI Systems:

Many properties of discrete-time LTI systems can be closely associated with the characteristics of H (z) in the z-plane and in particular with the pole locations and the ROC

1. **Causality:**
   For a causal discrete-time LTI system, we have Eq.
   $$h[n] = 0 \quad n < 0$$
   Since h[n] is a right-sided signal, the corresponding requirement on H(z) is that the ROC of H(z) must be of the form
   $|z| > r_{max}$
   That is, the ROC is the exterior of a circle containing all of the poles of H(z) in the z-plane. Similarly, if the system is anti-causal, that is,
   $h[n] = 0 \qquad n \geq 0$
   then h[n] is left-sided and the ROC of H(z) must be of the form
   $|z| < r_{min}$
   That is, the ROC is the interior of a circle containing no poles of H(z) in the z-plane.

2. **Stability**
   In Sec. we stated that a discrete-time LTI system is BIBO stable if and only if Eq.
   $$\sum_{n=\infty}^{\infty}|h[n]|<\infty$$
   The corresponding requirement on H(z) is that the ROC of H(z) contains the unit circle(that is, |z| = 1).

3. **Causal and Stable Systems**
   If the system is both causal and stable, then all of the poles of H (z) must lie inside the unit circle of the z-plane because the ROC is of the form $|z| > r_{max}$, and since the unit circle is included in the ROC, we must have $r_{max} < 1$.

Example

A causal discrete time LTI system is described by

$$y(n)-\frac{3}{4}y(n\text{-}1)+\frac{1}{8}y(n-2)=x(n)$$

When x (n) & y (n) are input and output of the system, respectively

a) Determine the system function H (z).
b) Find the impulse response h (n) of the system
c) Find the step response s (n) of the system.

Solution

a) $$Y(z)-\frac{3}{4}z^{-1}Y(z)\frac{1}{8}z^{-2}Y(z)=X(z)$$

$$r\left(1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right)Y(z)=X(z)$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1}{1-\frac{3}{4}z-1+\frac{1}{8}z-2}$$

$$= \frac{z^2}{z^2 - \frac{3}{4}z + \frac{1}{8}}$$

b) $$\frac{H(z)}{z} = \frac{z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{4}}$$

$$A = \left.\frac{Z}{z-\frac{1}{4}}\right|_{z=\frac{1}{4}} = 2$$

$$B = \left.\frac{Z}{z-\frac{1}{2}}\right|_{z=\frac{1}{4}} = -1$$

$$H(z) = \frac{2Z}{z-\frac{1}{2}} - \frac{Z}{z-\frac{1}{4}} \quad |z| > \frac{1}{2}$$

Talking the inverse Z – transform of H(z)

$$h(n) = 2\left(2\left[\frac{1}{2}\right]^n - \left[\frac{1}{4}\right]^n\right)u(n)$$

c) $$x(n) = u(n) \leftrightarrow X(z) = \frac{z}{z-1} |z| > 1$$

$$Y(z) = X(z)H(z)$$

$$= \frac{z^3}{(z-1)\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} \quad |z| > 1$$

$$\frac{Y(z)}{z} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{2}} + \frac{C}{z-\frac{1}{4}}$$

Where

$$A = \left.\frac{z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right| z = \frac{1}{2} = \frac{1}{\frac{1}{3}\times\frac{3}{4}} = \frac{8}{3}$$

$$B = \left.\frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)}\right| z = \frac{1}{2} = \frac{\frac{1}{4}}{-\frac{1}{2}\times+\frac{1}{4}} = -2$$

$$C = \left.\frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)}\right| z = \frac{1}{4} = \frac{\frac{1}{16}}{-\frac{3}{4}\times\frac{1}{4}} = \frac{1}{3}$$

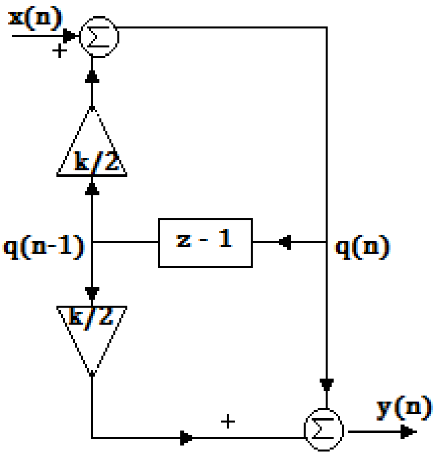
$$Y(z) = \frac{8}{3}.\frac{z}{z-1} - 2\frac{z}{z-\frac{1}{2}} + \frac{1}{3}\frac{z}{z-\frac{1}{4}} \quad |z|>1$$

Taking the increase Z–transformation Y (z)

$$y(n) = s(n) = \frac{8}{3}u(n) - \left[\frac{1}{2}\right]^n 2.\,u(n)\frac{1}{3}\left[\frac{1}{4}\right]^n u(n)$$

Example

The Discrete time system for what values k is the system BIBO stable?



Solution

$$q(n) = x(n) + \frac{k}{2}\ q(n-1)$$

$$y(n) = y(n) + \frac{k}{3}\ q(n-1)$$

$$Q(z) = X(z) + \frac{k}{2}z^{-1}\ Q(z)$$

$$Y(z) = Q(z) + \frac{k}{3}\ Q(z)\ z^{-1}$$

$$(1 - \frac{k}{3}z^{-1})\ Q(z) = X(z)$$

$$(1 + \frac{k}{3}z^{-1})\ Q(z) = Y(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{k}{3}z^{-1}}{1+\frac{k}{2}z^{-1}} = \frac{z+\frac{k}{3}}{z-\frac{k}{2}} \quad |z| > \left|\frac{k}{2}\right|$$

System has one zero at $z = -\frac{k}{3}$ one pole at $z = \frac{k}{2}$

The system will be BIBO stable if ROC contains the unit circle, $|z| = 1$,

Hence was the system is stable only if $|k| < 2$

5.6 THE UNILATERALZ-TRANSFORM

A. Definition:

The unilateral (or one-sided) z-transform X,(z) of a sequence x[n] is defined as equation.

$$X_1(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

and differs from the bilateral transform in that the summation is carried over only $n \geq 0$. Thus, the unilateral z-transform of x[n] can be thought of as the bilateral transform of x[n] u[n]. Since x[n]u[n] is a right-sided sequence, the ROC of X,(z) is always outside a circle in the z-plane.

B. Basic Properties:

Most of the properties of the unilateral z-transform are the same as for the bilateral z-transform. The unilateral z-transform is useful for calculating the response of a causal system to a causal input when the system is described by a linear constant-coefficient difference equation with nonzero initial conditions. The basic property of the unilateral z-transform that is useful in this application is the following time-shifting property which is different from that of the bilateral transform.

Time-Shifting Property

If x[n] ↔ $X_1(z)$, then for m ≥ 0,

$$x[n-m] \leftrightarrow z^{-m}X_1(z) + z^{-m+1}$$

$$x[-1] + z^{-m+2}x[-2] + ... + x[-m]$$

$$x[n+m] \leftrightarrow z^{m}X_1(z) - z^{m}x[0] -$$

$$z^{m-1}x[-2] + ... + x[1] - .... - zx[-m]$$

C. System Function

Similar to the case of the continuous-time LTI system, with the unilateral z-transform, the system function H (z) = Y (z)/X (z) is defined under the condition that the system is relaxed, that is, all initial conditions are zero.

STABILITY OF DISCRETE-TIME LTI SYSTEM

Let the LTI discrete-time system transfer function is

$$H(Z) = \frac{N(z)}{D(z)} = \frac{b_n z^m + b_{n-1}z^{m-1} + ... + b_1 z + b_0}{a_n z^n + a_{n-1}z^{n-1} + ... + a_1 z + a_0}$$

Where

$$D(z) = a_n z^n + a_{n-1}z^{n-1} + ... + a_1 z + a_n = 0$$

is characteristic equation of the system. Stability of system represented by equation is tested based on fulfillment of necessary and sufficient conditions.

5.7 RELATIONSHIP BETWEEN Z AND LAPLACE TRANSFORM

For a general signal x (t) is sampled at sampling rate 1/T to get discrete value x (kT) which has z-transform

$$X(z) = \sum_{k=-\infty}^{\infty} x(kT)z^{-k}$$

The same general signal x (t) can be considered as the impulse sample at the same rate 1/T and may be represented as

$$x(t) = \sum_{k=-\infty}^{\infty} x(kT)\delta(t-kT)$$

Equation is form the basic property that any arbitrary signal can be represented as weighted [x(kT)] sum of shifted delta sequence.

Laplace transform of Equation is

$$x(s) = \sum_{k=-\infty}^{\infty} r(kT)a - ksT$$

If e^{+sT} = z, Equation can be rewritten as

$$x(s) = \sum_{k=-\infty}^{\infty} r(k)z^{-k} = X(z)$$

Thus,

$$z = e^{sT}$$

$$\ln z = sT$$

$$s = \frac{1}{T}\ln Z$$

Relationship between Laplace transform and z-transform is summarized as

(i) $s = \frac{1}{T}\ln z$ (ii) $X(s) = X(z)\big|_z$

GATE QUESTIONS(EC)

Q.1 The region of convergence of the z-transform of a unit step function is
a) $|z| > 1$
b) $|z| < 1$
c)(real part of z)>0
d) (real part of z)<0
[GATE-2001]

Q.2 If the impulse response of a discrete-time system is $h[n] = -5^n u[-n-1]$, then the system function H(z) is equal to
a) $\frac{-z}{z-5}$ and the system is stable
b) $\frac{z}{z-5}$ and the system is stable
c) $\frac{-z}{z-5}$ and the system is unstable
d) $\frac{z}{z-5}$ and the system is unstable
[GATE-2002]

Q.3 A sequence x(n) with the z-transform $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$ is applied as an input to a linear, time-invariant system with the impulse response $h(n) = 2\delta(n-3)$ where

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

The output at n = 4 is
a) -6 b) zero
c) 2 d) -4
[GATE-2003]

Q.4 The z-transform of a system is $H(z) = \frac{z}{z-0.2}$
If the ROC is $|z| < 0.2$, then the impulse response of the system is
a) $(0.2)^n u[n]$
b) $(0.2)^n u[-n-1]$
c) $-(0.2)^n u[n]$
d) $-(0.2)^n u[-n-1]$
[GATE-2004]

Q.5 A causal LTI system is described by the difference equation
$2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$
The system is stable only if
a) $|\alpha| = 2, |\beta| < 2$
b) $|\alpha| > 2, |\beta| > 2$
c) $|\alpha| < 2$, any value of β
d) $|\beta| < 2$, any value of α
[GATE-2004]

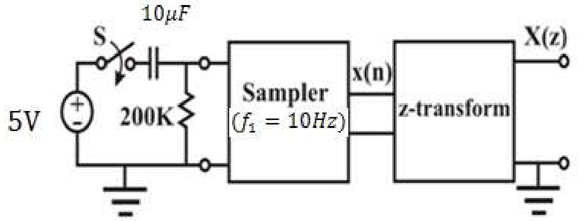
Q.6 The region of convergence of z-transform of the sequence $\left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$ must be
a) $|z| < \frac{5}{6}$ b) $|z| > \frac{5}{6}$
c) $\frac{5}{6} < |z| < \frac{6}{5}$ d) $\frac{6}{5} < |z| < \infty$
[GATE-2005]

Q.7 If the region of convergence of $x_1[n] + x_2[n]$ is $\frac{1}{3} < |z| < \frac{2}{3}\sqrt{a^2+b^2}$, then the region of convergence of $x_1[n] - x_2[n]$ includes
a) $\frac{1}{3} < |z| < 3$ b) $\frac{2}{3} < |z| < 3$
c) $\frac{3}{2} < |z| < 3$ d) $\frac{1}{3} < |z| < \frac{2}{3}$
[GATE-2006]

Q.8 The z – transform X[z] of a sequence $x[n]$ is given $X[z] = \frac{0.5}{1-2z^{-1}}$. It is given that the region of convergence of X[z] includes the unit circle. The value of $x[0]$ is
a) –0.5 b) 0
c) 0.25 d) 0.5
[GATE-2007]

Statement for Linked Answer Questions 9 and 10 :

In the following network, the switch is closed at $t = 0^-$ and sampling starts from t = 0. The sampling frequency is 10 Hz.



Q.9 The samples $x(n)$ (n = 0, 1, 2, …….) are given by

a) $5(1-e^{-0.05n})$ b) $5e^{-0.05n}$
c) $5(1-e^{-5n})$ d) $5e^{-5n}$

[GATE-2008]

Q.10 {x(n)} is real –valued periodic sequence with a period N. x(n) and X(k) form N-point Discrete Fourier Transform (DFT) pairs. The DFT Y(k) of the sequence $y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)x(n+r)$ is

a) $|X(k)|^2$
b) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)X^*(k+r)$
c) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)X(k+r)$
d) 0

[GATE-2008]

Q.11 The ROC of Z-Transform of the discrete time sequence $x(n) = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1)$ is

a) $\frac{1}{3} < |z| < \frac{1}{2}$ b) $|z| > \frac{1}{2}$
c) $|z| < \frac{1}{3}$ d) $2 < |z| < 3$

[GATE-2009]

Q.12 A system with transfer function H(z) has impulse response h(.) defined as $h(2) = 1$, $h(3) = -1$ and $h(k) = 0$ otherwise. Consider the following statements.

S_1: H(z) is a low-pass filter.
S_2: H(z) is an FIR filter.

Which of the following is correct?

a) Only S_2 is true
b) Both S_1 and S_2 are false
c) Both S_1 and S_2 are true, and S_2 is a reason for S_1
d) Both S_1 and S_2 are true, but S_2 is not a reason for S_1

[GATE-2009]

Q.13 Consider the z-transform $X(z) = 5z^2 + 4z^{-1} + 3;\ 0 < |z| < \infty$. The inverse z-transform $x[n]$ is

a) 5δ[n + 2] + 3δ[n] + 4δ[n-1]
b) 5δ[n-2] + 3δ[n] + 4δ[n +1]
c) 5u[n +2] + 3u[n] + 4u[n – 1]
d) 5u[n – 2] + 3u[n] + 4u[n + 1]

[GATE-2010]

Q.14 Two discrete time systems with impulse responses $h_1[n] = \delta[n-1]$ and $h_2[n] = \delta[n-2]$ are connected in cascade. The overall impulse response of the cascaded system is

a) δ[n-1] + δ[n – 2]
b) δ[n – 4]
c) δ[n – 3]
d) δ[n – 1]δ[n – 2]

[GATE-2010]

Q.15 The transfer function of a discrete time LTI system is given by

$$H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Consider the following statements:
S1: The system is stable and causal for ROC: $|z| > \frac{1}{2}$
S2: The system is stable but not causal for ROC: $|z| < \frac{1}{4}$
S3: The system is neither stable nor causal for ROC: $\frac{1}{4} < |z| < \frac{1}{2}$

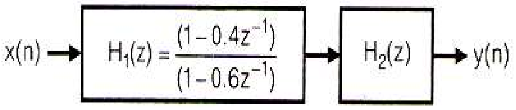
Which one of the following statements is valid?

a) Both S1 and S2 are true

b) Both S2 and S3 are true
c) Both S1 and S3 are true
d) S1, S2 and S3 are all true

[GATE-2010]

Q.16 Two system $H_1(z)$ and $H_2(z)$ are connected in cascade as shown below. The overall output $y(n)$ is the same as the input x(n) with a one unit delay. The transfer function of the second system $H_2(z)$ is



a) $\frac{(1-0.6z^{-1})}{z^{-1}(1-0.4z^{-1})}$
b) $\frac{z^{-1}(1-0.6z^{-1})}{(1-0.4z^{-1})}$
c) $\frac{z^{-1}(1-0.4z^{-1})}{(1-0.6z^{-1})}$
d) $\frac{(1-0.4z^{-1})}{z^{-1}(1-0.6z^{-1})}$

[GATE-2011]

Q.17 If $x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^{n} u[n]$ then the region of convergence (ROC) of its Z-transform in the Z-plane will be

a) $\frac{1}{3} < |Z| < 3$
b) $\frac{1}{3} < |Z| < \frac{1}{2}$
c) $\frac{1}{2} < |Z| < 3$
d) $\frac{1}{3} < |Z| < 2$

[GATE-2012]

Q.18 A discrete time all-pass system has two of its poles at $2\angle 30^0$ and $0.25\angle 0^0$. Which one of the following statements about the system is TRUE?

a) It has two more poles at $0.5\angle 30^0$ and $4\angle 0^0$.
b) It is stable only when the impulse response is two sided.
c) It has constant phase response over all frequencies.
d) It has constant phase response over the entire z-plane.

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
a	b	b	d	c	c	d	b	b	*	a	a	a	c
15	16	17	18										
c	b	c	b										

EXPLANATIONS

Q.1 (a)

$h(n) = u(n)$

$H(z) = \sum_{n=0}^{\infty} 1.z^{-n}$

For the convergence of H(z)

$\sum_{n=0}^{\infty} (z^{-1})^n < \infty$

ROC is the range of value of z for which

$|z^{-1}| or |z| > |1|$

Q.2 (b)

$h[n] = -5^n u[-n-1]$

$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$

$H(z) = \sum_{n=-\infty}^{\infty} -5^n z^{-n}$

Let $n = -k$

$= -\sum_{n=-1}^{-\infty} (5z^{-1})^{-k}$

$= 1 - \sum_{K=0}^{\infty} (5^{-1}Z)^k$

$= 1 - \frac{1}{1-5^{-1}z}$

$|5^{-1}Z| < 1, |z| < |5|$

Q.3 (b)

$Y(z) = H(z)X(z)$

$H(z) = 2z^{-3}$

$\therefore Y(z) = 2z^{-3}$

$(z^4 + z^2 - 2z + 2 - 3z^{-4})$

$= 2(z + z^{-1} - 2z^{-2}$

$+ 2z^{-3} - 3z^{-7})$

Taking inverse of z-transform

$y(n) = 2[\delta(n+1) +$

$\delta(n-1) - 2\delta(n-2)$

$+2\delta(n-3) - 3\delta(n-7)]$

At $n = 4, y(4) = 0$

Q.4 (d)

$H(z) = \frac{z}{z-0.2} = \frac{z}{z(1-0.2z^{-1})}$

$H(z) = \frac{z}{1-0.2z^{-1}}$

ROC is $|z| < 0.2$

Comparing with ,

$-a^n u(-n-1) \to \frac{1}{1-az^{-1}}, |z| < a$

We get, $h(n) = -(0.2)^n u(-n-1)$

Q.5 (c)

$2y[n] = \alpha y[n-2] - 2x[n]$

$+ \beta x[n-1]$

Taking z-transform

$2Y(z) = \alpha Y(z)z^{-2} - 2X(z)$

$+\beta X(z)z^{-1} Y(z)[2 - \alpha z^{-2}]$

$= X(z)[\beta z^{-1} - 2]$

$\frac{Y(z)}{X(z)} = \left(\frac{\beta z^{-1}-2}{2-\alpha z^{-2}}\right)$

For system to be stable, β can be of any value

$2 - \alpha z^{-2} > 0$

$2z^{-2} - \alpha = 0$

$z = \sqrt{\frac{\alpha}{2}} < 1$

$|\alpha| < 2$

For system to be stable all poles should be inside unity circle.

Q.6 (c)

z-transform

of $\left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$

$= \frac{1}{1-\frac{5}{6}z^{-1}} + \left[1 - \frac{1}{1-\left(\frac{6}{5}\right)^{-1} z}\right]$

ROC $= |z| > \frac{5}{6}$ ROC $= |z| < \frac{6}{5}$

$\frac{5}{6} < |z| < \frac{6}{5}$

Q.7 (d)

ROC remains the same for addition and subtraction in z-domain.

Q.8 (b)

$X[z] = \frac{0.5}{1-2z^{-1}}.$

$\because$ ROC includes unit circle

$\Rightarrow$ Left handed system

$\Rightarrow x(n) = -(0.5)(2)^n u(-n-1)$

$\Rightarrow x(0) = 0$

Q.9 (b)

$V_R(s) = \left(\frac{200\times10^3}{200\times10^3 + \frac{1}{10\times10^{-6}s}}\right)\frac{5}{S}$

$= \frac{5\times2\times10^5\times10\times10^{-6}}{2\times10^5\times10^{-5}s+1}$

$= \frac{10}{2s+1} = \frac{5}{s+0.5}$

$V_R(t) = 5e^{-0.5t}$

Therefore, samples

$x(n) = 5e^{-0.5n/10}$

$= 5e^{-0.05n}$

Q.10 (*)

Q.11 (a)

$x(n) = (1/3)^n u(n) - (1/2)^n u(-n-1)$

$(1/3)^n u(n)$ is right sided signal so ROC will be

$|z| > \frac{1}{3}$ i.e $\frac{1}{3} < |z|$ ...(i)

$-(1/2)^n u(-n-1)$ is left sided signal so ROC will be

$|z| < 1/2$ ...(ii)

From (i) and (ii) we see that ROC of the function will be

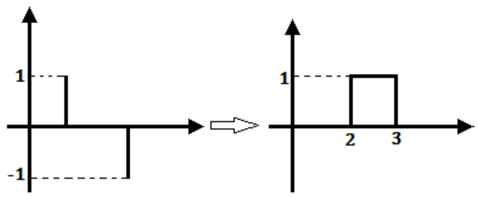
$1/3 < |z| < 1/2$

Q.12 (a)

$h(2) = 1$

$h(3) = -1$

$h(k) = 0$ *other wise*



It is finite impulse response . It is not low pass filter

Q.13 (a)

$\delta[n+n_0] \overset{z}{\leftrightarrow} z^{n_0}X(z)$

$= 5z^2 + 4z^{-1} + 3;\ 0 < |z < \infty.$

$x(n) = 5\delta(n+2) +$

$4\delta(n-1) + 3\delta(n)$

Q.14 (c)

$h_1[n] = \delta[n-1] \overset{Z}{\leftrightarrow} H_1(z) = z^{-1}$

$h_2[n] = \delta[n-1] \overset{Z}{\leftrightarrow} H_2(z) = z^{-2}$

Overall impulse response in z-domain,

$H(z) = H_1(z)H_2(z)$

$= z^{-1}z^{-2}$

$= z^{-3}$

Overall impulse response in discrete –time domain ,

$h[n]\delta[n-3]$

Q.15 (c)

1. A discrete –time LTI system is causal if and only if the ROC of its system function is the exterior of a circle, including infinity.
2. A discrete -time LTI system is stable if and only if the ROC of its system function includes the unit circle, |z|=1

$H(z) = \frac{\left(1-\frac{1}{4}z^{-1}\right)+\left(1-\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$

Or

$H(z) = \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$

for ROC: $|z| > \frac{1}{2}$, the system is stable and causal

for ROC: $|z| < \frac{1}{4}$, ROC does not include unit circle .So, system is not stable .

for ROC: $\frac{1}{4} < |z| < \frac{1}{2}$ ROC does not include unit circle .So, system is not stable .

Also ROC is not the exterior of $|z| = \frac{1}{2}$

So it is not causal.

Q.16 (b)

$y[n] = x[n-1]$

Taking z-transform of both sides

$Y(z) = z^{-1}X(z)$

$\frac{Y(z)}{X(z)} = z^{-1}$

For cascaded system

$H(z) = H_1(z).H_2(z)$

$z^{-1} = \frac{(1-0.4z^{-1})}{(1-0.6z^{-1})}H_2(z)$

$\therefore H_2(z) = \frac{z^{-1}(1-0.6z^{-1})}{(1-0.4z^{-1})}$

Q.17 (c)

Let $x_1[n] = \left(\frac{1}{3}\right)^{|n|}$

And $x_2[n] = \left(\frac{1}{2}\right)^n u[n]$

$\Rightarrow x_1[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{3}\right)^{-n} u[-n-1]$

$$\left(\frac{1}{3}\right)^n u[n] \overset{z}{\leftrightarrow} \frac{1}{1-\frac{1}{3}z^{-1}}; \text{ROC: } |z| > \frac{1}{3}$$

$$\left(\frac{1}{3}\right)^{-n} u[-n-1] \overset{z}{\leftrightarrow} \frac{-1}{1-\left(\frac{1}{3}\right)^{-1} z^{-1}}; \text{ROC: } |z| < 3$$

And

$$x_2[n] = \left(\frac{1}{2}\right)^n u[n] \overset{z}{\leftrightarrow} \frac{1}{1-\frac{1}{2}z^{-1}}; \text{ROC: } |z| > \frac{1}{2}$$

$$\therefore \text{ROC is } \frac{1}{2} < |z| < 3$$

Q.18 (b)

Location of poles are

$$P_1 = 2\angle 30^0 = 2e^{j30}$$

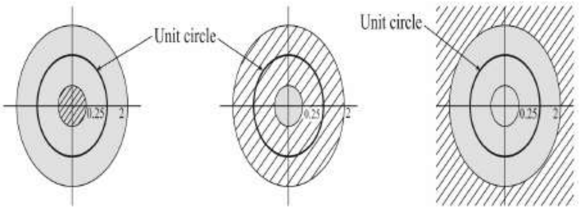
$$P_2 = 0.25\angle 0^0 = 0.25e^{j0}$$

According to concept of all pass filter, location of zeros are reciprocal conjugate of location of poles and vice-versa. So, location of zeros should be

$$z_1 = \frac{1}{P_1^*} = \frac{1}{\left(2e^{j30}\right)^*} = \frac{1}{2e^{-j30}} = 0.5e^{j30} = 0.5\angle 30^0$$

$$z_2 = \frac{1}{P_1^*} = \frac{1}{\left(0.25e^{j0}\right)^*} = \frac{1}{0.25e^{-j0}} = 4e^{j0} = 4\angle 0^0$$

But, in option (A), $0.5\angle 30^0$ and $4\angle 0^0$ are given as pole location. So, option is incorrect..



For ROC $0.25 < |z| < 2$

Impulse response corresponding to pole $0.25\angle 0^0$ will be right sided

Impulse response corresponding to pole $2\angle 30^0$ will be left sided.

So, for stability, impulse response should be two sided.

GATE QUESTIONS(EE)

Q.1 If u(t) is the unit step and δ(t) is the unit impulse function, the inverse z-transform of $F(z) = \frac{1}{z+1}$ for $k \geq 0$ is

a) $(-1)^k \delta(k)$
b) $\delta(k) - (-1)^k u(k)$
c) $(-1)^k u(k)$
d) $u(k) - (-1)^k \delta(k)$

[GATE-2005]

Q.2 A discrete real all pass system has a pole at $z = 2\angle 30^\circ$ it therefore

a) also has a pole at $0.5\angle 30^\circ$
b) has a constant phase response over the z-plane are $|H(z)| = const$
c) is stable only if it anti-causal
d) has a constant phase response over the unit circle are $|H(e^{j\Omega})| = const$

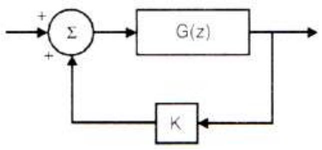
[GATE-2006]

Q.3 The discrete – time signal x[n] $\leftrightarrow X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$, where $\leftrightarrow$ denotes a transform-pair relationship, is orthogonal to the signal

a) $y_1[n] \leftrightarrow Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$
b) $y_2[n] \leftrightarrow Y_2(z) = \sum_{n=0}^{\infty} (5^n - n) z^{-(2n+1)}$
c) $y_3[n] \leftrightarrow Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$
d) $y_4[n] \leftrightarrow Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

[GATE-2006]

Q.4 Consider the discrete –time system shown in the figure where the impulse response of $G(z)$ is $g(0) = 0, g(1) = g(2) = 1, g(3) = g(4) = \cdots = 0$



This system is stable for range of value of K

a) $[-1, 1/2]$
b) $[-1, 1]$
c) $[-1/2, 1]$
d) $[-1/2, 2]$

[GATE-2007]

Q.5 $X(z) = 1 - 3z^{-1}, Y(z) = 1 + 2z^{-2}$ are Z – transmission of two signals x[n], y[n] respectively. A linear time invariant system has the impulse response h[n] defined by these two signals as

$h[n] = x[n-1] * y[n]$

where * denotes discrete time convolution. Then the output of the system for the input δ [n – 1]

a) Has Z – transform $z^{-1}X(z)\,Y(z)$
b) Equals $\delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$
c) Has Z – transform $1 - 3z^{-1} + 2z^{-2} - 6z^{-3}$
d) Does not satisfy any of the above three.

[GATE-2007]

Q.6 Given $X(z) = \frac{z}{(z-a)^2}$ with $|z| > a$, the residue of $X(z)z^{n-1}$ at $z = a$ for $n \geq 0$ will be

a) a^{n-1}
b) a^n
c) na^n
d) na^{n-1}

[GATE-2008]

Q.7 H(z) is a transfer function of a real system. When a signal $x[n] = (1+j)^n$ is the input to such a system, the output is zero. Further, the Region of Convergence (ROC) of $\left(1 - \frac{1}{2}z^{-1}\right)$H(z) is the entire Z-plane (except z =0). It can then be inferred that H (z) can have a minimum of

a) One pole and one zero
b) One pole and two zeros
c) Two poles and one zero

d) Two poles and two zeros

[GATE-2008]

Q.8 The z-transform of a signal x[n] is given by $4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$. It is applied to a system, with a transfer function $H(z) = 3z^{-1} - 2$. Let the output be y(n). Which of the following is true?

a) y(n) is non causal with finite support
b) y(n) is causal with infinite support
c) y(n) = 0: |n| > 3
d) $\mathrm{Re}[y(z)]_{z=e^{j\theta}} = -\mathrm{Re}[y(z)]_{z=e^{-j\theta}}$; $\mathrm{Im}[y(z)]_{z=e^{j\theta}} = \mathrm{Im}[y(z)]_{z=e^{-j\theta}}$; $-\pi \le \theta < \pi$

[GATE-2009]

Q.9 If $x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^{n} u[n]$ then the region of convergence (ROC) of its Z-transform in the Z-plane will be

a) $\frac{1}{3} < |Z| < 3$ b) $\frac{1}{3} < |Z| < \frac{1}{2}$
c) $\frac{1}{2} < |Z| < 3$ d) $\frac{1}{3} < |Z| < 2$

[GATE-2012]

Q.10 Let $X(Z) = \frac{1}{1-Z^{-3}}$ be the Z-transform of a causal signal x[n]. Then, the values of x[2] and x[3] are

a) 0 and 0 b) 0 and 1
c) 1 and 0 d) 1 and 1

[GATE-2014]

Q.11 Consider a discrete time signal given by $x[n]=(-0.25)^n u[n] + (0.5)^n u[-n-1]$ The region of convergence of its Z-transform would be

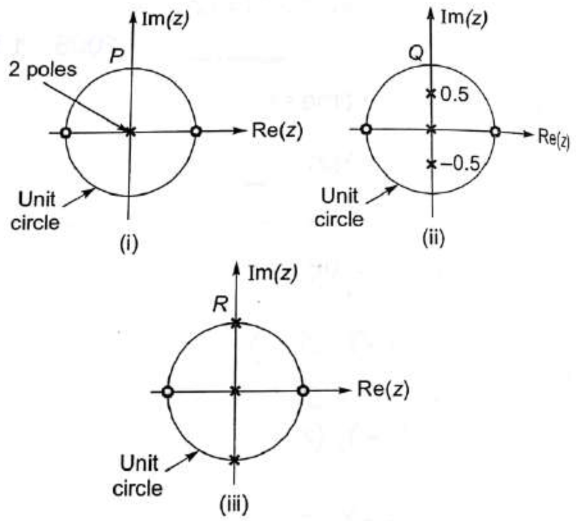
a) The region inside the circle of radius 0.5 and centered at origin.
b) The region outside the circle of radius 0.25 and centered at origin.
c) The annular region between the two circles, both centered at origin and having radii 0.25 and 0.5
d) The entire Z plane

[GATE-2015]

Q.12 Let $S= \sum_{n=0}^{\infty} n\alpha^n$ where $|\alpha|<1$. The value of α in the range $0< \alpha < 1$, such hat S=2a is__________

[GATE-2016]

Q.13 The pole-zero plots of three discrete-time systems P,Q and R on the z-plane are shown below.



Which of the following is the TRUE about the frequency selectivity of these systems?

a) All three are high-pass filters
b) All three are band-pass filters
c) All three are low-pass filters
d) P is a low-pass filter, Q is a band-pass filter and R is a high-pass filter.

[GATE-2017]

ANSWER KEY:

1	2	3	4	5	6	7
b	c	b	a	b	d	a
8	9	10	11	12	13	
a	c	b	c	0.29	b	

EXPLANATIONS

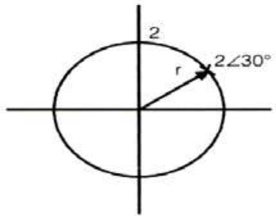
Q.1 **(b)**

$F(z) = 1 - \frac{z}{z+1} = 1 - \frac{1}{1+z^{-1}}$

$= \delta(k) - (-1)^k u(k)$

Q.2 **(c)**

For causal system, If all the poles are inside the



Unit Circle then system is stable, and converse in true anti-causal system.

Q.3 **(b)**

Q.4 **(a)**

Given

$g(1) = g(2) = 1$, otherwise 0

i.e. $g(n) = \delta[n-1] + \delta[n-2]$

therefore

$G(z) = z^{-1} + z^{-2}$

Therefore overall transfer function of closed loop system

$T(z) = \frac{G(z)}{1-kG(z)}$

$T(z) = \frac{z^{-1}+z^{-2}}{1-k(z^{-1}+z^{-2})}$

or $T(z) = \frac{z+1}{z^{-2}-k(z+1)}$

So the system will be stable if it's outer most pole will lie inside the unit circle

Location of poles

$z = \frac{+k \pm \sqrt{k^2+4k}}{2}$

$\frac{k \pm \sqrt{k^2+4k}}{2} < 1$

$k + \sqrt{k^2+4k} < 2$

$\sqrt{k^2+4k} < (2-k)$

$\Rightarrow k^2 + 4k < (2-k)^2$

$\Rightarrow k^2 + 4k < 4 + k^2 - 4k$

$\Rightarrow 8k < 4$

$k < \frac{1}{2}$

Q.5 **(b)**

$X(z) = (1 - 3z^{-1})$

$Y(z) = (1 + 2z^{-2})$

$h[n] = x(n-1) * y[n]$

$\Rightarrow H[z] = z^{-1}X(z).Y(z)$

$\Rightarrow H[z] = z^{-1}(1 - 3z^{-1}).(1 + 2z^{-2})$

$\Rightarrow H[z] = z^{-1}(1 + 2z^{-2} - 3z^{-1} - 6z^{-3})$

$\Rightarrow H[z] = (z^{-1} - 3z^{-2} + 2z^{-3} - 6z^{-4})$

When input

$I(n) = \delta[n-1]$

Then $I[z] = z^{-1}$

Therefore output

$P(n) = h[n] * I[n]$

$\Rightarrow P(z) = H(z)I(z)$

$P(z) = (z^{-1} - 3z^{-2} + 2z^{-3} - 6z^{-4}) \times z^{-1}$

$\Rightarrow P(z) = z^{-2} - 3z^{-3} + 2z^{-4} - 6z^{-5}$

$\therefore P(n) = \delta[n-2] - 3\delta[n-3] +$

$2\delta[n-4] - 6\delta[n-5]$

Q.6 **(d)**

$x(z) = \frac{z}{(z-a)^2}$

$\therefore z^{n-1}X(z) = \frac{z^n}{(z-a)^2}$

Since z=a is a pole of second order therefore residue at z=a

$= \frac{1}{\lfloor 1}\left[\frac{d}{dz}\left\{(z-a)^2.\frac{z^n}{(z-a)^2}\right\}\right]_{at\ z=a}$

$= [nz^{n-1}]_{z=a}$

$= na^{n-1}$

Q.7 **(a)**

Q.8 **(a)**

$y[n] = x[n] * H[n]$

$y[z] = x[z]H[z]$

$y[z] = (4z^{-3} + 3z^{-1} + 2-$

$6z^2 + 2z^2)(3z^{-1} - 2)$

$= 12z^{-4} + 9z^{-2} + 6z^{-1} - 18z +$

$6z^2 - 8z^{-3} - 6z^{-1} - 4 + 12z^2 - 4z^3$

$= -4z^3 + 18z^2 - 18z + 12z^{-4} +$

$9z^{-2} - 4 - 8z^{-3}$

$y[n] \neq 0$ for $n < 0$

Therefore it is non causal with finite support.

Q.9 (c)

Let $x_1[n] = \left(\frac{1}{3}\right)^{|n|}$

And $x_2[n] = \left(\frac{1}{2}\right)^{n} u[n]$

$\Rightarrow x_1[n] = \left(\frac{1}{3}\right)^{n} u[n] + \left(\frac{1}{3}\right)^{-n} u[-n-1]$

$\left(\frac{1}{3}\right)^{n} u[n] \overset{z}{\leftrightarrow} \frac{1}{1-\frac{1}{3}z^{-1}}$; ROC: $|z| > \frac{1}{3}$

$\left(\frac{1}{3}\right)^{-n} u[-n-1] \overset{z}{\leftrightarrow} \frac{-1}{1-\left(\frac{1}{3}\right)^{-1} z^{-1}}$; ROC: $|z| < 3$

And

$x_2[n] = \left(\frac{1}{2}\right)^{n} u[n] \overset{z}{\leftrightarrow} \frac{1}{1-\frac{1}{2}z^{-1}}$; ROC: $|z| > \frac{1}{2}$

$\therefore$ ROC is $\frac{1}{2} < |z| < 3$

Q.10 (b)

$$X(z) = \frac{1}{1-z^{-3}}$$

From z – transform definition

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$\begin{array}{r|l} & 1 + z^{-3} + \dots \\ \hline 1 - z^{-3} & 1 \\ & 1 - z^{-3} \\ \hline & z^{-3} \\ & z^{-3} \end{array}$$

$$= x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} \dots$$

$$x(z) = \frac{1}{1-z^{-3}} = 1 + 0\cdot z^{-1} + 0\cdot z^{-2} + 1\cdot z^{-3} \dots$$

By comparision $x(2) = 0$ and $x(3) = 1$

Q.11 (C)

$x[n] = (-0.25)^n u(n) + (0.5)^n u(-n-1)$

Signal x[n] is sum of two signals, one is right sided $[(-0.25)^n u(n)]$ and other is left sided $[(0.5)^n u(-n-1)]$.

The right sided signal will have pole at location with magnitude 0.25. So, ROC is $|Z| > 0.25$.

The left sided signal will have pole at location with the magnitude 0.5. So, ROC is $|Z| < 0.5$.

So, ROC of X(z) (Z transform of x(n) will be)

$0.25 < |Z| < 0.5$

Q.12 0.29

The Z-transform of

$$a^n u(n) \longrightarrow \frac{1}{(1-aZ^{-1})}$$

$$\text{and } na^n u(n) \longrightarrow \frac{aZ^{-1}}{(1-aZ^{-1})^2}$$

$$\text{so, } \frac{aZ^{-1}}{(1-aZ^{-1})^2} = \sum_{n=0}^{\infty} na^n Z^{-n}$$

If we put Z = 1 in above equation we get

$$\frac{a}{(1-a)^2} = \sum_{n=0}^{\infty} na^n$$

$$\text{Since, } \sum_{n=0}^{\infty} na^n = 2a = \frac{a}{(1-a)^2}$$

$$\text{so } \quad 2 = \frac{1}{(1-a)^2} \Rightarrow a = 0.29$$

Q.13 (b)

GATE QUESTIONS(IN)

Q.1 Given $x(z)=\frac{1/2}{1-az^{-1}}+\frac{1/3}{1-bz^{-1}}$ |a| and |b|<1 with the ROC specified as |a|<|z|<|b|, x[0] of the corresponding sequence is given by
a) 1/3 b) 5/6
c) 1/2 d) 1/6
[GATE-2004]

Q.2 The region of convergence of the z-transform of the discrete –time signal $x[n] = 2^n u[n]$ will be
a) $|z| > 2$ b) $|z| < 2$
c) $|z| > \frac{1}{2}$ d) $|z| < \frac{1}{2}$
[GATE-2008]

Q.3 $H(z)$ is a discrete rational transfer function. To ensure that both $H(z)$ and its inverse are stable its
a) Poles must be inside the unit circle and zeros must be outside the unit circle
b) pole and zeros must be inside the unit circle
c) poles and zeros must be outside the unit circle
d) poles must be outside the unit circle and zeros should be inside the unit circle
[GATE-2010]

Q.4 Consider the difference equation $y[n]-\frac{1}{3}y[n-1]=x[n]$ and suppose that $x[n]=\left(\frac{1}{2}\right)^n u[n]$. Assuming the condition of initial rest, the solution for $y[n], n\geq 0$ is
a) $3\left(\frac{1}{3}\right)^n-2\left(\frac{1}{2}\right)^n$ b) $-2\left(\frac{1}{3}\right)^n+3\left(\frac{1}{2}\right)^n$
c) $\frac{2}{3}\left(\frac{1}{3}\right)^n+\frac{1}{3}\left(\frac{1}{2}\right)^n$ d) $\frac{1}{3}\left(\frac{1}{3}\right)^n+\frac{2}{3}\left(\frac{1}{2}\right)^n$
[GATE-2011]

Q.5 The system function of an LTI system is given by $H(z)=\frac{1-\frac{1}{3}Z^{-1}}{1-\frac{1}{4}Z^{-1}}$
The above system can have stable inverse if the region of convergence of H(z) is defined as
a) $|z|<\frac{1}{4}$ b) $|z|<\frac{1}{12}$
c) $|z|>\frac{1}{4}$ d) $|z|<\frac{1}{3}$
[GATE-2014]

Q.6 The transfer function of a digital system is given by: $\frac{b_0}{1-z^{-1}+a_2z^{-2}}$;
where a_2 is real.
The transfer function is BIBO stable if the value of a_2 is:
a) −1.5 b) −0.75
c) 0.5 d) 1.5
[GATE-2014]

Q.7 The z- transform of $x[n]=\alpha^{|n|}$, 0|α| <1, is X(z). The region of convergence of X(z) is
a) $|\alpha| < |z| < \frac{1}{|\alpha|}$
b) $|z|>\alpha$
c) $|z| > \frac{1}{|\alpha|}$
d) $|z| < \min\left[|\alpha|, \frac{1}{|\alpha|}\right]$
[GATE-2015]

Q.8 The Region of Convergence (ROC) of the Z-transform of a causal unit step discrete-time sequence is
a) |z|<1 b) |z|≤ 1
c) |z|>1 d) |z|≥1
[GATE-2017]

Q.9 Consider two discrete-time signals: $x_1(n)$ = {1, 1} and $x_2(n)$ = {1, 2}, for n=0, 1. The Z-transform of the convoluted sequence $x_1(n) * x_2(n)$ is

a) $1+2z^{-1}+3z^{-2}$
b) z^2+3z+2
c) $1+3z^{-1}+2z^{-2}$
d) $z^{-2}+3z^{-3}+2z^{-4}$

[GATE-2017]

Q.10 Let $y[n]=x[n]*h[n]$, where $*$ denotes convolution and $x[n]$ and $h[n]$ are two discrete time sequence. Given that the z-transform of $y[n]$ is $\mathrm{Y}(\mathrm{z})=2+3\mathrm{z}^{-1}+z^{-2}$, the z-transform of $p[n]=x[n]*h[n-2]$ is

a) $2+3z+z^{-2}$

b) $3z+z^{-2}$

c) $2z^{2}+3z+1$

d) $2z^{-2}+3z^{-3}+z^{-4}$

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10
c	a	b	b	c	c	a	c	c	d

EXPLANATIONS

Q.1 (c)

$$X(z)=\frac{\frac{1}{2}}{\left(1-az^{-1}\right)}+\frac{\frac{1}{3}}{1-bz^{-1}}$$

ROC : $|a|<|z|<|b|$

$$\therefore\ x[n]=\frac{1}{2}(a)^{n}u[n]-\frac{1}{3}(b)^{n}u[-n-1]$$

$$x[0]=\frac{1}{2}$$

Q.2 (a)

$x[n]=2^{n}u[n]$

$x(z)=\sum_{n=-\infty}^{\infty}x[n]z^{-n}$

$=\sum_{n=-\infty}^{\infty}2^{n}u[n]\,z^{-n}$

$=\sum_{n=0}^{\infty}2^{n}.z^{-n}$

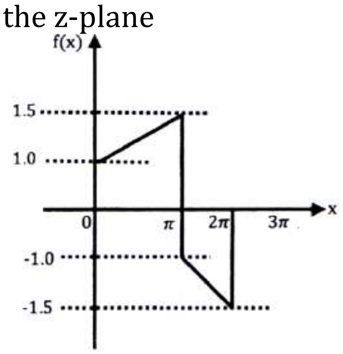
$=\sum_{n=0}^{\infty}\left(\frac{2}{z}\right)^{n}=\frac{1}{1-2z^{-1}}$

ROC $\Rightarrow |2z^{-1}|<1\Rightarrow |z|>2$

Q.3 (b)

The discrete time system with rational transfer function, $H(z)$ is stable if the poles of $H(z)$ lie inside the unit circle in the z-plane.

$\therefore$ For $H(z)$ and its inverse $1/H(z)$ to be stable, both the poles and zeros of $H(z)$ must lie inside the unit circle in the z-plane



Q.4 (b)

$y[n]-\frac{1}{3}y[n-1]=x[n]$

$\frac{Y(z)}{X(z)}=H(z)=\frac{1}{1-\frac{1}{3}z^{-1}}$

For input $x[n]=\left(\frac{1}{2}\right)^{n}u[n]$

$X(z)=\frac{z}{\left(z-\frac{1}{2}\right)},|z|>\frac{1}{2}$

$Y(z)=H(z)X(z)$

$=\frac{1}{\left(1-\frac{1}{3}z^{-1}\right)}\frac{z}{\left(z-\frac{1}{2}\right)}=\frac{z}{\left(1-\frac{1}{3}\right)\left(z-\frac{1}{2}\right)}$

$\frac{Y(z)}{z}=\frac{z}{\left(1-\frac{1}{3}\right)\left(z-\frac{1}{2}\right)}=\frac{-2}{\left(1-\frac{1}{3}\right)}+\frac{3}{\left(z-\frac{1}{2}\right)}$

$Y(z)=\frac{-2z}{z-\frac{1}{3}}+\frac{3z}{z-\frac{1}{2}}$

$\therefore y[n]=\left[-2\left(\frac{1}{3}\right)^{n}+3\left(\frac{1}{2}\right)^{n}\right]u(n)$

Q.5 (C)

If poles lie inside the unit circle then it will be stable only if R_{oc} include unit circle or R_{oc} is extension of circle whose radius is given by outer most pole.

In this case $|z|>\frac{1}{4}R_{oc}$ will give stable system.

Q.6 (C)

Transfer function of the digital system is,

$H(z)=\frac{b_0}{1-z^{-1}+a_2z^{-2}}$; a_2 is real

$=\frac{b_0z^2}{z^2-z+a_2};a=-1$

The stability of a two-pole system can be investigated by a stability criterion method called "schr–cohn" stability test".

In this approach the reflection coefficients K_1 and K_2 are to be calculated from the denominator polynomial. This method states that a second order system is stable if its reflection coefficients $|k_1| < 1$ and $|k_2| < 1$. It is also given that,

$K_2 = a_2$ and $K_1 = \frac{a}{1+a_2}$

Thus $-1 < a_2 < +1; -1 < \frac{a_1}{1+a_2} < +1$

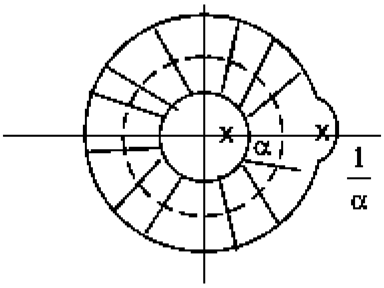
These two conditions are satisfied by 0.5

Note: An alternative approach to solve this problem is trial and error method. Each option is substituted for 'a_2' in the system function and poles are to be calculated. The value 'a_2' which gives both the poles inside the unit-circle is the suitable "a_2". Here 'a_2 = 0.5 satisfies the condition.

Q.7 (a)

$x(n)\alpha^{|n|} = \alpha^n u(n) + (1/\alpha)^n u(-n-1)$

$|z| > \alpha; |z| < \frac{1}{\alpha}$



Q.8 (C)

Q.9 (C)

$x_1[n] = \{1, 1\} \times x_2[n] = \{1, 2\}$

So, $x[n] = x_1[n] * x_2[n] = \{1, 3, 2\}$

$x(z) = 1 + 3z^{-1} + 2z^{-2}$

Q.10 (d)

$y[n] = x[n] * h[n]$

Convolution in linear shift invariant operation,

$x[n] * h[n-2] = y[n-2] = p[n]$

Apply time shifting property

$P(z) = z^{-2}Y(z) = 2z^{-2} + 3z^{-3} + z^{-4}$

6 DISCRETE – TIME FOURIER TRANSFORM

6.1 INTRODUCTION TO DTFT

1) The Discrete – Time Fourier Transform (DTFT) $X(e^{j\Omega})$ of a discrete – time signal x (n) is expressed as

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\Omega n} \quad \text{or}$$

DTFT x (n) = $X(e^{j\Omega})$

And Inverse Discrete – Time Fourier Transform (IDTFT) is expressed as

$$x(n) = \frac{1}{2}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega$$

Or IDTFT $X(e^{j\Omega}) = x(n)$

2) From equations, it is clear that x (n) and $X(e^{j\Omega})$ are a symbolical representation, this may be expressed as

$$x(n) \xleftrightarrow{DTFT} X(e^{j\Omega})$$

Discrete Time Fourier Transform (DTFT)

$x(n) \underleftrightarrow{DTFT} X(\Omega)$

$x(n) \leftrightarrow X(\Omega)$

$x(n) \underleftrightarrow{IDFT} X(\Omega)$ Fourier Transform Pair

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\Omega n} \quad \text{Analysis equation}$$

$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(\Omega)e^{j\Omega n}d\Omega \quad \text{Synthesis equation}$$

Where x (n) → non periodic sequence

Fourier Spectra

$X(\Omega) = |X(\Omega)|\ e^{j\phi(\Omega)}$ →phase spectrum
↓
Magnitude spectral

$|X(\Omega)|$ → an even function of Ω

If x (n) real Φ (Ω) → odd function of Ω

Convergence of X (Ω): Same as continuous time, the sufficient condition for the convergence of X (Ω) is that x (n) is absolutely summable, that is

$$\sum_{n=-\infty}^{\infty} |X(n)| < \infty$$

If ROC of X (z) contains the unit circle, then the Fourier transform X (Ω) of x (n) equals X (z)

$X(\Omega) = X(z)\,|_{z = e^{j\Omega}}$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\Omega n},$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

In DTFT time is discrete where as frequency is continuous. Since Discrete time signal can uniquely be represented over 2π units. It may be "0 to 2π" or "–π to π"

$F[\delta(n)] = 1$

$Z[\delta(n)] = 1$

$$F[\delta(n)] = \sum_{n=-\infty}^{\infty} \delta(n)\, e^{-j\Omega n} = 1$$

δ (n) is absolutely summable and ROC of Z-transform of δ f (t) contains unit circle.

Example

Find the Discrete Time Fourier Transform of $x(n) = a^n u(n)\ |a| < 1$

Solution:

$$X(\Omega) = \sum_{n=-\infty}^{\infty} \delta(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} (ae^{-j\Omega n})^n$$

$= 1 + ae^{-j\Omega} + (ae^{-j\Omega})^2 + \ldots\ldots\ldots\ldots$

$$X(\Omega) = \frac{1}{1-ae^{-j\Omega}}$$

$a^n u(n)$ → absolutely summable

$$X(\Omega) = \frac{1}{1 - a\cos\Omega + aj\sin\Omega}$$

$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty$$

$$|X(\Omega)| = \frac{1}{\sqrt{(1 - a\cos\Omega)^2 + (a\sin\Omega)^2}}$$

$$X(\Omega) = X(\tau) \Big|_{z = e^{j\Omega}}$$

$$|X(\Omega)| = \frac{1}{\sqrt{1 - 2a\cos\Omega + a^2}}$$

Put $a = \frac{1}{2}$

$$x(n) = \left[\frac{1}{2}\right]^2 u(n),\ |x(\Omega)| = \frac{1}{\sqrt{1.25 - \cos\Omega}}$$

Example: Find the Discrete Time Fourier Transform of

$x(n) = -a^n u(-n-1)$ a is real

Solution:

$$X(z) = \frac{1}{1 - a\tau^{-1}} \qquad |z| < |a|$$

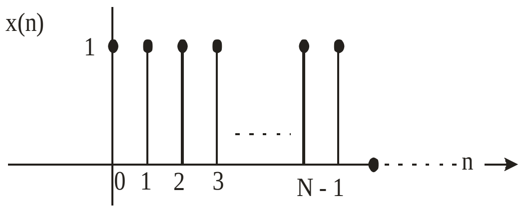
$X(e^{j\Omega})$ exists for $|a| > 1$ because the ROC of X (z) then contains the unit circle. Thus,

$$X(\Omega) = X(e^{j\Omega}) = \frac{1}{1 - ae^{-j\Omega}} \quad |a| > 1$$

Example: Find the Discrete Time Fourier Transform of me rectangular pulse sequence

$x(n) = u(n) - u(n - N)$

Answer:



$$X(z) = \sum_{n=0}^{N-1} z^{-n} = \frac{1 - z^{-N}}{1 - z^{-1}},\ |z| > 0$$

Thus $X(e^{j\Omega})$ exists because the ROC of X (z) includes the unit circle.

Hence

$X(\Omega) = X(e^{j\Omega})$

$$= \frac{1 - e^{-j\Omega N}}{1 - e^{-j\Omega}}$$

$$= \frac{e^{-j\Omega N/2}}{e^{-j\Omega/2}} \frac{(e^{+j\Omega N/2} - e^{-j\Omega N/2})}{(e^{j\Omega/2} - e^{-j\Omega/2})}$$

$$= e^{-j\Omega(N-1)/2} \frac{\sin\left(\frac{\Omega N}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)}$$

6.2 PROPERTIES OF DISCRETE TIME FOURIER TRANSFORM

A. Periodicity :

$X(\Omega + 2\pi) = X(\Omega)$

Values of Ω (radians) only over the range $0 \le \Omega < 2\pi$ or $-\pi \le \Omega < \pi$ while in continuous ω (radians/second) over the entire range $-\infty < \omega < \infty$

B. Linearity : $a_1 x_1(n) + a_2 x_2(n) \leftrightarrow a_1 X_1(\Omega) + a_2 X_2(\Omega)$

C. Time Shifting : $x(n - n_0) \leftrightarrow e^{-j\Omega n_0} X(\Omega)$

D. Frequency Shifting : - $e^{-j\Omega n_0} x(n) \leftrightarrow X(\Omega - \Omega_0)$

E. Time Scaling : - $x(at) \leftrightarrow \frac{1}{|a|} \times \left(\frac{\omega}{a}\right)$ in continuous

Let m → positive integer

$$x_m[n] = \begin{cases} x(n/m) = x[k] & \text{if } n = km,\ k = \text{integer} \\ 0 & \text{if } n \ne km \end{cases}$$

$$x_m[n] \leftrightarrow X(m\Omega)$$

As signal spread in time (m > 1), its Fourier Transform is compressed.

- The $X(m\Omega)$ is periodic with period $\frac{2\pi}{m}$, since $X(\Omega)$ is periodic with period 2π

F. Conjugation : - $x^*(n) = x^*(-\Omega)$

G. Time Reverse : - $x(-n) = X(-\Omega)$

H. Duality :- $X(t) \leftrightarrow 2\pi x(-\omega) \rightarrow$ continuous

$x(n) \leftrightarrow X(\Omega)$

$X(t)$ Fourier Series $C_k = x[-k]$ ↔

$X(t)$ Fourier Series $C_k = x(-k)$ ↔

I. Differentiation in Frequency :

$$nx(n) \leftrightarrow \frac{jdx(\Omega)}{d\Omega}$$

J. Differencing :

$$x(n) - x(n-1) \leftrightarrow (1-e^{-j\Omega})X(\Omega)$$

K. Accumulation :

$$\sum_{k=-\infty}^{n} x[k] \leftrightarrow \pi X(0)\,\delta(\Omega) + \frac{1}{1-e^{-j\Omega}} X(\Omega),\quad |\Omega| \le \pi$$

The accumulation is the discrete time counter part of integration.

L. Convolution :

$$x_1(n) \otimes x_2(n) \leftrightarrow x_1(\Omega)\ x_2(\Omega)$$

M. Multiplication :

$$x_1(n)\,x_2(n) \leftrightarrow \frac{1}{2\pi} x_1(\Omega) \otimes x_2(\Omega)$$

$\otimes$ denotes the periodic convolution

$$x_2(\Omega) = \int_{2\pi} X_1(\theta)\, x_2(\Omega - \theta)\, d\theta$$

The multiplication property is dual property of convolution.

N. Addition Properties :-

If x(t) is real.

$x(n) = x_e(n) + x_o(n)$

$x(n) \leftrightarrow X(\Omega) = A(\Omega) + jB(\Omega) = |X(\Omega)|\, e^{j\Omega}$

$X(-\Omega) = X^*(\Omega)$

O. Parseval's Relations : -

$$\sum_{n=-\infty}^{\infty} X_1(n)\, x_2(-n) = \frac{1}{2\pi}\int_{2\pi} X_1(\Omega)\, x_2(-\Omega)\, d\Omega$$

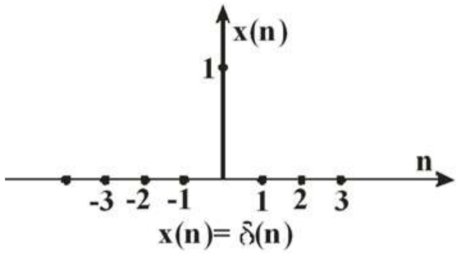
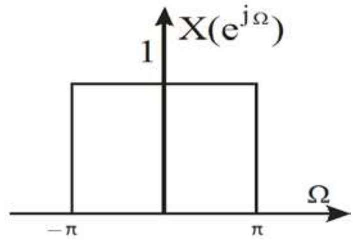
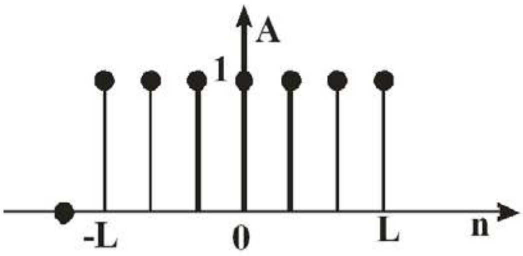
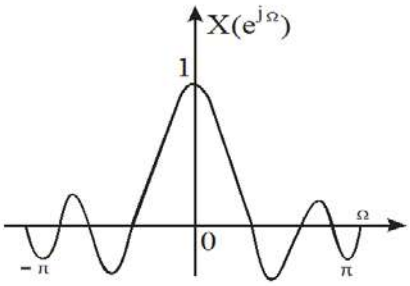
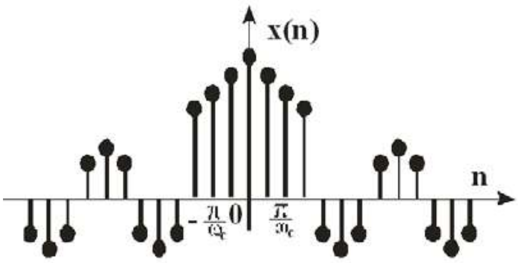
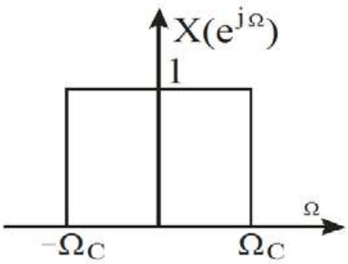
$$\Rightarrow \sum_{n=-\infty}^{\infty} |X(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(\Omega)|^2 d\Omega$$

As parseval's theorem (or identity) for Discrete Time Fourier Transform

Table 6.1 PROPERTIES OF DISCRETE TIME FOURIER TRANSFORM

S. No.	Name of Property	Time – Domain Expression	Frequency – Domain Expression
1	Notation	$x(n)$	$X(e^{j\Omega})$
2	Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(e^{j\Omega}) + a_2X_2(e^{j\Omega})$
3	Time –shifting	$x(n - n_0)$	$e^{-j\Omega n_0}X(e^{j\Omega})$
4	Frequency –shifting	$e^{j\Omega_0 n}x(n)$	$X(e^{j\Omega-\Omega_0})$
5	Frequency-differentiation	$nx(n)$	$j\dfrac{dX(e^{j\Omega})}{d\Omega}$
6	Conjugation	$x^*(n)$	$X^*(e^{j\Omega})$
7	Time – reversal	$x(-n)$	$X(e^{j\Omega})$
8	Parseval's theorem	$\sum_{n=-\infty}^{\infty} x_1(n)x_2^*(n)$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\Omega})X_2^*(e^{j\Omega})d\Omega$
9	Multiplication	$x_1(n)\ x_2(n)$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\lambda)X_2(e^{j\Omega-\lambda})d\lambda$
10.	Modulation	$x(n)\cos\Omega_0 n$	$\frac{1}{2}X(e^{j\Omega+\Omega_0})+\frac{1}{2}X(e^{j\Omega+\Omega_0})$
11	Scaling	$x(pn)$	$X\left(\frac{\Omega}{P}\right)$
12.	Convolution	$x(n)\otimes y(n)$	$X(e^{j\Omega})Y(e^{j\Omega})$

Table 6.2 FEW USEFUL DISCRETE TIME FOURIER TRANSFORM PAIRS

S. No.	Discrete Time signal x[n]	Discrete Time Fourier Transform
1		
2		
3		
4	$x(n) = \begin{cases} a^n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$	$X(e^{j\Omega}) = \dfrac{1}{1-ae^{-j\Omega}}$

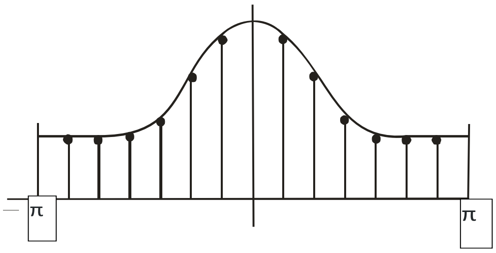
6.3 DISCRETE FOURIER TRANSFORM (DFT)

Discretizing in frequency of Discrete Time Fourier Transform is resulting Discrete Fourier Transform therefore in Discrete Fourier Transform both **time** & **frequency** are discrete.

DFT

$x(n) \xleftrightarrow{\text{DFT}} X(k)$

Sampled spectrum of Discrete Time Fourier Transform is Discrete Fourier Transform.



Ω X k

$\frac{2\pi}{N} X k$

$k = \frac{0 \text{ to } N-1}{N - \text{point}}$

$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} X(n) e^{-j\Omega n}$

Discrete Time Fourier Transform

$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \; k = 0, 1, \; ... \;, N-1, W = e^{-j2\pi}$

$W_N = e^{-j\frac{2\pi}{N}} \rightarrow$ phase factor

→ Z- Transform on unit circle at equidistant point $\left(\frac{2\pi}{N}\right)$ is called as Discrete Fourier Transform

→ Frequency resolution = $\frac{2\pi}{N} = \frac{f_s}{N}$

→ Zero padding is used for error correction & error detection.

The inverse Discrete Fourier Transform (IDFT) is given by

$x(n) = \sum_{n=0}^{N-1} x(k) W_N^{-kn} \quad n = 0, 1, \ldots\ldots, \; N-1$

Discrete Fourier Transform pair x (n) ↔ X (k)

A. Important features of Discrete Fourier Transform are as follows

1. There is a one to one correspondence between x(n) & X[k]
2. There is an externally fast algorithm, FFT (Fast Fourier Transform) for its calculation.
3. Discrete Fourier Transform is closely related to Discrete Fourier Series & Discrete Time Fourier Transform
4. The Discrete Fourier Transform is the appropriate Fourier representation for digital computer realization because it is discrete and of finite length in both the time & frequency domains.

B. Relationship between Discrete Fourier Transform & Discrete Fourier series

$X[k] = NC_k$

C. Relation between Discrete Fourier Transform & Discrete Time Fourier Transform

$X[k] = X[\Omega] \big|_{\Omega = \frac{2\pi}{N}k}$

6.4 PROPERTIES OF DISCRETE FOURIER TRANSFORM

$[m] \bmod N = m + iN \rightarrow$ Integer

$0 \leq [m]_{\bmod N} < N$

Example

$x(n) = \delta(n-3), x[(n-4)]_{\bmod 6}$

Solution

$x[(n-4)]_{\bmod 6} = \delta(n-7)_{\bmod 6}$

$= \delta(x - 7 + 6) = \delta(n-1)$

1. Linearity:

$a_1x_1(n) + a_2x_2(n) \leftrightarrow a_1x_1(k) + a_2x_2(k)$

2. Time Shifting:

$$x[n - n_0]_{\text{mod } N} \leftrightarrow W_N^{-kn0} X[k], W_N = e^{-j\left(\frac{2\pi}{N}\right)}$$

3. Frequency Shifting:

$$W_N^{-kn0}\, x[n] \leftrightarrow X[k - k_0]_{\text{ mod N}}$$

4. Conjugation:

$$x^*(n) = X^*[-k]_{\text{ mod N}}$$

5. Time Reversal:

$$x[-n]_{\text{ mod N}} \leftrightarrow X[-k]_{\text{mod N}}$$

6. Duality:

$$X[n] \leftrightarrow Nx[-k]_{\text{mod N}}$$

7. Circular Convolution:

$$x_1(n) \otimes x_2(n) \leftrightarrow X_1[k]\, X_2[k]$$

Where

$$x_1(n) \otimes x_2(n) = \sum_{n=0}^{N-1} x_i(i)\, x_2(n - i)_{\text{ mod N}}$$

8. Multiplication:

$$x_1(n)\, x_2(n) \leftrightarrow \frac{1}{N} X_1[k] \otimes X_2[k]$$

Where

$$X_1[k] \otimes X_2[k] = \sum_{i=1}^{N-1} x_i(i)\, x_2(k - i)_{\text{ mod N}}$$

9. Additional: -

When x(n) is real

$x(n) = x_e(n) + x_o(n)$

$x[n] \leftrightarrow X[k] = A[k] + jB[k] = |X[k]|\ e^{j\theta[k]}$

Then,

$X[-k]_{\text{ mod N}} \leftrightarrow X^*[k]$

$x_e[n] \leftrightarrow \text{Re}\ [X[k]] = A[k]$

$x_o[n] \leftrightarrow \text{Ima.}\ [X[k]] = B[k]$

$A[k] = A[-k]_{\text{ mod N}}, B[-k]_{\text{ mod N}} = -B[k]$

$X[-k]\ |_{\text{ mod N}} = |X[k]|,$

$Q[-k]_{\text{mod N}} = -Q[k]$

10. Parseval's theorem:

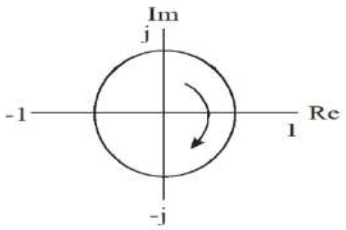
$$\sum_{n=0}^{N-1} |(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} x(k)|^2$$

(Parseval's Identity)

TWIDDLE FACTOR ($W = e^{-j2\pi}$) phase factor

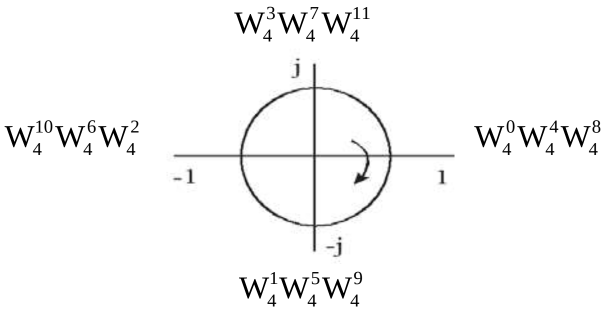
$W = e^{-j2\pi}, W_N = e^{-j\frac{2\pi}{N}}$

$W_N^{nk} = e^{-j\left(\frac{2\pi}{N}\right)nk}$



Evaluation of twiddle factor Divide circle in 4 equal parts

For n = 4



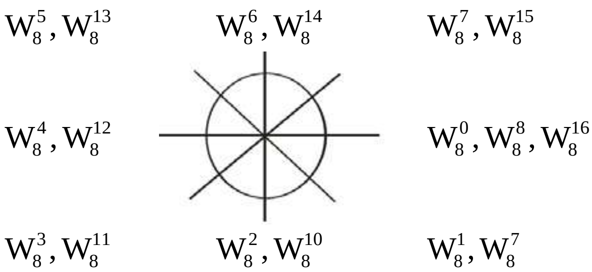
W_4^0, W_4^4, W_4^8 .... = 1

W_4^1, W_4^5, W_4^9 .... = - j

W_4^2, W_4^6, W_4^{10} .... = - 1

W_4^3, W_4^7, W_4^{11} .... = + j

For N = 8



$W_8^0\ ,\ W_8^8$ ....1, $W_8^1 \rightarrow \frac{1-j}{\sqrt{2}}$, $W_8^2 \rightarrow -j$

$W_8^3 \rightarrow \frac{-1-j}{\sqrt{2}}$, $W_8^4 = -1$, $W_8^5 = \frac{-1+j}{\sqrt{2}}$

$W_8^6 = j$, $W_8^7 \rightarrow \frac{1+j}{\sqrt{2}}$, $W_8^8 = 1$

TWIDDLE FACTOR MATRIX

W_{NXN}

$$\begin{matrix} W_N^0 & W_N^0 & W_N^0 & \text{--------} & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & \text{--------} & W_N^{N-1} \\ W_N^0 & W_N^2 & W_N^4 & \text{--------} & W_N^{2(N-1)} \\ W_N^0 & W_N^3 & W_N^6 & \text{--------} & W_N^{3(N-1)} \\ W_N^0 & W_N^4 & W_N^8 & \text{--------} & W_N^{4(N-1)} \\ \text{--} & \text{--} & \text{--} & \text{--------} & \text{--} \\ \text{--} & \text{--} & \text{--} & \text{--------} & \text{--} \\ W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \text{--------} & W_N^{(N-1)^2} \end{matrix}$$

$$\begin{matrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{matrix} = \begin{matrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & J \\ 1 & -1 & 1 & -1 \\ 1 & J & -1 & -j \end{matrix}$$

N- Point Discrete Fourier Transform

$X[k] = [W]_{NXN}\ [x(n)]$ N – point Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$$

k = 0, 1, 2, 3… N – 1

For N – point IDFT

$$x[n] = \frac{1}{N} [W]_{NXN}\ [X(k)]$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} x(k)\, W_N^{-nk}$$

n = 0, 1, 2, 3 … N – 1

Matrix Multiplication of Discrete Fourier Transform is applicable for sequence whose length is in power of Two …… 2, 4, 8, 16…

	M A S
Compute DFT→	Multiply→Add→Store

Direct computation of N-point Discrete Fourier Transform requires N^2 multiplication and N(N-1) number of addition.

Example

Find 4 – point Discrete Fourier Transform of x (n) = [1, 2, 3, 4]

Solution

$X[k] = [W_{4X4}]\ [x(n)]$

$$\begin{matrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{matrix} = \underset{4\,X\,4}{\begin{matrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & J \\ 1 & -1 & 1 & -1 \\ 1 & J & -1 & -j \end{matrix}} \quad \underset{4\,X\,1}{\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}}$$

[10 +, 1 -2j -3 + 4j, 1 -2 +3 -4, 1 +2j -3 -4j]

X[k] = [10, 1 -2 + 2j, -2, -2 -2j]

$|X[k]| = [10, \sqrt[2]{2}, 2, \sqrt[2]{2}]$

$\measuredangle X[k] = [0^0, 135, 180, 225]$

Example

Find Inverse Discrete Fourier Transform (IDFT) of the sequence

[10, -2 +2j -2 -2 -2j]

$[x(n)] = \frac{1}{4} [W_{4X4}]^*\ [X[k]]$

Solution:

$$x(n) = \frac{1}{4} \begin{matrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & J \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{matrix} * \begin{matrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{matrix}$$

$$= \frac{1}{4} \begin{matrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & J \end{matrix} \begin{matrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{matrix} = \begin{matrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{matrix} = \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

Example

Consider two sequence x (n) & h(n) of length 4 given us

$$x(n) = \cos\left(n\frac{\pi}{2}\right)$$

n = 0, 1, 2, 3…

$h(n) = \left[\frac{1}{2}\right]^n$

n = 0, 1, 2, 3…

a) Calculate y[n] =x[n]*h[n] by doing circular convolution directly

b) Calculate y[n] by Discrete Fourier Transform

$x[n] = [1, 0, -1, 0]$ and $h[n] = \left[1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right]$

Example

N-point Discrete Fourier Transform of following sequence x[n]

a) x (n) = δ[n]

b) x (n) = 4(n) – 4(n – N)

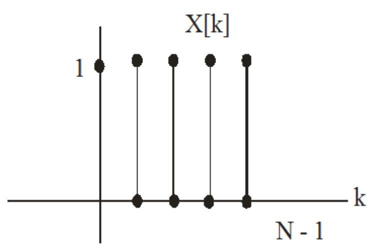
$x[n] = \sum_{n=0}^{N-1} \delta(n)\, W_N^{kn} = 1$

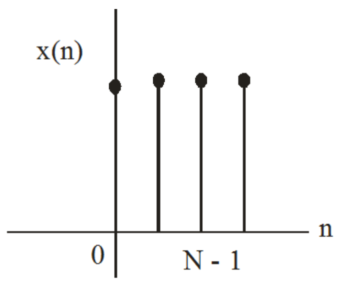
k = 0, 1, 2, 3 … N – 1

Solution:

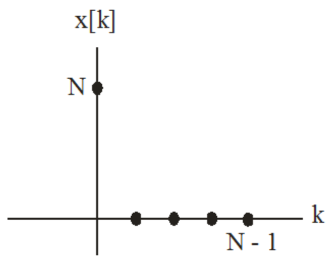
a)

δ(n) ↔





↔



b) $x[k] = \sum_{n=0}^{N-1} W_N^{kn} = \frac{1 - W_N^{kn}}{1 - W_N^{k}} = 0$

$k \neq 0$ ↺ since $W_N^{kn} = e^{-j\frac{2\pi}{N}kN} = e^{-j2\pi k} = 1$

$k \neq 0$

$x[0] = \sum_{n=0}^{N-1} W_N^{0} = \sum_{n=0}^{N-1} 1 = N$

Linear Convolution:

a)

	x(n)				
h(n)		1	0	-1	0
	1	1	0	-1	0
	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
	$\frac{1}{4}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	0
	$\frac{1}{8}$	$\frac{1}{8}$	0	$-\frac{1}{8}$	0

x (n) * h(n)

$\left[1, \frac{1}{2}, \frac{-3}{4}, \frac{-3}{8}, \frac{-1}{4}, \frac{-1}{8}, 0\right]$

Wrap around

N – Point $1, \frac{1}{2}, \frac{-3}{4}, \frac{-3}{8}, \frac{-1}{4}, \frac{-1}{8}, 0$

N – Point circular convolution

$\frac{3}{4}, \frac{3}{8}, \frac{-3}{4}, \frac{-3}{8}$

$y(n) = \frac{3}{4}, \frac{3}{8}, \frac{-3}{4}, \frac{-3}{8}$

b) $X[k] = \sum_{n=0}^{3} x(n)\, W_4^{kn} = 1 - W_4^{2k}$ k = 0, 1, 2, 3

$X[k] = \sum_{n=0}^{3} h(n)\, W_4^{kn}$

$= 1 + \frac{1}{2} W_4^{k} + \frac{1}{4} W_4^{2k} + \frac{1}{8} W_4^{3k}$ k = 0,1,2,3

$Y[k] = X[k]\, H[k]$

$= 1 - W_4^{2k} \quad 1 + \frac{1}{2} W_4^{k} + \frac{1}{4} W_4^{2k} + \frac{1}{8} W_4^{3k}$

$= \; 1 + \frac{1}{2} W_4^{k} + \frac{1}{4} W_4^{2k} + \frac{1}{8} W_4^{3k} + \frac{1}{2} W_4^{2k}$

$- \frac{1}{2} W_4^{3k} - \frac{1}{4} W_4^{4k} - \frac{1}{8} W_4^{5k} \quad W_4^{5k} = W_4^{k}$

$= 1 - \frac{1}{4} + \frac{3}{8} W_4^{k} + \frac{1}{8} W_4^{3k} + \frac{3}{4} W_4^{2k} - \frac{3}{8} W_4^{3k}$

Thus by the definition of Discrete Fourier Transform

We get

$y[n] = \frac{3}{4}, \frac{3}{8}, \frac{-3}{4}, \frac{-3}{8}$

Example

Consider the finite length complex exponential sequence

$$x[n] = \begin{cases} e^{j\Omega o n} & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

a) Find x (Ω)

b) Find N–point Discrete Fourier Transform X[k] of x (n)

Solution

$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{j\Omega o n}$

$= \sum_{n=0}^{N-1} e^{j\Omega o n}\, e^{-j\Omega n} = \sum_{n=0}^{N-1} e^{-j(\Omega-\Omega o)n}$

$= \frac{1-e^{-j(\Omega-\Omega o)n}}{1-e^{-j(\Omega-\Omega o)}}$

$= \frac{e^{-j(\Omega-\Omega o)N/2}}{1-e^{-j(\Omega-\Omega o)/2}} \frac{e^{-j(\Omega-\Omega o)N/2} - e^{-j(\Omega-\Omega o)N/2}}{e^{-j(\Omega-\Omega o)N/2} - e^{-j(\Omega-\Omega o)N/2}}$

$= e^{-j(\Omega-\Omega o)} \frac{(N-1)}{2} \frac{\sin[(\Omega-\Omega 0)N/2]}{\sin[(\Omega-\Omega 0)/2]}$

$X[k] = X[\Omega]_{\Omega} = \frac{2\pi k}{N} = X\left[\frac{k.2\pi}{N}\right]$

$X[k]\, e^{-j\left[\left(\frac{2\pi}{N}\right)k-\Omega 0\right]} \frac{N-1}{2} \frac{\sin\left[\left(\frac{2\pi}{N}k-\Omega 0\right)\frac{N}{2}\right]}{\sin\left[\left(\frac{2\pi}{N}k-\Omega 0\right)\frac{1}{2}\right]}$

Important Property

$W_N = e^{-j2\pi/N} \rightarrow$ phase factor

Periodicity: $W_N^{k+N} = W_N^{k}$

Symmetry: $W_N^{k+N/2} = -W_N^{k}$

Example

Let set x (n) = [A, 2, 3, 4, 5, 6, 7, B] if X (0) = 20 and X (4) = 0, N = 8

Find A & B?

Solution

$X(0) = \sum_{n=0}^{1} x(n)$

20 = A + 1 + 3 + 4 + 5 + 6 +7 + B

20 = 21 + A + B

A + B = -7

$X(4) = \sum_{n=0} x(n)\,(-1)^n$

=A [(+1)] +2(-1) +3(1) +4(-1) +5(1) +6(-1) +7 (1) + 8(-1)

0 = A – B + 3

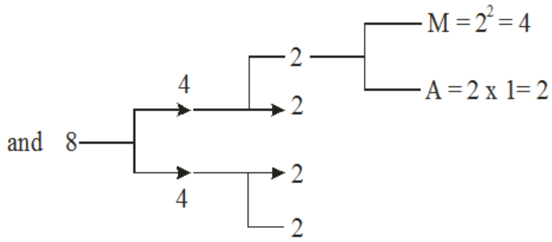
A + B = -7

A – B = -3

2A = -10

A = -5, B = -2

Direct Computation of 8 – point Discrete Fourier Transform	Multiplication $8^2 = 64\ N^2$
	Addition N(N−1)=56 8×7



So total M = 4 x 4 = 16

A = 2 x 4 = 8

The complexity associated with direct computation of Discrete Fourier Transform can be reduced by Fast Fourier Transform

FAST FOURIER TRANSFORM

1) Decimation in Time Fast Fourier Transform (DITFFT)
2) Decimation in Frequency Fast Fourier Transform (DIFFFT)

GATE QUESTIONS(EC)

Q.1 Let $x(n) = \left(\frac{1}{2}\right)^2 u(n), y(n) = x^2(n)$ and $Y(e^{jw})$ be Fourier transform of y(n). Then $Y(e^{j0})$ is

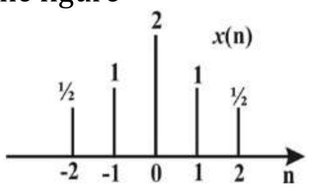
a) $\frac{1}{4}$ b) 2

c) 4 d) $\frac{4}{3}$

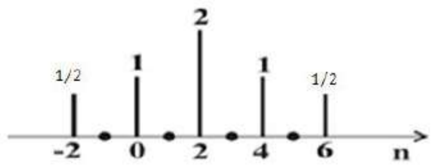
[GATE-2005]

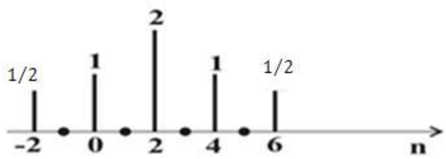
Statement of Linked Answer Questions 2 and 3:

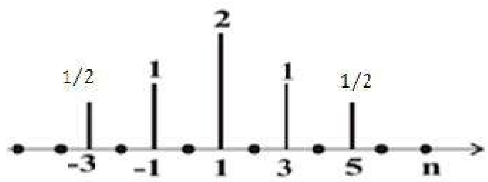
A sequence $x(n)$ has non-zero values as shown in the figure

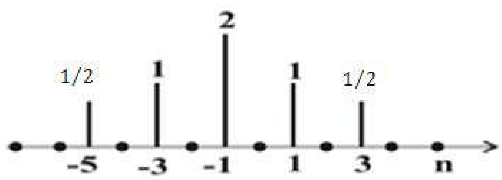


Q.2 The sequence $y(n) = x\left(\frac{n}{2} - 1\right)$ for n even will be =0, for n odd

a)


b)


c)


d)


[GATE-2005]

Q.3 The Fourier transform of y (2n) will be

a) $e^{-j2\omega}[\cos 4\omega + 2\cos 2\omega + 2]$

b) $[\cos 2\omega + 2\cos\omega + 2]$

c) $e^{-j\omega}[\cos 2\omega + 2\cos\omega + 2]$

d) $e^{\frac{-j\omega}{2}}[\cos 2\omega + 2\cos\omega + 2]$

[GATE-2005]

Q.4 A signal $x(n) = \sin(\omega_0 n + f)$ is the input to a linear time-invariant system having a frequency response $H(e^{j\omega})$. If the output of the system $Ax(n - n_0)$, then the most general form of $\angle H(e^{j\omega})$ will be

a) $-n_0\omega_0 + \beta$ for any arbitrary real

b) $-n_0\omega_0 + 2\pi k$ for any arbitrary integer k.

c) $n_0\omega_0 + 2\pi k$ for any arbitrary integer k.

d) $-n_0\omega_0\Phi$.

[GATE-2005]

Q.5 A 5 – point sequence x [n] is given as

$x[-3] = 1$, $x[-2] = 1$, $x[-1] = 0$, $x[0] = 5$, $x[1] = 1$

Let $X(e^{j\omega})$ denote the discrete – time Fourier transform of x [n]. The value of $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$ is

a) 5 b) 10 π

c) 16 π d) 5 + j10 π

[GATE-2007]

Q.6 {x(n)} is real –valued periodic sequence with a period N. x(n) and X(k) form N-point Discrete Fourier Transform (DFT) pairs. The DFT Y(k) of the sequence $y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)x(n+r)$ is

a) $|X(k)|^2$

b) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)X^*(k+r)$

c) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)X(k+r)$

d)0

[GATE-2008]

Q.7 The 4-point Discrete Fourier Transform (DFT) of a discrete time sequence {1, 0, 2, 3} is

a) [0, -2 + 2j, 2, -2 -2j]

b) [2, 2 + 2j, 6, 2-2j]

c) [6, 1-3j, 2, 1 + 3j]

d) [6, - 1 + 3j, 0, -1 -3j]

[GATE-2009]

Q.8 For an N-point FFT algorithm with $N = 2^m$, which one of the following statements is TRUE?

a) It is not possible to construct a signal flow graph with both input and output in normal order

b) The number of butterflies in the m^{th} state is N/m

c) In –place computation requires storage of only 2N node data

d) Computation of a butterfly requires only one complex multiplication.

[GATE-2010]

Q.9 The first five points of the 8 –point DFT of a real valued sequence are 5, 1-j3, 0, 3-j4 and 3+j4 the last two points of the DFT are respectively

a) 0,1-j3 b)0,1+j3

c) 1+j3,5 d)1-j3,5

[GATE-2011]

Q.10 The DFT of a vector [a b c d] is the vector [α β γ δ]. Consider the product

$$[p \quad q \quad r \quad s]=[a \quad b \quad c \quad d]\begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}$$

The DFT of the vector [p q r s] is a scaled version of

a) $[\alpha^2 \quad \beta^2 \quad \gamma^2 \quad \delta^2]$

b) $[\sqrt{\alpha} \quad \sqrt{\beta} \quad \sqrt{\gamma} \quad \sqrt{\delta}]$

c) $[\alpha+\beta \quad \beta+\delta \quad \delta+\gamma \quad \gamma+\alpha]$

d) $[\alpha \quad \beta \quad \gamma \quad \delta]$

[GATE-2013]

Q.11 Let $X[k]=k+1,\ 0\le k\le 7$ be 8-point DFT of a sequence $x[n]$, where $X[k]=\sum_{n=0}^{N-1}x[n]e^{-j2\pi nk/N}$ .The value (correct to two decimal places) of $\sum_{n=0}^{3}x[2n]$ is _____.

[GATE-2018]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10
d	a	c	b	b	*	d	d	a	a
11									
3									

EXPLANATIONS

Q.1 (d)

$x(n) = \left(\frac{1}{2}\right)^n u(n)$

$y(n) = x^2(n) = \left(\frac{1}{2}\right)^{2n} u^2(n)$

$= [(\frac{1}{2})^2]^n u(n)$

$y(n) = \left(\frac{1}{4}\right)^n u(n)$

$y(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$

Put $z = e^{j\omega}$

$Y(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$

$Y(e^{j0}) = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$

Q.2 (a)

$y(n) = x(\frac{n}{2} - 1)$,n even

$= n$ for n odd

$n = 0, y(n) = x(-1) = 1$

$n = 2, y(n) = x(0) = 2$

$n = 4, y(n) = x(1) = 1$

$n = 6, y(n) = x(2) = 1/2$

Q.3 (c)

$y(2n) = x(n-1)$, seeing in graph

$f(n) = y(2n)$

$= \frac{1}{2}\delta(n+1) + \delta(n) + 2\delta(n-1)$

$+\delta(n-2) + \frac{1}{2}\delta(n-3)$

Taking z-transform

$F(z) = \frac{1}{2}z + 1 + 2z^{-1}$

$+z^{-2} + \frac{1}{2}z^{-3}$

$z = e^{j\omega}$

$F(e^{j\omega}) = \frac{1}{2}e^{j\omega} + 1 + 2e^{-j\omega}$

$+e^{-2j\omega} + \frac{1}{2}e^{-3j\omega}$

$= e^{-j\omega}(\frac{1}{2}e^{2j\omega} + e^{j\omega} + 2 +$

$e^{-j\omega} + \frac{1}{2}e^{-j\omega})$

$= e^{-j\omega}[\frac{e^{2j\omega}+e^{-2j\omega}}{2} + e^{j\omega}$

$+e^{-j\omega} + 2]$

$f(n) = e^{-j\omega}[\cos 2\omega + 2\cos\omega + 2]$

Q.4 (b)

$y(n) = Ax(n - n_0)$

$A\sin[\omega_0(n - n_0) + \Phi]$

$-\frac{d\theta(\omega)}{d\omega} = n_0 (= t_g)$

$\theta(\omega) = -n_0 \int d\omega$

$= -n_0\omega_0 + k$

To avoid phase change k should be an integral multiple of 2π

$\therefore\ \theta(\omega) = -n_0\omega_0 + 2\pi k$

Q.5 (b)

$X(e^{j\omega}) = e^{3j\omega} + e^{2j\omega} + 0 + 5 + e^{-j\omega}$

$\because \int_{-\pi}^{\pi} e^{aj\omega} d\omega = 0$ if $a \neq 0$

$\int_{-\pi}^{\pi} X(e^{j\omega})d\omega = \left|\frac{e^{3j\omega}}{3j} + \frac{e^{2j\omega}}{2j} + 5\omega + \frac{e^{-j\omega}}{-j}\right|_{-\pi}^{\pi}$

$= 5\pi + 5\pi = 10\pi$

Q.6 (*)

Q.7 (d)

4-point DFT of sequence {1,0,2,3} is given as

$$\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & -1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\end{bmatrix}\begin{bmatrix}1\\0\\2\\3\end{bmatrix}$$

$$= \begin{bmatrix}1+2 & +3\\ 1-2 & +3j\\ 1+2 & -3\\ 1-2 & -3j\end{bmatrix} = \begin{bmatrix}6\\ -1+3j\\ 0\\ -1-3j\end{bmatrix}$$

Q.8 (d)

For an N-point FFT algorithm with $N = 2^m$, computation of a butterfly requires only one complex multiplication and two complex additions.

Q.9 (a)

The given function $f(x)$ is periodic function with period , $T = 2\pi$ F(x) is shown over one period from 0 to 2π in fig

The d.c value of

$f(x) = a_0 = \frac{1}{T}\int_0^T f(x)\,dx$

$a_0 = \frac{Area\ under\ f(x)}{T} = 0$

As can be seen from Fig.

Q.10 (a)

DFT of vector [a, b, c, d] is [α β γ δ]

i.e. $\begin{bmatrix}\alpha\\ \gamma\\ \beta\\ \delta\end{bmatrix} = \begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & j & -1 & -j\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}$

$= \begin{bmatrix}a+b & + & c+d\\ a-j & b- & c+jd\\ a-b & + & c-d\\ a+j & b-c & -jd\end{bmatrix}$ ....(i)

Now, it is given that

$[p \quad q \quad r \quad s] =$

$[a \quad b \quad c \quad d]\begin{bmatrix}a & b & c & d\\ d & a & b & c\\ c & d & a & b\\ b & c & d & a\end{bmatrix}$

$= [a^2 + bd + c^2bd \;\; ab + ab +]$

$cd + cd \;\; 2ac + b^2 + d^2 \;\; 2ad + 2bc$ ...(ii)

DFT of $[p \quad q \quad r \quad s]$ is given as

$[p \quad q \quad r \quad s]\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & j & -1 & -j\end{bmatrix}$

$= \begin{bmatrix}(p+q+r+s)(p-jq-pr+js)\\ (p-q+r-s)(p+jq-r-js)\end{bmatrix}$

$= [\alpha^2 \; \beta^2 \; \gamma^2 \; \delta^2]$

Check:

$P + q + r + s = (a^2 + c^2 + 2bd)$

$+(2ab + 2cd) + (b^2 + d^2 + 2ac)$

$+(2ad + 2bc)$

...[From (ii) equation]

$= (a+b+c+d)^2$

$= a^2$ ...[From (i) equation]

Q.11 3

$X[k] = k+1, \; 0 \le k \le 7$

$X[k] = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Find

$$\sum_{n=0}^{3} x[2n] = x[0] + x[2]x[4] + x[6]$$

N point Discrete Fourier Transform (DFT)

$$X(K) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}$$

For $K = 0$

$$X(0) = \sum_{n=0}^{7} x(n)$$

$$= x(0)+x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)$$

(i)

For $K = 4$

$$X(4) = \sum_{n=0}^{7} x(n)(-1)^n$$

$$= x(0)-x(1)+x(2)-x(3)+x(4)-x(5)+x(6)-x(7)$$

(ii)

Adding (i) and (ii)

$$X(0)+X(4) = 2[x(0)+x(2)+x(4)+x(6)]$$

$$x(0)+x(2)+x(4)+x(6) = \frac{X(0)+X(4)}{2}$$

$$= \frac{1+5}{2} = 3$$

GATE QUESTIONS(IN)

Q.1 4- point DFT of a real discrete –time signal $x[n]$ of length 4 is gives by $X[k], n = 0,1,2,3, \text{and } k = 0,1,2,3$. It is given that $x[0] = 5, X[1] = 1 + j1,$ $X[2] = 0.5, X[3]$ and $x[0]$ respectively are

a) $1 - j, 1.875$ b) $1 - j, 1.500$

c) $1 + j, 1.875$ d) $0.1 - j0.1, 1.500$

[GATE-2010]

Q.2 X(k) is the Discrete Fourier Transform of a 6-point real sequence x(n). If X(0)=9 + j0, X(2) = 2 + j2, X(3)= 3- j0, X(5)=1- j1, x(0) is

a) 3 b) 9

c) 15 d) 18

[GATE-2014]

Q.3 Three DFT coefficients, out of the DFT coefficients of a five-point real sequence are given as: X(0)=4, X(1)=1-1j and X(3)=2+j2. The zeroth value of the sequence x(n).

a) 1 b) 2 c) 3 d) 4

ANSWER KEY:

1	2	3
a	a	b

EXPLANATIONS

Q.1 **(a)**

As signal is real,

$X[k] = X^*(N-k)$

$\therefore X[3] = 1 - J$

and $X[0] = \frac{1}{N}\sum_k X(k)$

$= \frac{1}{4}(5 + 1 + J1 + 0.5 + 1 - J1)$

= 1.875

Q.2 **(a)**

Given 6 point DFT of x[n] with

X (0)9+ j0

X(1)= 1

X(2)=2+ j2

X (3)=3- j0

X(4)= 2

X (5)=1- j

⇒ By symmetry properties

X(1) =x* (5)=1+j

X(4) =x* (2)=2-j2

X[0]=

$$\frac{1}{6}[x(0)+x(1)+x(2)+x(3)+x(4)+x(5)]$$

$$=\frac{1}{6}[9+1+2+3+2+1]=3$$

Q.3 **(b)**

Since $x[n]$ is real

so , $X[k]=X^*[N-k]$

$X[2]=X^*[3]=2-j2$

$X[4]=X^*[1]=1+1j$

So , $X[4]=\frac{1}{N}\sum_{k=0}^{4}X[k]$

$$=\frac{4+1+2+2+1}{5}=2$$

7 SAMPLING THEOREM

Sampling is a process to convert a continuous–time signal to a discrete–time signal.

7.1 SAMPLING THEOREM

"A band-limited signal can be reconstructed exactly if it is sampled at a rate atleast twice the maximum frequency component in it."

$f_s \geq 2\, f_m$ $f_m \rightarrow$ Modulating freq.

$\frac{1}{T_s} \geq \frac{2}{f_m}$ $f_s \rightarrow$ Sampling freq.

$\frac{T_m}{2} \geq T_s$

Sampling – Different types of sampling are

1. Ideal sampling
2. Practical sampling

(a) Natural sampling

(b) Flat top sampling

Ideal Sampling:

Sampling though Ideal impulse train

Impulse Train:

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Where T_s = Sampling time

Analog continuous to Analog discrete

$$F \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \leftrightarrow \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\,\omega_s)$$

Fourier transform of impulse train is another impulse train with amplitude suppressed by $\frac{1}{T_s}$ with same period.

7.2 IDEAL SAMPLING

x (t) →X → X_s (t)

δ_{T_s} (t)

$$X_s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - n\,T_s)$$

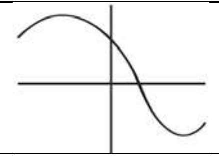
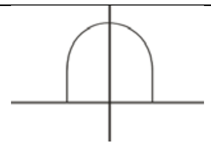
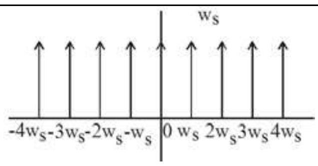
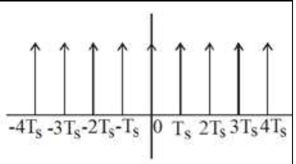
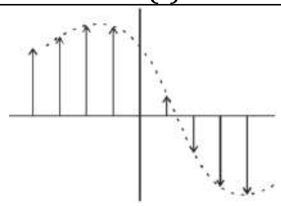
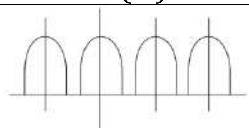
Fourier Transform

$$X_s(\omega) = \frac{1}{2\pi} X(\omega) * \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta(\omega - n\,\omega_s)$$

$$X_s(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\,\omega_s),\ \omega_s = \frac{2\pi}{T_s}$$

$\omega_s \rightarrow$ sampling rate

Assume band limited signal (spectrum)

x(t)		X(ω)
	↔	
δ_{T_s} (t)		δ_{T_s} (ω)
	↔	
X_s (t)		X_s (ω)
	↔	 $-\omega_m, \omega_m$ $-\omega_s\ +\omega_s\ +2\omega_s$

$$X_s(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\,\omega_s)$$

$= \frac{1}{T_s}$ [X(ω)+X(ω- ω_s)+X(ω- 2ω_s)+X(ω- ω_s)

+ X(ω- 2ω_s)+ …….]

X_s (ω) is a periodic function of ω, consisting of a super position of shifted replicas of X (ω) scaled by $\frac{1}{T_s}$

Output frequencies are:

A. ω_m
B. $\omega_s \pm \omega_m$
C. $2\omega_s \pm \omega_m$
D. $3\omega_s \pm \omega_m$

i) Thus as long as the sampling frequency f_s is greater than twice the signal bandwidth B, $X_s(\omega)$ will consist of non overlapping repetition of X (ω).
ii) x(t) can be recovered from its sample X_s (t) by passing the sampled signal through an ideal low pass filter of Bandwidth B Hz.
iii) The minimum sampling rate f_s = 2B required to recover x (t) from its samples X_s (t) is called the Nyquist rate.

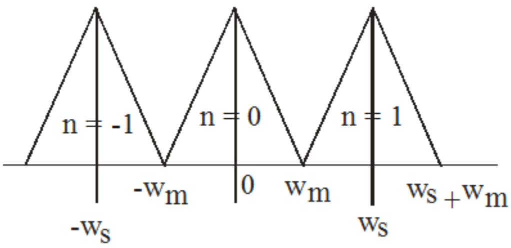
f_s = 2B = Nyquist rate

$T_s = \frac{1}{2B}$ = Nyquist interval

Case 1.
$\omega_s = 2\,\omega_m$ → critical sampling
Case 2.
$\omega_s > 2\,\omega_m$ → over sampling
Case 3.
$\omega_s < 2\,\omega_m$ → under sampling

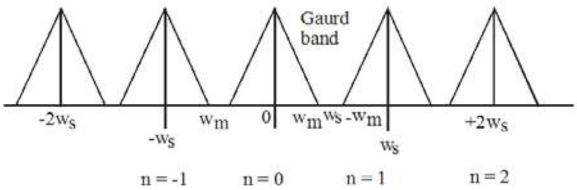
Case 1.
$\omega_s = 2\,\omega_m$

$$X_s(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - 2\omega_m n)$$



Critical sampling no overlapping takes place.
Case 2. $\omega_s = 2\,\omega_m,\ \omega_s = 3\omega_m$

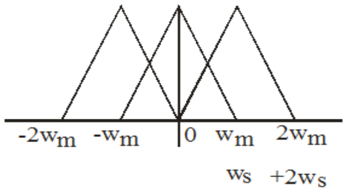
$$X_s(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - 3\omega_m n)$$



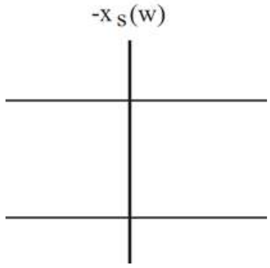
To avoid overlapping of signals $\omega_s - \omega_m > \omega_m$
i.e. $\omega_s > 2\omega_m$

Case 3.
$\omega_s < 2\,\omega_m,\ \omega_s < \omega_m$

$$\therefore X_s(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_m)$$



Aliasing (overlapping of nearby spectral component), spectral folding



Aliasing → replacing by retransmission with proper sampling frequency.
$\omega_s \geq 2\omega_m$: to avoid aliasing
Minimum Sampling rate → Nyquist rate = $2\omega_m$

Example
If the continuous time signal x (t) = cos 1250 πt is sampled at sampling frequency f_s = 10 Hz, then find the discrete time sequence x (n) ?
Solution:
$x(t) = x(nT_s) = \cos(1250\,\pi\, nT_s)$

$= \cos\left(\frac{1250\pi n}{f_s}\right)$

$f_s = \frac{1}{T_s}$, f_s = 10 Hz

$x(n) = \cos\left[\frac{1250\pi n}{10}\right] = \cos 125\pi n$

7.3 ALIASING

$(\omega_s > 2\omega_m)$ $X_s(\omega) \leftrightarrow X_s(t)$ no longer contain all info of X (ω). Hence x (t) is not recoverable from sample X_s (t).

$\omega_s < 2\omega_m$ (loss of information) is due to superimposition of high frequency component to low frequency component. This phenomenon is called frequency folding of Aliasing.

The Aliasing can be minimized or avoided by

(1) Selecting high sampling frequency. $(\omega_s > \omega_m)$

(2) Using anti aliasing filter before sampling.

Anti Aliasing Filter:

An analog filter is used before sampler to alternate signals with frequency higher than $\frac{\omega_s}{2}$ $(\omega_s \rightarrow \text{sampling frequency})$

Example

Calculate minimum sampling frequency for following signals?

a) $x_1(t) = \frac{\text{Sin}200\pi t}{\pi t}$ $\omega_m = 200\pi$

$\omega_s = 2\omega_m = 2 \times 200\pi$

Nyquist Rate = 400π

Nyquist frequency ← f_s = 200 Hz T_s

$= \frac{1}{f_s} = 0.05$ sec. (T_s = Nyquist interval)

b) $x_2(t) = \left[\frac{\sin 200\pi t}{\pi t}\right]^2$

$= \left[\frac{1-\cos 400\pi t}{(\pi t)^2 2}\right]$

$\omega_s = 2 \times \omega_m = 2 \times 400\pi = 800\pi$

f_s = 400 Hz

c) $x_3(t) = 5\cos 1000\pi t \cos 4000\pi t$

$= 5\cos 5 \times 10^3\pi t + \sin 3 \times 10^3\pi t$

$\omega_m = 5 \times 10^3\pi$

N.R. = 10000π = 2 × ω_m

f_s = 5000 Hz

d) $x_4(t) = e^{-6t}u(t) * \frac{\sin at}{\pi t}$

$$\text{rect}\left(\frac{t}{T}\right) \leftrightarrow 2T\frac{\sin\frac{\omega T}{2}}{\omega T}$$

$$2\frac{T\sin\frac{tT}{2}}{tT} \leftrightarrow 2\pi\,\text{rect}\left(\frac{-\omega}{T}\right), \quad T/2 = a$$

F.T

$$x_4(\omega) = \frac{1}{6+j\omega}\text{rect}\left(\frac{\omega}{2a}\right) \quad (\omega = \text{width})$$

$$\frac{\sin at}{\pi t} \leftrightarrow \text{rect}\left(\frac{\omega}{2a}\right)$$

$\omega_m = a$

Nyquist rate = 2a

Example

Calculate minimum sampling time for following signals?

a) x (t) = cos 2πt + cos 5πt

$\omega_m = 5\pi$, $\omega_s = 10\pi$, f_s = 5Hz

$2\pi, 5\pi\; T_s = \frac{1}{5}\text{sec} = \text{sec}$

b) $x(t) = \cos(2\pi t)\cdot\frac{\sin \pi t}{\pi t} + \cos(2\pi t)\cdot\frac{\cos \pi t}{\pi t}$

$\omega_m = 3\pi$

Hence, minimum sampling frequency

$\omega_s \geq 2\omega_m = 2 \times 3\pi = 6\pi$

$\frac{2\pi}{T_s} \geq 2 \times 3\pi$

$\frac{1}{T_s} \geq 3$

$T_s \leq \frac{1}{3}$ sec.

Example:

Two signals $x_1(t)$ & $x_2(t)$ are band limited to 3 kHz & 4 kHz respectively. Find the Nyquist rate for the following signals.

(a) $x_1(3t)$

(b) $x_2(t - 3)$

(c) $x_1(2t) + x_2(t)$

(d) $x_1(t) \cdot x_2(t)$

(e) $x_1(t) * x_2(t)$

Solution

a) $x_1(3t)$ Fourier Transform $\frac{1}{3} X_1\left(\frac{\omega}{3}\right) \leftrightarrow$

expand in freq. domain by factor 3.
$(BW)x_1$ = 3 kHz, BW (x_2) = 4 kHz
$\omega_{max}\,(x_1(3t))$ = 3 × 3 = 9 kHz,
ω_s = 2 × 9 = 18 kHz

b) x_2 (t - 3) Fourier Transform $e^{-j3\omega t}$
$X_2(\omega)$ only phase shift
$(BW)x_2$ = 4 kHz
$\omega_{max}\,(x_2(t-3))$ = 4 kHz,
ω_s = 4 × 2 = 8 kHz

c) $x_1(2t) + x_2(2t)$ Fourier Transform $\frac{1}{2} x_1\left(\frac{\omega}{2}\right) + x_2(\omega)$

$(BW)x_1$ = 2 ×3 = 6 kHz, BW (x_2) = 4 kHz
$\omega_{max}\,(x_1(2t))$= 6 kHz, ω_s =2×6 = 12 kHz

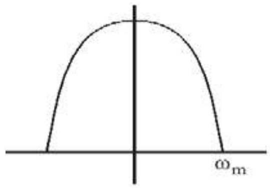
d) $x_1(t) \,.\, x_2(t)$ Fourier Transform $X_1(\omega) * X_2(\omega)$
The spectrum of both signals when multiplied (convolution in frequency domain) has the sum and difference frequencies terms.
3 + 4 = 7 kHz, (4 – 3) = 1 kHz
ω_s = 2 × BW Highest = 2 × 7 = 14 kHz

e) The spectrum of x_1 (t) * x_2 (t) (multiplication in freq. domain) extends to 3 kHz. (Lowest among the terms.); Hence the Nyquist rate is 6 kHz = 2 × 3kHz.

7.4 SAMPLING OF BAND PASS SIGNALS

Low pass signals

$X(\omega) = 0 \;\; |\omega| > B$ or ω_m

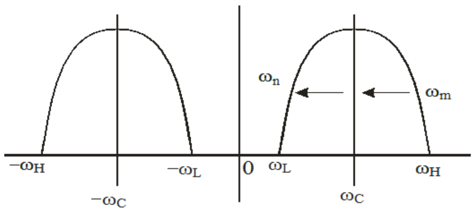


ω highest = ω_m

$X(\omega) = 0$

Band pass signals

$\omega_L > \omega$ and $\omega_H < \omega$



ω highest = $\omega_H = \omega_c + \omega_m$

Case 1
If either ω_H or ω_L is harmonic of sampling frequency
ω_s, then $\omega_s = 2\,(\omega_H - \omega_L)$ $(\omega_H - \omega_L)$

Example
ω_H = 10.2 Mhz, ω_L = 10 Mhz
ω_H = 51 x . 2 $\quad \omega_s$ = 2(.2) = 0.4 Mhz
ω_L = 50 X . 2 $\quad T_s = \frac{.1}{.4X10^6}$ = 2.5 μsec.

Case 2
If ω_L or ω_H is not harmonic of sampling frequency
ω_s, $\omega_s = \frac{2(\omega_c + \omega_m)}{[M]}$, or $f_s = \frac{2f_H}{(M)}$

Largest integer → $[M] = \frac{\omega_c + \omega_m}{2\omega_m} = \frac{f_H}{f_H - f_L}$

Example
Given the signal m (t) = 5cos 1000πt cos 7000πt. Find the minimum sampling rate based on

a) The low pass uniform sampling theorem and

b) The band pass sampling theorem.

Solution

a) m (t) = 5cos 1000πt cos7000πt

= 2.5 cos 5000πt + 2.5 cos 7000πt

2.5 kHz 3.5 kHz

f_m = 3.5 kHz

f_s = 2 X 3.5 = 7 kHz

b) f_H = 3.5 kHz f_L = 2.5 kHz

$$f_s = \frac{2X3.5khz}{[M]}$$

$$M = \frac{3.5}{1} = 3.5 \rightarrow \text{truncate}$$

$$[M] = 3$$

$$f_s = \frac{2X3.5khz}{3} = 2.333 \text{ kHz}$$

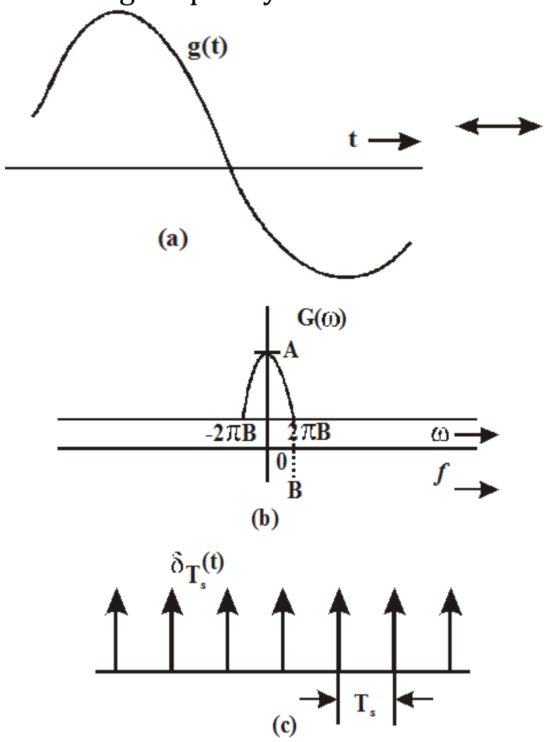
Summary

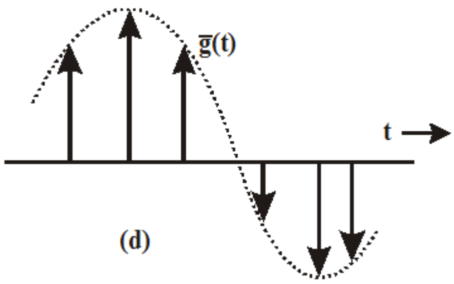
Sampling Theorem

"A band-limited signal can be reconstructed exactly if it is sampled at a rate atleast twice the maximum frequency component in it."

$$f_s \geq 2f_m$$

Where f_s is sampling frequency, f_m is modulating frequency





Impulse train

$$\delta_{T_S}(t) = \sum_{n=\infty}^{\infty} \delta(t - nT_S)$$

Where T_s = Sampling Time

Analog continuous to Analog discrete conversion

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_S) = \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_S)$$

Aliasing Effect:

Overlapping between nearby spectral Component

Type of Sampling

i) Ideal Sampling (Realize by Ideal Switch)

ii) Natural Sampling (Realize by switch)

iii) Flat top Sampling (Realize by Sample & Hold circuit)

GATE QUESTIONS(EC)

Q.1 A 1 KHz sinusoidal signal is ideally sampled at 1500 samples/sec and the sampled signal is passed through an ideal low-pass filter with cut-off frequency 800 Hz. The output signal has the frequency

a) Zero KHz b) 0.75 KHz
c) 0.5 KHz d) 0.25 KHz

[GATE-2004]

Q.2 A signal m(t) with bandwidth 500 Hz is first multiplied by a signal g(t) where

$$g(t) = \sum_{R=-\infty}^{\infty} (1)^k \delta(t - 0.5 \times 10^{-4} k)$$

The resulting signal is then passed through an ideal low pass filter with bandwidth 1 kHz. The output of the low pass filter would be

a) δ (t) b) m(t)
c) 0 d) m(t) δ(t)

[GATE-2006]

Q.3 An LTI system having transfer function $\frac{s^2+1}{s^2+2s+1}$ and input x(t) = sin(t + 1) is in steady state. The output is sampled at a rate ω_s rad/s to obtain the final output {y(k)}. Which of the following is true?

a) y(.) is zero for all sampling frequencies ω_s
b) y(.) is nonzero for all sampling frequencies ω_s
c) y(.) is nonzero for $\omega_s > 2$, but zero for $\omega_s < 2$
d) y(.) is zero for $\omega_s > 2$, but nonzero for $\omega_s < 2$

[GATE-2009]

Q.4 A band –limited signal with a maximum frequency of 5 kHz is to be sampled. According to the sampling theorem, the sampling frequency in which is not valid is

a) 5kHz b) 12kHz
c) 15kHz d) 20kHz

[GATE-2013]

ANSWER KEY:

1	2	3	4
c	b	a	a

EXPLANATIONS

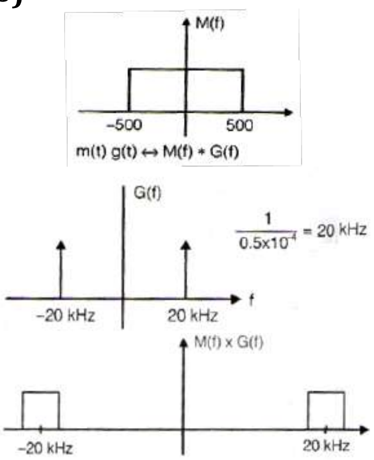
Q.1 (c)

$1k \times 1.5k = 2.5k\ or 0.5k$

LPF has $f_c = 0.8k$

∴ Only 0.5 k will appear at output.

Q.2 (b)



After low pass filtering with $f_c = 1kHz$ output is zero

Q.3 (a)

$$X(s) \rightarrow \boxed{H(s) = \frac{s^2+1}{s^2+2s+1}} \rightarrow X(s)$$

$x(t) = sin(t+1)$

$\omega = 1rad/sec$

$X(s) = \frac{\omega}{\omega^2+s^2} e^s = \frac{1}{1^2+s^2} e^s = \frac{e^s}{s^2+1}$

$H(s) = \frac{Y(s)}{X(s)}$

$Y(s) = H(s)X(s)$

$Y(s) = \frac{s^2+1}{s^2+2s+1} \times \frac{e^s}{s^2+1} = \frac{e^s}{s^2+2s+1}$

$Y(s) = \frac{e^s}{(s+1)^2}$

$Y(t) = (t+1)e^{-(t+1)}$

$Y(\infty) = \lim_{s\to 0} sy(s)$

$= \lim_{s\to 0} \frac{se^s}{(s+1)^2} = 0$

So at steady state y(.) remains zero for all sampling frequencies ω_s.

Q.4 (a)

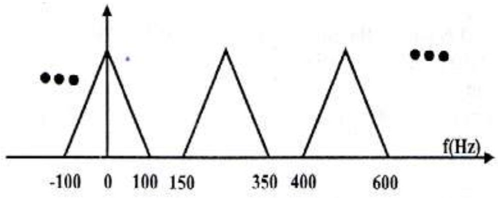
$(f_s)_{min} = 2f_m$

$(f_s)_{min} = 2 \times 5 = 1 = kHz$

So, $f_s \geq 10kHz$

GATE QUESTIONS(IN)

Q.1 The spectrum of a band limited signal after sampling is shown below. The value of the sampling interval is



a) 1ms b) 4ms
c) 2ms d) 8ms
[GATE-2006]

Q.2 Let $x(t)$ be a continuous –time real, valued signal band –limited to F Hz. The Nyquist sampling rate , in Hz , for $y(t) = x(0.5t) + x(t) - x(2t)$ is
a)F b) 2F
c)4F d)8F
[GATE-2007]

Q.3 The continuous time signal $x(t) = \cos(100\pi t) + \sin(300\pi t)$ is sampled at the rate 100Hz to get the signal $x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s), T_s =$ Sampling period The signal $x_s(t)$ is passed through an ideal low pass filter with cutoff frequency 100Hz. The output of the filter is proportional to
a) $\cos(100\pi t)$
b) $\cos(100\pi t) + \sin(100\pi t)$
c) $\cos(100\pi t) - \sin(100\pi t)$
d) $\sin(100\pi t)$
[GATE-2011]

Q.4 The highest frequency present in the signal x(t) is f_{max}. The highest frequency present in the signal y(t) = x^2(t) is
a) $\frac{1}{2} f_{Max}$ b) f_{max}
c) $2f_{max}$ d) $4f_{max}$
[GATE-2015]

Q.5 If a continuous time signal x(t)=cos(2πt) is sampled at 4Hz, the value of the discrete time sequence x(n)=5 is
a) -0.707 b) -1
c) 0 d) 1
[GATE-2017]

ANSWER KEY:

1	2	3	4	5
b	c	b	c	c

EXPLANATIONS

Q.1 (b)

Let X(f) be the spectrum of the band limited signal, x(t) . $X_s(f)$ be the spectrum of the sampled signal $X_s(t)$

$X_s(f) = f_s \sum_{k=-\infty}^{\infty} X(f - kf_s)$

Where f_s is the sampling frequency i.e. X(f) is repeated at

$\pm f_s, \pm 2f_s, \pm 3f_s \dots \dots \dots . etc$

From the given spectrum of $X_s(f)$ in the question, $f_s = 250\ s/sec$

∴Sampling interval,

$T_s = \frac{1}{f_s} = \frac{1}{250} \times 100ms = 4ms$

Q.2 (c)

$x(t)$ band limited to FHz

i.e. $f_m = FHz$

$y(t) = x(0.5t) + x(t) - x(2t)$ is

$x(t)$ – maximum frequency

$= 2FHz$.

$x(0.5t)$ –maximum frequency

$= \frac{F}{2} Hz.$

$x(2t)$ – maximum frequency

$= 2FHz.$

$y(t) - maximum\ frequency$

$= \max\left(F, \frac{F}{2}, 2F\right) = 2F$

Nyquist rate

$= 2 \times \text{maxfrequency}$

$= 2 \times 2F = 4FHz$

Q.3 (b)

Highest frequency component in x(t) is 150 Hz. So, the Nyquist sampling rate is 300Hz.But x(t) is sampled at 100Hz. While cost $(100\pi t)$ with frequency 50 Hz will be recovered satisfactorily after passing through the low-pass and $\sin(300\pi t)$ will get aliased resulting in filter output $\sin(100\pi t)$. $\cos(100\pi t)$ doesn't contribute to aliasing

Q.4 (c)

Multiplication in time domain corresponds to convolution in frequency domain

$x^2(t) \to x(f) * x(f)$

Using limit property of convolution $x^2(t)$ have maximum frequency $2f_m$.

Q.5 (C)

$$x(nT_s) = \cos 2\pi \times \frac{n}{4} = \cos\frac{\pi}{2} n = x(n)$$

$$x(n)_{n=5} = \cos\frac{5\pi}{2} = 0$$

GATE QUESTIONS(IN)

Q.1 The bilinear transformation $\omega=\frac{z-1}{z+1}$

a) maps the inside of the unit circle in the z-plane to left half of the ω-plane
b) maps the outside of the unit circle in the z-plane to left half of the ω-plane
c) maps the inside of the unit circle in the z-plane to right half of the ω-plane
d) maps the outside of the unit circle in the z-plane to right half of the ω-plane

[GATE-2002]

Q.2 A linear time invariant system with system function $H(z)=1+z^{-1}+z^{-2}$ is given an input signal sampled at 18kHz. The frequency of the analog sinusoid which cannot pass through the system is

a) $\frac{12}{\pi}$ kHz b) 6kHz
c) 12kHz d) $\frac{6}{\pi}$ kHz

[GATE-2003]

Q.3 Bilinear transformation avoids the problems of aliasing connected with the use of impulse invariance through

a) mapping the entire imaginary axis of the s-plane on the unit circle in the z-plane
b) pre-filtering the input-signal to impose bank-limitedness
c) mapping zeros of the left half of the s-plane inside the unit circle in the z-plane
d) up-sampling the input signal so that the bandwidth is reduced.

[GATE-2003]

Q.4 A digital notch filter with a notch frequency of 60Hz is to be transformed into one operating at a new notch frequency is 400Hz. A low-pass to low-pass transformation $z^{-1}=\frac{1-\alpha\hat{z}}{\hat{z}-\alpha}$ is used on the first to obtain the new notch filter. Then the value of α should be

a) -0.95 b) 0.95
c) -0.46 d) 0.46

[GATE-2004]

Q.5 A digital filter has the transfer function:

$$H(z)=\frac{z^2+1}{z^2+0.81}$$

If this filter has to reject a 50 Hz interference from the input, then the sampling frequency for the input signal should be

a) 50Hz b) 100Hz
c) 150Hz d) 200Hz

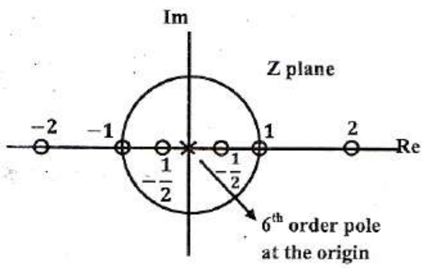
[GATE-2006]

Q.6 A digital filter having a transfer function $H(z)=\frac{p_0+p_1z^{-1}+p_3z^{-3}}{1+d_3z^{-3}}$ is implemented using Direct Form -I and Direct Form –II realization of IIR structure. The number of delay units required in Direct Form –I and Direct Form –II realizations are , respectively

a) 6 and 6 b) 6 and 3
c) 3 and 3 d) 3 and 2

[GATE-2010]

Q.7 Shown below is the pole –zero plot of a digital filter



Which one of the following statements is TRUE?

a) This is a low pass filter
b) This is high pass filter
c) This is an IIR filter
d) This is an FIR filter

[GATE-2011]

Q.8 A discrete-time signal x[n] is obtained by sampling an analog signal at 10 kHz.
The signal x[n] is filtered by a system with impulse response h[n] = 0.5{δ[n]+ δ
[n-1]}.The 3dB cutoff frequency of the filter is:

a) 1.25 kHz
b) 2.50 kHz
c) 4 .00 kHz
d) 5.00 kHz

[GATE-2014]

Q.9 Let 3 + 4j be zero of a fourth order linear-.phase FIR filter. The complex number Which is NOT a zero of this filter is

a) 3 -4j
b) $\frac{3}{25}+\frac{4}{25}j$
c) $\frac{3}{25}-\frac{4}{25}j$
d) $\frac{1}{3}-\frac{1}{4}j$

[GATE-2015]

Q.10 Consider a low-pass filter module with a pass-band ripple of in the gain magnitude. If M such identical modules are cascaded, ignoring the loading effects, the pass-band ripple of the cascade is

a) $1-(1-\delta)^M$
b) δ^M
c) $(1-\delta^M)$
d) $(1-\delta)^M$

[GATE-2015]

Q.11 The signal x[n]= sin (πn /6)/(π n) is processed through a linear filter with the impulse response h[n] = sin(ω_cn)/(π n) where $\omega_c > \pi/6$.
The output of the filter is

a) sin(2 ω_c n)/(π n)
b) sin(π n/3)/(π n)
c) $[\sin(\pi n/6)/(\pi n)]^2$
d) sin(π n/6)/(π n)

[GATE-2015]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11
a	c	b	c	a	b	d	b	d	a	d

EXPLANATIONS

Q.1 (a)

The bilinear transformation $\omega = \dfrac{z-1}{z+1}$

maps inside

the unit circle in the z-plane to left half of the $\omega-$plane.

For *z* inside of the unit circle O < z < 1

for O < z < 1

$$\omega = \frac{z-1}{z+1} = \frac{(-)\text{ve}}{(+)\text{ve}} = (-)\text{ve}$$

$\Rightarrow \omega-$ plane if lies left half of w-plane for *z* < 1.

Q.2 (c)

Given, f_s = 18 kHz and we know that,

$$f_s \geq 2f_m$$

$$f_m \leq \frac{f_s}{2}$$

$$f_m \leq \frac{18}{2}\text{kHz} = 9\,\text{kHz}$$

So, the system will pass the signal which is having frequency components less than 9 kHz,

So, the correct option is (c) which is having frequency 12 kHz is greater than 9 kHz, which cannot pass through the given system.

Q.3 (b)

Q.4 (c)

Q.5 (a)

Q.6 (b)

Given $H(z) = \frac{p_0+p_1z^{-1}+p_3z^{-3}}{1+d_3z^{-3}}$

This T.F corresponds to a 3rd order digital filter

D F –I realization requires 2N delays

D F –II realization or canonic realization with minimum number of delay elements requires N delays.

Where N is the order of the order of the filter

∴ D F –I requires 6 delay

D f - II requires 3 delays

Q.7 (d)

From the given pole-zero plot of the digital filter, the system function

$H(z) = \frac{(z+1)(z-1)\left(z+\frac{1}{2}\right)\left(z-\frac{1}{2}\right)(z+2)(z-2)}{z^6}$

$= \frac{1}{z^6}[(z^2-1)(z^2-\frac{1}{4})(z^2-4)]$

$= z^{-6}[(z^4-1.25z^2+\frac{1}{4})(z^2-4)]$

$= z^{-6}[z^6-4z^4-1.25z^4+6z^2+\frac{1}{4}z^2-1]$

$= z^{-6}[z^6-5.25z^4+6.25z^2-1]$

$= 1-5.25z^{-2}6.25z^{-4}-1z^{-6}$

$h(n) = (\ldots\ldots\ldots 0,0,\underset{\uparrow}{1},0,-5.25,0,6.25,0-1,0,0\ldots\ldots..)$

As the impulse response, h(n)=Inverse Z.T of H(z) has only finite duration =7 samples, the given digital is an FIR filter.

Q.8 (b)

Clearly, period of the signal x(t) is 3

So, T =3

And $a_0 = \frac{1}{T}\int_0^T x(t)\,dt$

$= \frac{1}{3}\int_0^3 x(t)dt$

$= \frac{1}{3}(1-1)$

$= 0$

So $a_k = 0$ for k even integer

Q.9 **(d)**

The property of a FIR filter is that if Zo is a zero then the remaining zeros are

So option (D) is not matching with any of this.

$$\frac{1}{z_0} \qquad [Z_0]^* \qquad \left[\frac{1}{z_0}\right]^*$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\left[\frac{3}{25} - j\frac{4}{25}\right] \quad [3-4j,] \quad \left[\frac{3}{25} + j\frac{4}{25}\right]$$

So options (D) is not matching with any of this .

Q.10 **(a)**

Q.11 **(d)**

$$\left[\frac{\sin \alpha n}{\pi n}\right] \times \left[\frac{\sin\beta n}{\pi n}\right] = \left[\frac{\sin\gamma n}{\pi n}\right]$$

Where $\gamma = \min(\alpha, \beta)$

Output g(n) = x(n)* h(n)

In frequency domain they will be multiplied.

ASSIGNMENT QUESTIONS

Q.1 An input x[n] with length 3 is applied to a linear time invariant system having an impulse response h[n] of length 5, and Y(ω) is the DTFT of the output y[n] of the system. If |h[n]| ≤ L and |x[n]| ≤ B for all n, the maximum value of Y(0) can be

a) 15 LB b) 12 LB
c) 8 LB d) 7 LB

Q.2 The two-sided Laplace transform of $x(t) = e^{-3t}u(t) + e^{2t}u(-t)$ is

a) $X(s) = \dfrac{-5}{s^2+s-6}, -3 < \sigma < 2$

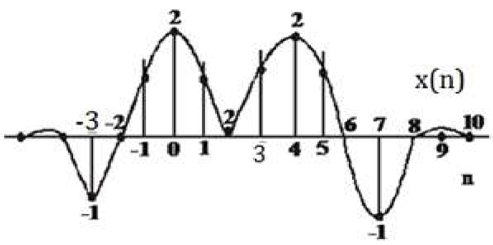
b) $X(s) = \dfrac{-5}{s^2+s-6}, 2 < \sigma < 3$

c) $X(s) = \dfrac{-5}{s^2+s-6}, -3 < \sigma < -2$

d) $X(s) = \dfrac{-5}{s^2+s-6}, -3 < \sigma < -2$

Q.3 For the signal x[n] shown in figure $x[n] = 0$ for n < -3 and n > 7

If X(ω) is the Fourier transform of x[n], which one of the following is TRUE?



a) X (0) = 5

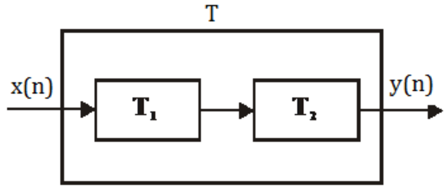
b) $\int_{-\pi}^{\pi} X(\omega)d\omega = 2\pi$

c) the phase $\angle X(\omega)$ odd function

d) X(ω) = X(-ω)

Q.4 Two systems T_1 and T_2 are cascaded to get the system T as shown in fig

Which one of the following statements is TRUE?

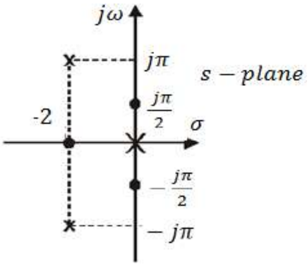


a) If both T_1 and T_2 are linear then T is NOT necessarily linear
b) If both T_1 and T_2 are time invariant then T is NOT necessarily time invariant
c) If both T_1 and T_2 are non-linear then T is NOT necessarily non-linear
d) If both T_1 and T_2 are causal then T is NOT necessarily causal

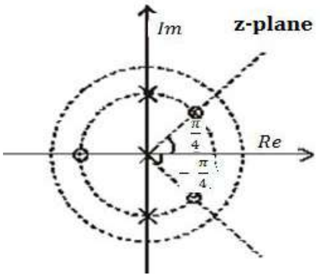
Q.5 If $x[n] = \begin{cases} \dfrac{2}{\pi}, & n = 0 \\ \dfrac{\sin 2n}{\pi n}, & n \neq 0 \end{cases}$, the energy of x[n] is

a) $\dfrac{2}{\pi}$ b) $\dfrac{1}{\pi}$
c) $\dfrac{1}{2\pi}$ d) $\dfrac{3}{\pi}$

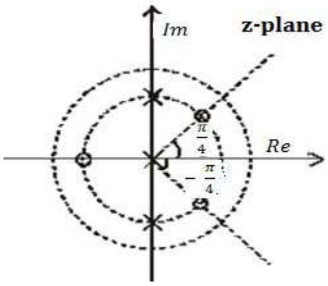
Q.6 The pole-zero plot of the transfer function ($H_a(s)$) of a linear time invariant system in s-plane is shown in Fig. The corresponding impulse response $h_a(t)$ is sampled at 2Hz to get the discrete-time impulse response sequence h[n]. If the right half of the s-plane is mapped into the outside of the unit circle, which one of the following shows the equivalent pole-zero plot of H(z) in the z-plane (the concentric circles are $|z| = \dfrac{1}{e}$ and $|z| = 1$)?



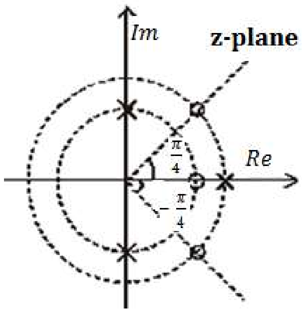
a)



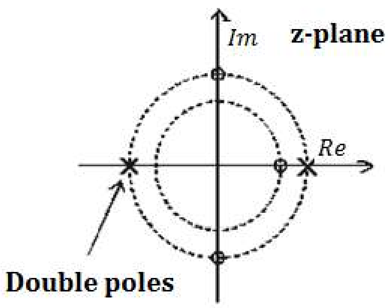
b)



c)



d)



Q.7 The z-transform x(z) of a sequence x[n] is given by

$$x(z)=\frac{z^{30}}{\left(z-\frac{1}{2}\right)(z-2)(z+3)}$$

If X(z) converges for |z|=1 then x[-18] is

a) $-\frac{1}{9}$ b) $-\frac{2}{21}$

c) $-\frac{1}{10}$ d) $-\frac{2}{27}$

Q.8 The z-transform X(z) of a real and right-sided sequence x[n] has exactly two poles and one of them is at $z = e^{j\pi/2}$ and there are two zeros at the origin. If X(1) =1, which one of the following is TRUE?

a) $X(z)=\frac{2z^2}{(z-1)^2+2}$, ROC is $\frac{1}{2}<|z|<1$

b) $X(z)=\frac{2z^2}{z^2+1}$, ROC is $|z|>\frac{1}{2}$

c) $X(z)=\frac{2z^2}{(z-1)^2+2}$, ROC is $|z|>1$

d) $X(z)=\frac{2z^2}{z^2+1}$, ROC is $|z|>1$

Q.9 The impulse response h[n] of a linear time invariant system is real. The transfer function H(z) of the system has only one pole and it is at $z=\frac{3}{4}$. The zeros of H(z) are non-real and located at $|z|=\frac{3}{4}$. The system is

a) Stable and causal
b) Unstable and ant-causal
c) Unstable and causal
d) Stable and anti-causal

Q.10 A signal x(t) is band-limited to W Hz, and $y(t)=x^3(t)+x(t)+1$. The Nyquist sampling frequency of y(t) is

a) 3 W b) 6 W
c) 12 W d) 27 W

Q.11 A signal $x(t)=\cos(10\,t)\cos(100t)$ is passed through a system whose impulse response is
$H(\omega)=\exp(-j100-j2(\omega-100))$.
If y(t) is the system output, then

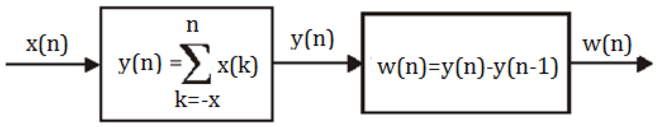
a)
$$y(t)=\cos\left(10\left(t-\frac{1}{2}\right)\right)\cos(100(t-1)$$

b)
$$y(t)=\cos(10(t-1))\cos\left(100\left(t-\frac{1}{2}\right)\right)$$

c) $y(t)=\cos(10(t-2))\cos(100(t-1))$

d) $y(t)=\cos(10(t-1))\cos(100(t-2))$

Q.12 The output w[n] of the system shown in fig. is



a) x[n]
b) x[n-1]
c) x[n] – x [n-1]
d) $\frac{1}{2}$ (x[n−1]+ [n])

Q.13 Which one of the following is a periodic signal?

a) $x_1(t)=2e^{j\left(t+\frac{\pi}{4}\right)}u(t)$

b) $x_2[n]=u[n]+u[-n]$

c)
$$x_3[n]=\sum_{k=-\infty}^{\infty}\{\delta[n-4k]-\delta[n-1-4k]\}$$

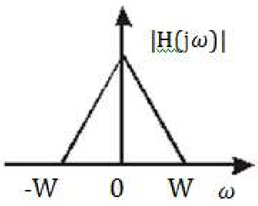
d) $x_4(t)=e^{(-1+j)t}$

Q.14 If the input-output relation of a system is $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$, then the system is

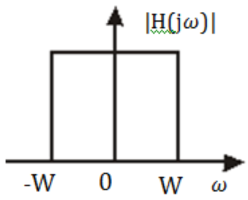
a) linear, time invariant and unstable
b) linear, non-causal and unstable
c) linear, causal and time invariant
d) Non-causal, time invariant and unstable

Q.15 Which one of the following can be the magnitude of the transfer function $|H(j\omega)|$ of a causal system?

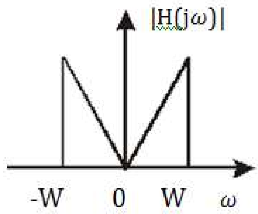
a)



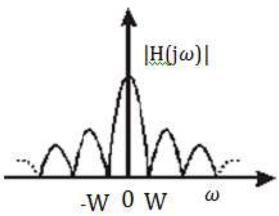
b)



c)

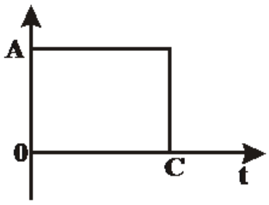


d)



Q.16 Consider the function $H(j\omega) = H_1(\omega) + jH_2(\omega)$, where $H_1(\omega)$ is an odd function and $H_2(\omega)$ is an even function. The inverse Fourier transform of $H(j\omega)$, is
a) a real and odd function
b) a complex function
c) a purely imaginary function
d) a purely imaginary and odd function

Q.17 The Laplace transform of the signal given in Figure, is



a) $-A\left(\frac{1-e^{CS}}{s}\right)$ b) $A\left(\frac{1-e^{CS}}{s}\right)$

c) $A\left(\frac{1-e^{-CS}}{s}\right)$ d) $-A\left(\frac{1-e^{-CS}}{s}\right)$

Q.18 If X(z) is the z-transform of x[n]= $\left(\frac{1}{2}\right)^{|n|}$, the ROC of X(z) is

a) $|z| > 2$

b) $|z| < 2$

c) $\frac{1}{2} < |z| < 2$

d) The entire z-plane

Q.19 In a linear phase system, τ_g the group delay and τ_p the phase delay are

a) Constant and equal to each other

b) τ_g is a constant and $\tau_p \alpha \omega$

c) a constant and $\tau_g \alpha \omega$

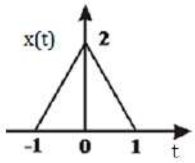
d) $\tau_g \alpha \omega$ and $\tau_p \alpha \omega$

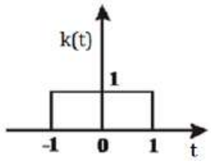
Q.20 A signal m(t), band-limited to a maximum frequency of 20 kHz is sampled at a frequency f_s kHz to generate s(t). An ideal low pass filter having cut-off frequency 37 kHz is used to reconstruct m (t) from s(t). The minimum value of f_s required to reconstruct m (t) without distortion is

a) 20 kHz b) 40 kHz

c) 57 kHz d) 77 kHz

Q.21 If the signal x(t) shown in figure (right sided) is fed to an LTI system having impulse response k(t) as shown in figure (left sided) the value of the DC component present in the output y(t) is





a) 1 b) 2

c) 3 d) 4

Q. 22 The impulse response h[n] of an LTI system is

h[n] = u[n + 3] + u[n – 2] – 2u[n – 7].

Then the system is:

1) Stable 2) Casual

3) Unstable 4) Not casual

Which of these are correct?

a) 1 and 2 only b) 2 and 3 only

c) 3 and 4 only d) 1 and 4 only

Q.23 Consider the following statements: Fourier series of any periodic function X(t) can be obtained if

1. $\int_0^1 |x(t)|\,dt \angle \infty$

2. Finite number of discontinuous exist within finite time interval t.

Which of the above statements is/are correct?

a) 1 only b) 2 only

c) Both 1 and 2 d)neither 1 nor 2

Q. 24 The Fourier transform of unit step sequence is

a) $\pi\delta(\Omega)$ b) $\frac{1}{1-e^{-j\Omega}}$

c) $\pi\delta(\Omega)+\frac{1}{1-e^{-j\Omega}}$ d) $1-e^{-j\Omega}$

Q.25 If $X(\omega) = \delta(\omega - \omega_0)$ then X(t) is

a) $e^{-j\omega_0 t}$ b) δ(t)

c) $\frac{1}{2\pi}e^{j\omega_0 t}$ d) 1

Q.26 Which one of the following is a Dirichlet's condition?

a) $\int_{t_1}^{\infty} |X(t)| \angle \infty$

b) Signal x(t) must have a finite number of maxima and minima in the expansion interval

c) x(t) can have an infinite number of finite discontinuities in the expansion interval

d) x^2(t) must be absolutely summable

Q. 27 The output of a linear system for step input is, $t^2 e^{-t}$. Then the transfer function is

a) $\frac{s}{(s+1)^2}$ b) $\frac{2s}{(s+1)^3}$

c) $\frac{s}{s^2(s+1)}$ d) $\frac{1}{(s+1)^3}$

Q. 28 The frequency response H(Ω) of a system for impulse sequence response
h [n] = δ [n] + δ [n – 1] is

a) H (Ω) = 2 cos $\left(\frac{\Omega}{2}\right)\angle -\frac{\Omega}{2}$

b) H (Ω) = cos $\Omega\angle -\Omega$

c) H (Ω) = 2 cos $\Omega\angle -\frac{\Omega}{2}$

d) H (Ω) = $2\angle -\frac{\Omega}{2}$

Q.29 Z and Laplace transform are related by

a) $s=\ln z$ b) $s=\frac{\ln z}{T}$

c) $s=z$ d) $s=\frac{T}{\ln z}$

Q.30 Convolution of two sequences $X_1[n]$ and X_2 [n] is represented as

a) $X_1(z) * X_2(z)$ b) $X_1(z) \bullet X_2(z)$

c) $X_1(z) + X_2(z)$ d) $X_1(z) / X_2(z)$

Q.31 The Z-transform of – u (-n -1) is:

a) $\frac{Z}{Z-1}$ with $|Z|>1$

b) $\frac{Z}{Z-1}$ with $0<|Z|<1$

c) $\frac{Z}{Z-1}$ with $|Z|=1$

d) $\frac{Z}{Z-1}$ with $|Z|=0$

Q.32 The Z-transform of X(K) is given by

$$X(Z)=\frac{(1-e^{-T})Z^{-1}}{(1-Z^{-1})(1-e^{-T}Z^{-1})}$$

The initial value x (0) is

a) Zero b) 1

c) 2 d) 3

Q.33 Unit step response of the system described by the equation y (n) + y (n – 1) = x (n) is

a) $\frac{Z^2}{(Z+1)(Z-1)}$ b $\frac{Z}{(Z+1)(Z-1)}$

c) $\frac{Z+1}{Z-1}$ d) $\frac{Z(Z-1)}{(Z+1)}$

Q.34 A function of one or more variables which conveys information on the nature of physical phenomenon is called

a) Noise b) Interference

c) System d) Signal

Q.35 The output y(t) if a continuous-time system S for the input x(t) is given by:

$$y(t)=\int_{-\infty}^{t} x(\lambda)d\lambda.$$

Which one of the following is correct?

a) S is linear and time-invariant

b) S is linear and time-varying

c) S is non-linear and time-invariant

d) S is non linear and time-varying

Q.36 What is the period of the sinusoidal signal x(n) = 5 cos [0.2 πn]?

a) 10 b) 5

c) 1 d) 0

Q.37 When $y(t) \xrightarrow{FT} Y(j\bar{\omega})$; $x(t) \xrightarrow{FT} X(j\bar{\omega})$; $h(t) \xrightarrow{FT} H(j\bar{\omega})$. What is $Y(j\bar{\omega})$?

a) $\frac{X(j\bar{\omega})}{H(j\bar{\omega})}$

b) $X(j\bar{\omega})H(j\bar{\omega})$

c) $X(j\bar{\omega})+H(j\bar{\omega})$

d) $X(j\bar{\omega})-H(j\bar{\omega})$

Q.38) For good quality signal transmission all frequency components should have the same transmission delay, t_d and same phase shift- φ_s. What can be said about the statement?
a) Correct
b) True for t_d but true for $\emptyset_z$
c) Not true for t_d but true for $\emptyset$
d) Both t_d and $\emptyset_z$ are not involved in quality

Q.39 Consider the function $F(s)=\frac{\varpi}{s^2+\varpi^2}$ where F(s) is the Laplace transform of f (t). What is the steady- state value of f (t)?
a) Zero
b) One
c) Two
d) A value between -1 and +1

Q.40 The transfer function of a linear-time invariant system is given as $\frac{1}{(s+1)}$

What is the steady-state value of the unit-impulse response?

a) Zero b) One
c) Two d) Infinite

Q.41 An audio signal is band limited to 4 kHz. It is sampled at 8 kHz. What will be the spectrum of the sampled signal?
a) – 4kHz to 4 kHz
b) – 8 kHz to 8 kHz
c) Every 4n kHz and repeating
d) Every ± 8 kHz and repeating as well as at zero (k integer)

Q.42 A signal occupies a band 5 kHz to 10 kHz. For proper error free reconstruction at what rate it should be sampled?
a) 10 kHz b) 20 kHz
c) 5 kHz d)(10 + 5)× 2 kHz

Q.43 Laplace transform of sin(ωt +α) is

a) $\frac{s\cos\alpha+\omega\sin\alpha}{s^2+\omega^2}$

b) $\frac{\omega}{s^2+\omega^2}\cos\alpha$

c) $\frac{s}{s^2+\omega^2}\sin\alpha$

d) $\frac{s\sin\alpha+\omega\cos\alpha}{s^2+\omega^2}$

Q.44 A system defined by $y[n]=\sum_{k=-\infty}^{n} x[k]$ is an example of
a)invertible system
b) Memory-less system
c) non–invertible system
d) Averaging system

Q.45 Which one of the following functions is a periodic one?
a) sin(10 πt) + sin(20 πt)
b) sin(10t)+ sin(20 πt)
c) sin(10 πt) + sin(20t)
d) sin(10t) + sin(25t)

Q.46 What is the average power for periodic non – sinusoidal voltages and currents?
a) The average power of the fundamental component alone
b) The sum of the average powers of the harmonics excluding the fundamental
c) The sum of the average powers of the sinusoidal components including the fundamental

d) The sum of the root mean square power of the sinusoidal components including the fundamental

Q.47 Which one of the following is the correct relation?
a) F(at) ↔ aF(ω/a)
b) F(at) ↔ aF(aω)
c) F(t/a) ↔ aF(ω/a)
d) F(at) ↔ (1/a)F(ω/a)

Q.48 The Fourier transform of a function is equal to its two – sided Laplace transform evaluated
a) on the real axis of the s- plane
b) on a line parallel to the real axis of the s – plane
c) on the imaginary axis of the s – plane
d) on a line parallel to the imaginary axis of the s – plane

Q.49 If the Fourier transform of f(t) is F(jω), then what is the Fourier transform of f(t)?
a) F (jω)
b) F(-jω)
c) –F(jω)
d) Complex conjugate of F(jω)

Q.50 If f(t) is an even function, then what is its Fourier transform F(jω)?
a) $\int_0^{\infty} f(t)\cos(2\omega t)dt$
b) $2\int_0^{\infty} f(t)\cos(\omega t)dt$
c) $2\int_0^{\infty} f(t)\sin(\omega t)dt$
d) $\int_0^{\infty} f(t)\sin(2\omega t)dt$

Q.51 What is the Laplace transform of $\cos\omega_0 t$?
a) $\frac{\omega_0}{s^2+\omega_0^2}$ b) $\frac{s}{s^2+\omega_0^2}$
c) $\frac{\omega_0}{(s+\omega_0)^2}$ d) $\frac{s}{(s+\omega_0)^2}$

Q.52 Which one of the following is the impulse response of the system whose step response is given as $c(t)=0.5(1-e^{-2t})u(t)$?
a) $e^{-2t}u(t)$
b) $0.5\delta(t) + e^{-2t}u(t)$
c) $0.5\delta(t) - 0.5e^{-2t}u(t)$
d) $0.5e^{-2t}u(t)$

Q.53 If X(z) is $\frac{1}{1-z^{-1}}$ with |z| > 1, then what is the corresponding x(n)?
a) e^{-n} b) e^{n}
c) u(n) d) δ(n)

Q.54 Multiplexing is possible if signals are sampled. Two signals have bandwidths A = 0 to 4 kHz and B = 0 to 8 kHz respectively. The sampling frequency chosen is 12 kHz. Which one of the following is correct? This choice of the sampling frequency
a) is correct since A and B have an integral relationship of 2
b) will not lead to aliasing
c) does not obey sampling theorem
d) can never lead to multiplexing

Q.55 Match List – I with List – II and select the correct answer using the code given below the Lists:

List-I [Function in time Domain transform F(s)]	List-II [Corresponding Laplac f(t)]
A. $\sin\omega_0 t u(t-t_0)$	1. $\frac{\omega_0}{s^2+\omega_0^2}$
B. $\sin\omega_0(t-t_0)u(t-t_0)$	2. $\left\{\frac{\omega_0}{s^2+\omega_0^2}\right\}e^{-t_0 s}$
C. $\sin\omega_0(t-t_0)tu(t)$	3. $\frac{e^{-t_0 s}}{\sqrt{s^2+\omega_0^2}}\sin\left(\omega_0 t_0-\tan^{-1}\frac{\omega_0}{s}\right)$
D. $\sin\omega_0 tu(t)$	4. $\frac{1}{\sqrt{s^2+\omega_0^2}}\sin$

$\left(\omega_0 t_0 - \tan^{-1}\frac{\omega_0}{s}\right)$

Code:	A	B	C	D
a)	3	1	4	2
b)	4	2	3	1
c)	3	2	4	1
d)	4	1	3	2

Q.56 Energy of a power signal is
a) Finite b) Zero
c) Infinite d) Between 1and 2

Q.57 The discrete time signal x(n) is defined by $x(n) = \begin{cases} 1 & n=1 \\ -1 & n=-1 \\ 0 & n=0 \end{cases}$ And [n] > 1
Which one of the following is the composite signal y(n) = x(n) + x(-n) for all integer values of n?
a) 0 b) 2
c) ∞ d) $-\infty$

Q.58 Which one of the following is the mathematical representation for the average power of the signal x(t)?
a) $\frac{1}{T}(t)\,dt$
b) $\frac{1}{T}\int_0^T x^2(t)dt$
c) $\int_{-T/2}^{T/2} x(t)\,dt$
d) $\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x^2(t)dt$

Q.59 Which one of the following systems described by the following input – output relations is non-linear?
a)y(n)=nx(2n)
b)$y(n)=x(n^2)$
c) $y(n) = n^2x(n)$ d) $y(n) = x^2(n)$

Q.60 The output of a linear system for any input can be computed in which one of the following ways?
a) Only by summation of impulse response by convolution integral
b) Only by summation of step responses by superposition integral
c) Neither (a) nor (b)
d) Using either (a) or (b)

Q.61 A discrete-time signal x[n] has Fourier transform $X(e^{j\omega})$.
Match List-I with List-II and select the correct answer using the code given below the lists:

List I (Signal)	List II (Fourier Transform)
A,x[-n]	1. $X^*(e^{-j\omega})$
B.nx [n]	2. $X(e^{-j\omega})$
C.x*[n]	3. $e^{-j\omega}X(e^{j\omega})$
D.x[n-1]	4. $j\frac{d}{d\omega}X(e^{j\omega})$

Code:	A	B	C	D
a)	1	3	2	4
b)	2	4	1	3
c)	1	4	2	3
d)	2	3	1	4

Q.62 A real signal x(t) has Fourier transform X(f). Which one of the following is correct?
a) Magnitude of X(f) has even symmetry while phase of X(f) has odd symmetry
b)Magnitude ofX(f) has odd symmetry while phase of X(f) has even symmetry
c) Both magnitude and phase of X(f) have even symmetry
d)Both magnitude and phase of X(f) have odd symmetry

Q.63 What is the output as $t\to\infty$ for a system that has a transfer function $G(s)=\frac{2}{s^2-s-2}$; when subjected to a step input?
a) –1 b) 1
c) 2 d) Unbounded

Q.64 What is the Laplace transform of a delayed unit impulse function $\delta(t-1)$?

a) 1 b) zero
c) exp (-s) d) s

Q.65 Algebraic expression for Z transform of x[n] is X(z). What is the algebraic expression for Z transform of $^{n}X[n]$?

a) $X(z - z_o)$ b) $X(^{\omega_0 z})$
c) $X^{0z})$ d) $X(z)$

Q.66 Which one of the following rules determines the mapping of s-plane to z-plane?

a) Right half of the s-plane maps into outside of the unit circle in z-plane
b) Left half of the s-plane maps into inside of the unit circle
c) Imaginary axis in s-plane maps into the circumference of the unit circle
d) All of the above

Q.67 What is the Z-transform of the signal $x[n] = \alpha^{n} u(n)$?

a) $X(z) = \dfrac{1}{z-1}$

b) $X(z) = \dfrac{1}{1-z}$

c) $X(z) = \dfrac{z}{z-\alpha}$

d) $X(z) = \dfrac{1}{z-\alpha}$

Q.68 Which one of the following is the correct statement? The system characterized by the equation y(t) = ax(t) + b is

a) Linear for any value of b
b) Linear if b > 0
c) Linear if b < 0
d) Non-linear

Q.69 x[n] is defined as

$$x[n] = \begin{cases} 0 & \text{for } n < -2 \text{ or } n > 4 \\ 1, & \text{otherwise} \end{cases}$$

Determine the value of n for which x[-n-2] is guaranteed to be zero.

a) n < 1 and n > 7
b) n < -4 and n > 2
c) n < -6 and n > 0
d) n < -2 and n > 4

Q.70 Which one of the following is the correct statement? The continuous time system described by $y(t) = x(t^2)$ is

a) Causal, linear and time-varying
b) Causal, non-linear & time-varying
c)Non-causal,non-linearand time invariant
d) Non-causal and time-variant

Q.71 The discrete-time signal [n] is given as

$$x[n] = \begin{cases} 1, & n = 1, 2 \\ -1, & n = -1, -2 \\ 0, & n = o \text{ and } |n| > 2 \end{cases}$$

Which one of the following is the time-shifted signal y[n] = x[n+3]

a)
$$y[n] = \begin{cases} 1, & n = -1, -2 \\ -1, & n = -4, -5 \\ 0, & n = -3 n < -5 \text{ and } n > -1 \end{cases}$$

b)
$$y[n] = \begin{cases} 0, & n = -1, -2 \\ -1, & n = -4, -5 \\ 1, & n = -3 n < -5 \text{ and } n > -1 \end{cases}$$

c)
$$y[n] = \begin{cases} 1, & n = -1, -2 \\ 0, & n = -4, -5 \\ -1, & n = -3 n < -5 \text{ and } n > -1 \end{cases}$$

d)
$$y[n] = \begin{cases} -1, & n = 1, 2 \\ 1, & n = 4, 5 \\ 0, & n = 3 n > 5 \text{ and } n < 1 \end{cases}$$

Q.72 For continuous-time systems state variable representation is

$\frac{d}{dt}q(t) = Aq(t) + bx(t)$ What is the corresponding representation for discrete – time system?

a) $\frac{d}{dt}$ q[n] = A q[n] + bx[n]

b) q[n + 1] = A q[n] + bx[n]

c) q[n] = A q[n – 1] + bx[n]

d) $\frac{d}{dn}[q(n+1)]$ = A qs[n – 1] + bx [n – 1]

Q.73 The impulse response of a system $h(n) = a^n u(n)$. What is the condition for the system to be BIBO stable?

a) a is real and positive

b) A is real and negative

c) | a | > 1

d) | a | < 1

Q.74 Match the List-I (CT Function) with List-II (CT Fourier Transform) and select the correct answer using the code given below the lists:

List I (CT Function)	**List II (CTFourier Transform)**
A) $e^{-t}u(t)$	1. $\frac{2}{1+\omega^2}$
B) $x(t) = \begin{cases} 1, \lvert t\rvert \le 1 \\ 0, \lvert t\rvert > 1 \end{cases}$	2. $j\omega X(j\omega)$
C) $\frac{dx(t)}{dt}$	3. $\frac{1}{1+j\omega}$
D) $\frac{2}{1+t^2}$	4. $\frac{2\sin\omega}{\omega}$

Code:	**A**	**B**	**C**	**D**
a)	1	4	2	3
b)	3	2	4	1
c)	1	2	4	3
d)	3	4	2	1

Q. 75 If the Fourier transform of x(t) is $\frac{2}{\omega}$ sin ($\pi\omega$), then what is the Fourier transform of e^{j5t} x(t)?

a) $\pi\omega$

b) $\frac{2}{\omega}\sin\{\pi(\omega-5)\}$

c) $\frac{2}{\omega+5}\sin\{\pi(\omega+5)\}$

d) $\frac{2}{\omega-5}\sin\{\pi(\omega-5)\}$

Q.76 What is the inverse Fourier transform of u(ω)?

a) $\frac{1}{2}\delta(t) + \frac{j}{\pi t}$ b) $\frac{1}{2}\delta(t)$

c) $2\delta(t) + \frac{j}{\pi t}$ d) $2\delta(t) + j\sin(t)$

Q.77 What is the Laplace transform of x (t) = $-e^{2t}$ u(t) * (tu(t))?

a) $\frac{-1}{s^2(s+2)}$ b) $\frac{-1}{s^2(s-2)}$

c) $\frac{1}{s^2(s-2)}$ d) $\frac{-1}{s^2(s-2)}$

Q.78 What is F(s) = $\frac{8s+10}{(s+1)(s+2)^3}$ equal to?

a) $\frac{2}{s+1} + \frac{4}{(s+2)^3} - \frac{4}{(s+2)^2} - \frac{2}{s+2}$

b) $\frac{2}{s+1} + \frac{6}{(s+2)^3} - \frac{2}{(s+2)^2} - \frac{2}{s+2}$

c) $\frac{2}{s+1} + \frac{6}{(s+2)^3} - \frac{2}{(s+2)^2} + \frac{2}{s+2}$

d) $\frac{4}{s+1} + \frac{6}{(s+2)^3} - \frac{2}{(s+2)^2} - \frac{4}{s+2}$

Q.79 For the function x(t), X(s) is given by:

$$X(s) = e^{-s}\left[\frac{-2}{s(s+2)}\right]$$

Then, what are the initial and final values of x(t), respectively?

a) 0 and 1 b) 0 and –1

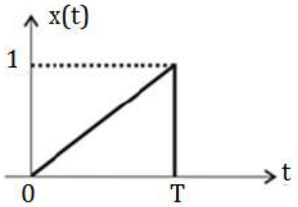
c) 1 and 1 d) –1 and 0

Q.80 The Laplace transform X(s) of a function x(t) is

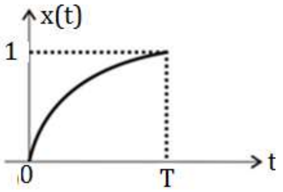
$$X(s)=\left(\frac{1-e^{-sT}}{s}\right)$$

What is the wave shape of x(t)?

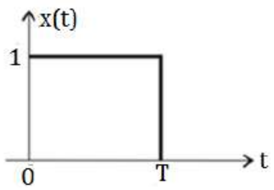
a)



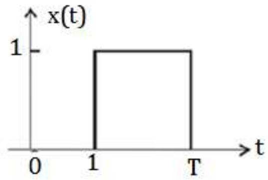
b)



c)



d)



Q.81 What is the inverse z transform of X (z)?

a) $\frac{1}{2\pi j}\oint X(z)z^{n-1}dz$ b) $\frac{1}{2\pi j}\oint X(z)z^{n+1}dz$

c) $\frac{1}{2\pi j}\oint X(z)z^{1-n}dz$ d) $2\pi j\oint X(z)z^{-(n+1)}dz$

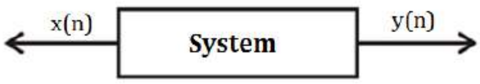
Q.82 Which one of the following is the correct statement?
The region of convergence of z transform x | n | consists of the values of z for which x | n | r^{-n} is.

a) Absolutely integrable
b) Absolutely summalble
c) unity
d) < 1

Q.83 For the system shown, x[n] = k δ[n], and y[n] is related to x[n] as

$$y[n]-\frac{1}{2}y[n-1]=x[n]$$

What is y[n] equal to?



a) k b) $(1/2)^n k$
c) nk d) 2^n

Q.84 Match List-I (Application of Signals) with List-II (Definition) and select the correct answer using the code given below the lists:

List-I (Application of Signals)	List-II (Definition)
A. Reconstruction	1. Sampling rate is chosen significantly greater than the Nyquist rate
B. Over Sampling	2. A mixture of continuous and discrete time signal
C. Interpolation	3. To convert the discrete time sequence back to a continuous time signal and then resample
D. Decimation	4. Assign values between samples and signals

Code:	A	B	C	D
a)	3	4	1	2
b)	2	1	4	3
c)	3	1	4	2
d)	2	4	1	3

Q.85 The governing differential equations connecting the output y(t) and the input x(t)
of four continuous systems are given in the List I and List II respectively. Match List I (Equation) with List II (System Category) and select the correct answer using the codes given below the lists:

List 1
(Equation)

A $2t\frac{dy}{dt}+4y=2tx$

B.$y\frac{dy}{dt}+4y=2x$

C.$4\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y$

D.$\left(\frac{dy}{dt}\right)^2+2ty=4\frac{dx}{dt}$

List II
(System category)

1.Linear ,time Invariant and dynamic

2.Non Linear, time invariant dynamic

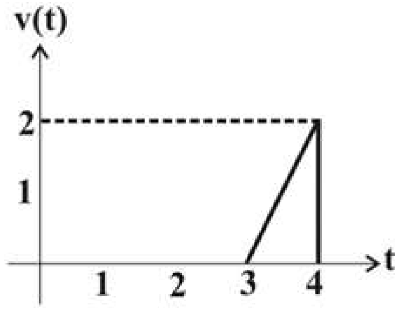
3.Linear time–variable and $=3\frac{dx}{dt}$ dynamic

4. Non linear time variable and dynamic

5.Non linear time invariant non dynamic

Code:	A	B	C	D
a)	3	2	1	4
b)	4	1	5	3
c)	3	1	5	4
d)	4	2	1	3

Q.86 In the graph shown, which one of the following expresses v(t)?



a) (2t + 6)[u(t - 3) + 2u(t - 4)]
b) (-2t - 6)[u(t - 3) + u(t - 4)]
c) (-2t + 6)[u(t - 3) + u(t - 4)]
d) (2t - 6)[u(t - 3) - u(t - 4)]

Q.87 Consider the following statements about linear time invariant (LTI) continuous time systems:
1) The output signal in an LTI system with known input and known impulse response can always be determined.
2) A causal LTI system is always stable.
3) A stable LTI system has an impulse response, h(t) which has a finite value when integrated over whole of the time axis, i.e. $\int_{-\infty}^{+\infty} h(\lambda)d\lambda$ is finite.
Which of the statements given above are correct?

a) 1 and 3 b) 1 and 2
c) 2 and 3 d) 1, 2 and 3

Q.88 The unit sample response of a discrete system is 1 1/2 1/4 0 0 0 ....For and input sequence 101000..., what is the output sequence?

a) 1 1/2 1/4 1/2
1/4 0 0 ......
b) 1 0 1/4 0 0
- ...... -
c) 2 1/2 5/4 0 0
- ...... -
d) 1 1/2 5/4 1/2
1/4 0 0 ......

Q.89 Let $x[n] = a^n u[n]$ $h[n] = b^n u[n]$
What is the expression for y[n], for a discrete time system?

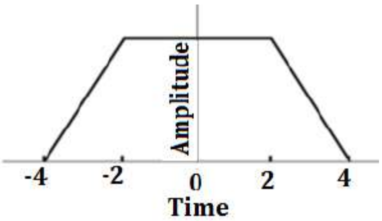
a) $\sum_{k=-\infty}^{\infty} a^k u[k] b^{n-k} u[n-k]$

b) $\sum_{k=-\infty}^{\infty} a^n u[k] b^{n-k} u[n-k]$

c) $\sum_{k=-\infty}^{\infty} a^k u[n-k] b^n u[k]$

d) $\sum_{k=-\infty}^{\infty} a^{n-k} u[k] b^{n-k} u[n-k]$

Q.90 The graph shown represents a waveform obtained by convolving two rectangular waveforms of duration



a) Four units each
b) Four and two units respectively
c) Six and three units respectively
d) Six and two units respectively

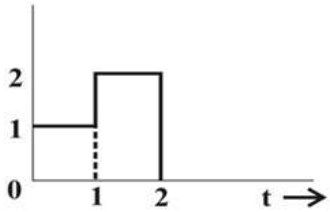
Q.91 Match List I(Time Function) with List II (Fourier Spectrum/Fourier Transform) and select the correct answer using the codes given below the lists :

List I
(Time Function)
A.Periodic Function
B.Aperiodic Function
C.Unit Impulse $\delta(t)$
D.$\sin \omega t$

List II
(Fourier Spectrum/ FourierTransform)
1.Continuous spectrum at all frequencies
2.$\delta(\omega)$
3.Line discrete spectrum

Code:	A	B	C	D
a)	4	2	3	1
b)	3	1	4	2
c)	4	1	3	2
d)	3	2	4	1

Q.92 What is the Laplace transform of the waveform shown.



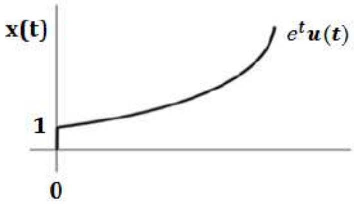
a) $F(s) = \frac{1}{s} + \frac{1}{s}e^{-s} - \frac{2}{s}e^{-2s}$

b) $F(s) = \frac{1}{s} - \frac{1}{s}e^{-s} + \frac{2}{s}e^{-2s}$

c) $F(s) = \frac{1}{s} + \frac{1}{s}e^{-s} + \frac{2}{s}e^{-2s}$

d) $F(s) = \frac{1}{s} - \frac{1}{s}e^{-2s} - \frac{2}{s}e^{-s}$

Q.93 For the signal shown



a) Only Fourier transform exists.

b) Only Laplace transform exists
c) Both Laplace and Fourier transforms exists
d) Neither Laplace transform nor Fourier transform exists.

Q.94 What does the transfer function of a system describe for the system ?
a) Only zero-input response

b) Only zero state response
c)Both zero-input and zero-state responses
d) Neither zero-input response nor zero-state response

Q.95 The output y[n] of a discrete time LTI system is related to the input x[n] as given $y[n]=\sum_{k=0}^{\infty}x[k]$ Which one of the following correctly relates the z-transform of the input and output, denoted by X(z) and Y(z), respectively ?

a) $Y(z)=(1-z^{-1})X(z)$

b) $Y(z)=z^{-1}X(z)$

c) $Y(z)=\frac{X(z)}{1-z^{-1}}$

d) $Y(z)=\frac{dX(z)}{dz}$

Q.96 Which one of the following is the inverse of z – transform of

$X(z)=\frac{z}{(z-2)(z-3)}|z|<2$?

a) $\left[2^n-3^n\right]u(-n-1)$

b) $\left[3^n-2^n\right]u(-n-1)$

c) $\left[2^n-3^n\right]u(n+1)$

d) $\left[2^n-3^n\right]u(n)$

Q.97 Match List I (Discrete Time Signal) with List II (Transform) and select the correct answer using the code given below the lists :

ListI(DiscreteTimeSignal) ListII(Transform)

A.Unit Step Function 1.1

B.Unit Impulse function 2. $\frac{z-\cos\omega T}{z^2-2z\cos\omega T+1}$

C.sin $\omega t, t=0,T,2T.....$ 3. $\frac{z}{z-1}$

D.cos $\omega t, t=0,T,2t.....$ 4. $\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$

Code:	**A**	**B**	**C**	**D**
a)	2	4	1	3
b)	3	1	4	2
c)	2	1	4	3
d)	3	4	1	2

Q.98 A signal represented by x(t) = 5 cos 400πt is sampled at a rate 300 sample/s. The resulting samples are passed through an ideal low pass filter of cut-off frequency 150 Hz. Which of the following will be contained in the output of the LPF?

a) 100 Hz

b) 100 Hz, 150 Hz

c) 50 Hz, 100 Hz

d) 50 Hz, 100 Hz, 150 Hz

Q.99 Which one of the following must be satisfied if a signal is to be periodic for -∞< t <∞ ?

a) $x(t+T_0)=x(t)$

b) $x(t+T_0)=\frac{dx(t)}{dt}$

c) $x(t+T_0)=\int_t^{T0}x(t)d(t)$

d) $x(t+T_0)=x(t)+kT$

Q.100 $y[n]=\sum_{k=-\infty}^{n}x[k]$ .Which one of the following systems is inverse of the system given

a) x[n] = y[n] - y[n-1] b) x[n] = y[n]

c) x[n] = y[n+4] d) x[n]=ny[n]

Q.101 If $\left(\frac{27s+97}{s^2+33s}\right)$ is the Laplace transform of f(t), then $f(0^+)$ is :

a) Zero b) $\frac{97}{33}$

c) 27 d) Infinity

Q.102 Match List I (Equation Connecting Input x(n)) and Output y(n) of four discrete time systems with List II (Systems Category) and select the correct answer using the codes given below the lists :

ListI (Equation connecting category) **List II Input x(n)And Output y(n)**

A. $y(n+2)+(n+1)+y(n) = 2x(n+1)+x(n)$ — 1.linear,time variable dynamic

B. $n^2y^2(n)+y(n)=x^2(n)$ — 2.Linear,time-invariant dynamic

C $y(n+1)+ny(n)= 4nx(n)$ — 3.Non-linear,time variable,,dynamic

D. $y(n+1)y(n)=4x(n)$ — 4.Nonlinear, Time invariant,dynamic

5.Nonlinear,time variable memoryless

Code:	A	B	C	D
a)	3	5	2	1
b)	3	2	5	4
c)	2	3	5	1
d)	2	5	1	4

Q.103 A discrete time system has impulse response $h(n)=a^n u(n+2), |a|<1$. Which one of the following statements is correct?
The system is
a) Stable, casual and memory less
b) Unstable, casual and has memory
c) Stable, non-casual and has memory
d) Unstable, non-casual and memory less

Q.104 The impulse response of a linear time-invariant system is a rectangular pulse of duration T. It is excited by an input of a pulse of duration T. What is the filter output waveform?
a) Rectangular pulse of duration T
b) Rectangular pulse of duration 2T
c) Triangular pulse of duration T
d) Triangular pulse of duration 2T

Q.105 For half-wave (odd) symmetry with T_0 = period of x(t), which one of the following is correct ?
a) $x(t \pm T_0/2) = -x(t)$
b) $x(t \pm T_0/2) = x(t)$
c) $x(t \pm t_0) = -x(t)$
d) $x(t \pm T_0) = x(t)$

Q.106 A square wave is defined by

$$x(t)=\begin{cases} A, & 0<t<T_0/2 \\ -A, & T_0/2<t<T_0 \end{cases}$$

It is periodically extended outside this interval. What is the general coefficient a_n in the Fourier expansion of this wave ?
a) 0
b) $\frac{2A(1-\cos n\pi)}{n\pi}$
c) $\frac{2A(1+\cos n\pi)}{n\pi}$
d) $\frac{2A(1-\cos n\pi)}{[(n+1)\pi]}$

Q.107 Match List I (Time Domain Property) with List II (Frequency Domain Property) pertaining to Fourier Representation Periodicity and select the correct answer using the codes given below the lists :

List I(Time Domain Property)	**ListII(Frequency Property)**
A.Continuous	1.Periodic
B.Discrete	2.Continuous
C.Periodic	3.Non-Periodic
D.Non-periodic	4.Discrete

Code:	A	B	C	D
a)	3	4	1	2
b)	2	4	1	3
c)	2	1	4	3
d)	3	1	4	2

Q.108 Given that $x_1(t)=e^{k_1 t}u(t)$ and $x_2(t)=e^{-k_2 t}u(t)$. Which one of the following gives their convolution ?

a) $\frac{\left[e^{k_1 t}-e^{-k_2 t}\right]}{\left[k_1+k_2\right]}$

b) $\frac{\left[e^{k_1 t}-e^{-k_2 t}\right]}{\left[k_2-k_1\right]}$

c) $\frac{\left[e^{k_1 t}+e^{-k_2 t}\right]}{\left[k_1+k_2\right]}$

d) $\frac{\left[e^{k_1 t}+e^{-k_2 t}\right]}{\left[k_2-k_1\right]}$

Q.109 To which one of the following difference equations, the impulse response $h(n)=\delta(n+2)-\delta(n-2)$ corresponds?

a) y(n+2)=x(n)-x(n-2)
b) y(n-2)=x(n)-x(n-4)
c) y(n) = x(n+2)+x(n-2)
d) y(n) = -x(n+2)+x(n-2)

Q.110 Laplace transforms of f(t) and g(t) are F(s) and G(s), respectively. Which one of the following expressions gives the inverse Laplace transform of F(s) G(s) ?

a) $f(t)g(t)$ b) $\frac{f(t)}{g(t)}$
c) $f(t)+g(t)$ d) $f(t)*g(t)$

Q.111 Which one of the following represents the Fourier Transform X(jω) of the signal $X(t) = t\,e^{-at}u(t)$?

a) $X(jw)=\frac{jw}{(a+jw)}$

b) $X(jw)=\frac{jw}{(a+jw)^2}$

c) $X(jw)=\frac{(a+jw)^2}{jw}$

d) $X(jw)=\frac{1}{(a+jw)^2}$

Q.112 What is the inverse Laplace transform of $\frac{e^{-as}}{s}$?

a) e^{-at} b) $u(t-a)$
c) $\delta(t-a)$ d) $(t-a)u(t-a)$

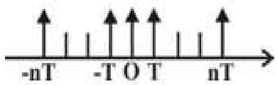
Q.113 Two rectangular waveforms of duration T1 and T2 seconds are convolved. What is the shape of the resulting waveform?

a) Triangular b) Rectangular
c) Trapezoidal d) Semi-circular

Q.114 Match List I (Nature of Periodic Function) with List II (Properties of Spectrum Function) and select the correct answer using the codes given below the lists :

List I (Natural of Periodic)

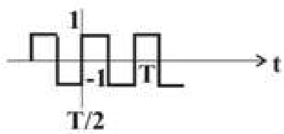
A. Impulse train



B. Full wave rectified since function

C. $\sin\frac{2\pi t}{6}.\cos\frac{4\pi t}{6}$

D.



List II (Properties of Spectrum Function)

1. Only even harmonics are present
2. Impulse train with strength :
3. $\alpha_3=\frac{1}{2j};\alpha_{-3}-\frac{1}{2j};$ $\alpha_1=\frac{1}{2j};\alpha_{-1}\frac{1}{2j}$
4. Only odd harmonics are present
5. Both even and odd harmonics are present

Code:	**A**	**B**	**C**	**D**
a)	5	2	3	4
b)	2	1	4	3

c)	5	2	4	3
d)	2	1	3	4

Q.115 A discrete LTI system is non-causal if its impulse response is :

a) $a^n u(n-2)$ b) $a^{n-2}u(n)$
c) $a^{n+2}u(n)$ d) $a^n u(n+2)$

Q.116 **Assertion A:**In the exponential Fourier representation of a real-valued periodic function f(t) of frequency f_0, the coefficient of the terms $e^{j2\pi n_0 t}$ and $e^{-j2\pi n f_0 t}$ are negative of each other.

Reason R: The discrete magnitude spectrum of f(t) is even and the phase spectrum
is odd.

a) Both A and R are individually true and R is the correct explanation of A.
b) Both A and R are individually true but R is NOT the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Q.117 Match List I with List II and select the correct answer using the codes given below the list :

List I	**List II**
A.Free and Forced response	1.Discrete time system
B.Z-transforms	2.Dirichlet conditions
C.Probability	3.Non-homogenous differential equation
D.Fourier series	4.Random processes

Code:	**A**	**B**	**C**	**D**
a)	1	3	2	4
b)	3	1	2	4
c)	1	3	4	2
d)	3	1	4	2

Q.118 The inverse Fourier transform of $\delta(f)$ is

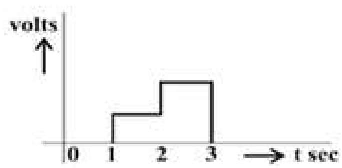
a) u(t) b) 1
c) $\delta(t)$ d) $e^{j2\pi t}$

Q.119 The relationship between the input x(t) and the output y(t) of a system is $\frac{d^2y}{dt^2} = x(t-2)u(t-2) + \frac{d^2x}{dt^2}$ The transfer function of the system is

a) $1+\frac{s^2}{e^{2s}}$ b) $1+\frac{s^{-2s}}{s^2}$

c) $1+\frac{e^{2s}}{s^2}$ d) $1+\frac{s^2}{e^{-2s}}$

Q.120 The Laplace transform of the waveform shown in the figure is :



a) $\frac{1}{s}\left[e^{s}+e^{2s}+2e^{3s}\right]$

b) $\frac{1}{s}\left[e^{s}+e^{2s}+2e^{-3s}\right]$

c) $\frac{1}{s}\left[e^{-s}+e^{-2s}+2e^{-3s}\right]$

d) $\frac{1}{s}\left[e^{-s}+e^{-2s}-2e^{-3s}\right]$

Q.121 Match List I (F(s)) with List II (f(t)) and select the correct answer using the codes given below the lists :

List I(F(s))	**List II (f(t))**
A. $\frac{10}{s(s+10)}$	1. $10\delta(t)$
B. $\frac{10}{s(s^2+10)}$	2. $(e^{-10t}\cos 10t).u(t)$
C. $\frac{(s+10)}{(s+10)^2+100}$	3. $(\sin 10t).u(t)$
D.10	4. $(1-e^{-10t}).u(t)$

Code:	A	B	C	D
a)	3	4	1	2
b)	4	3	1	2
c)	3	4	2	1
d)	4	3	2	1

Q.122 The Laplace transform of sin ωt is :

a) $\frac{s}{s^2+\omega^2}$ b) $\frac{\omega^2}{s^2+\omega^2}$

c) $\frac{s^2}{s^2+\omega^2}$ d) $\frac{\omega}{s^2+\omega^2}$

Q.123 Given, $Lf(t)=F(s)=\int_0^{\infty} f(t)e^{-st}dt$

Which of the following expression are correct ?

1) **L** $[f(t-a)u(t-a)]=F(s)e^{-sa}$

2) **L** $f(t)=\frac{-dF(s)}{ds}$

3) **L** $(t-a)f(t)=asF(s)$

4) **L** $\frac{dF(t)}{dt}=sF(s)-f(o_+)$

Select the correct answer using the codes given below :

a) 1,2 and 3 b) 1, 2 and 4
c) 2, 3 and 4 d) 1, 3 and 4

Q.124 Match List I (Type of signal) with List II (Property of Fourier transform) and select the correct answer using the codes given below the lists:

ListI(Type of Signal)	**List II (Property of Fourier transform)**
A.Real and even symmetric	1.Imaginary and even symmetric
B.Real and odd symmetric	2.Real and even symmetric
C.Imaginary and even symmetric	3.Real odd symmetric
D.Imaginary and odd symmetric	4.Imaginary odd symmetric

Code:	A	B	C	D
a)	1	4	2	3
b)	2	4	1	3
c)	1	3	2	4
d)	2	3	1	4

Q.125 Match List I (Fourier series and Fourier transforms) with List II (Their properties) and select the correct answer
using the codes given below the lists
:

List I (Fourier Series and Fourier transform)	**List II (Their Properties)**
A.Fourier series	1.Discrete ,periodic
B.Fourier transform	2.Continuous,periodic
C.Discrete time Fourier	3.Discrete ,aperiodic
D.Discrete Fourier transfrom	4.Continuous ,aperiodic

Code:	A	B	C	D
a)	3	4	2	1
b)	1	2	4	3
c)	3	2	4	1
d)	1	4	2	3

Q.126 Match List I(Functions in the time domain) with List II (Fourier transform of the function) and select the correct answer using the codes given below the lists

List I (Functions in the time domain)	**List II (Fourier transform of the function)**
A.Delta function	1.Delta function
B.Gate function	2.Gaussian function
C.Normalized Gaussian function	3.Constant function

D.Sinusoidal function	4.Sampling function

Code:	A	B	C	D
a)	1	2	4	3
b)	3	4	2	1
c)	1	4	2	3
d)	3	2	4	1

Q.127 Which one of the systems described by the following input-output relations is time invariant?
a) y(n)=n x(n)
b) y(n)=x(n)-x(n-1)
c) y(n)=x(-n)
d) $y(n)=x(n)\cos 2\pi f_0 n$

Q.128 Match List I (Input-output relation) with List II (Property of the system) and select the correct answer using the codes given below the list.

List I(input – output relation)	List II(Property)
A.y(n)=x(n)	1.Nonlinear,non-casual
B.$y(n)=x(n^2)$	2.Linear ,non-casual
C.$y(n)=x^2(-n)$	3.Linear ,casual
D.$y(n)=x^2(n)$	4.Nonlinear ,casual

Code:	A	B	C	D
a)	1	4	3	2
b)	3	2	1	4
c)	1	2	3	4
d)	3	4	1	2

Q.129 The range of values of a and b for which the linear time invariant system with impulse response will be disable is

$$h(n) = a^n, n \geq 0$$
$$b^n, n < 0$$

a) |a|>1, |b|>1
b) |a|<1, |b|<1
c) |a|<1, |b|>1
d) |a|>1,|b|<1

Q.130 The Fourier transform X(f) of the periodic delta functions,

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT) \text{ is :}$$

a) $T\sum_{k=-\infty}^{\infty} \delta(f-kT)$
b) $T\sum_{k=-\infty}^{\infty} \delta(f-\frac{k}{T})$
c) $\frac{1}{T}\sum_{k=-\infty}^{\infty} \delta(f-\frac{k}{T})$
d))$\frac{1}{T}\sum_{k=-\infty}^{\infty} \delta(f-kT)$

Q.131 Consider the following statements related to Fourier series of a periodic waveform.
1) It expresses the given periodic waveform as a combination of d.c. component, sine and cosine waveforms of different harmonic frequencies.
2) The amplitude spectrum is discrete.
3) The evaluation of Fourier coefficients gets simplified if waveform symmetries are used.
4)The amplitude spectrum is continuous.

Which of the above statements are correct?
a) 1,2 and 4 b) 2, 3 and 4
c) 1, 3 and 4 d) 1,2 and 3

Q. 132) Match List I(Functions) with List II (Fourier Transforms) and select the correct answer using the codes given below the lists :

List I (Functions)	ListII (Fourier transforms)
A.exp $(-\alpha t)u(t), \alpha > 0$	1.$\frac{1}{(\alpha + j2\pi f)^2}$

B. $\exp(-\alpha|t|), \alpha > 0$ 2. $\frac{1}{\alpha + j2\pi f}$

C. $t\exp(-\alpha t)u(t), \alpha > 0$ 3. $\left(f - \frac{\alpha}{t_0}\right)$

D. $\exp(j2\pi\alpha t / t_0)$ 4. $\frac{2\alpha}{a^2 + (2\pi f)^2}$

Code:	A	B	C	D
a)	3	1	4	2
b)	2	4	1	3
c)	3	4	1	2
d)	2	1	4	3

Q.133 Let x(t) be a real signal with the Fourier transformX(t).Let X*(f) denote the complex conjugate of X(f). Then

a) X(-f) = X* (f) b) X(-f)=X(f)
c) X(-f)= -X(f) d) X(-f) =-X *(f)

Q.134 Let the transfer function of a network be H(f) = |H(f)| $e^{j\theta(f)} = 2e^{-j4\pi f}$. If a signal x(t) is applied to such a network, the output y(t) is given by

a) 2x(t) b) x (t – 2)
c) 2x (t – 2) d) 2x (t - 4π)

Q.135 The response of a linear, time-invariant, discrete-time system to a unit step input u(n) is the unit impulse δ(n). The system response to a ramp input n u(n) would be

a) u(n) b) u(n - 1)

c) n δ (n) d) $\sum_{k=0}^{\infty} k\,\delta(n-k)$

Q.136 For a Z-transform

$$X(z) = \frac{z\left(2z - \frac{5}{6}\right)}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}$$

Match List I (The sequences) with List II (The region of convergence) and select the correct answer using the codes given below the lists:

List I (The sequences)	**List II (The region of convergence)**
A. $\left[(1/2)^n + (1/3)^n\right]u(n)$	1. $(1/3) < \lvert z\rvert < (1/2)$
B. $(1/2)^n u(n) - (1/3)^n u(-n-1)$	2. $\lvert z\rvert < 1/3$
C. $(1/2)^n u(-n-1) + (1/3)^n u(n)$	3. $\lvert z\rvert < 1/3$ and $\lvert z\rvert > 1/2$
D. $\left[(1/2)^n + (1/3)^n\right] u(-n-1)$	4. $\lvert z\rvert > 1/2$

Code:	A	B	C	D
a)	4	2	1	3
b)	1	3	4	2
c)	4	3	1	2
d)	1	2	4	3

Q.137 The poles of an analog system are related to the corresponding poles of the digital system by the relation $Z = e^{ST}$. Consider the following statements :

1) Analog system poles in the left half plane map on to digital system poles inside the circle |Z|=1.
2)Analog system zeros in the left half S-plane map on to digital system zeros inside the circle |Z|=1.
3)Analog system poles on the imaginary axis of S-plane map on to digital system poles on the unit circle |Z|=1.
4) Analog system poles on the imaginary axis of S-plane map on to digital system zeros on the unit circle |Z|=1.

Which of these statements are correct?

a) 1 and 2 b) 1 and 3
c) 3 and 4 d) 2 and 4

Q.138 Assertion A: The stability of the system is assured if the Region of Convergence (ROC) includes the unit circle in the Z-plane.

Reason R : For a casual stable system all the poles should be outside the unit circle in the Z-plane.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R true but R is NOT a correct explanation of A.
c) A is true but R is false
d) A is false but R is true.

Q.139 Assertion A: The signals $a^n u(n)$ and $a^n u(-n-1)$ have the same Z-transform, $\frac{Z}{Z-a}$

Reason R : The Region of Convergence (ROC) for $a^n u(n)$ is $|Z| > |a|$, whereas the ROC for $a^n u(-n-1)$ is $|Z| < |a|$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R true but R is NOT a correct explanation of A.
c) A is true but R is false
d) A is false but R is true.

Q.140 Consider the following statements
1) Fourier transform is special case of Laplace transform.
2) Region of convergence need not be specified for Fourier transform.
3) Laplace transform is not unique unless the region of convergence is specified.
4) Laplace transform is a special case of Fourier transform.
Which of these statements are correct?

a) 2 and 4   b) 4 and 1
c) 4, 3 and 2   d) 1, 2 and 3

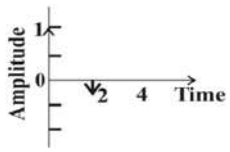
Q.141 The signal $x(t) = A\cos(\omega_0 t + \phi)$ is
a) An energy signal
b) A power signal
c) An energy as well as a power signal
d) Neither an energy nor a power signal

Q.142 If $x_1(t) = 2\sin \pi t + \cos 4\pi t$ and $x_2(t) = \sin 5\pi t + 3\sin 13\pi t$, then
a) x_1 and x_2 both are periodic
b) x_1 and x_2 both are not periodic
c) x_1 is periodic, but x_2 is not periodic
d) x_1 is not periodic, but x_2 is periodic

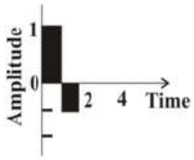
Q.143 If $y(t) + \int_0^{\infty} y(\tau)x(t-\tau)d\tau = \delta(t) + x(t)$, then y(t) is
a) u(t)   b) $\delta(t)$
c) r(t)   d) 1

Q.144 The impulse response of a system is $h(t)=\delta(t-0.5)$ If two such systems are cascaded, the impulse response of the overall system will be
a) $0.5\delta(t-0.25)$ b) $\delta(t-0.25)$
c) $\delta(t-1)$ d) $0.5\delta(t-1)$

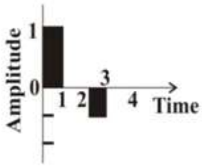
Q.145 The impulse response of a system consists of two delta functions as shown in the given figure. The input to the system is a unit amplitude square pulse of one unit time duration. Which one of the following diagrams depicts the correct output?



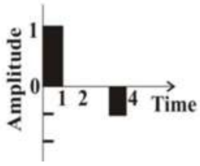
a)



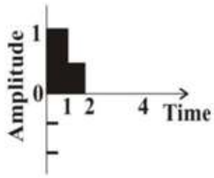
b)



c)



d)



Q.146 The Fourier transform of a double-sided exponential signal $x(t)=e^{-b|t|}$

a) Is $\dfrac{2b}{(b^2+\omega^2)}$

b) Is $\dfrac{e^{-j\tan^{-1}\left(\frac{\omega}{b}\right)}}{(b^2+\omega^2)}$

c) Does not exist

d) Exist only when it is single sided

Q.147 The Fourier transform of u(t) is

a) $\dfrac{1}{j\omega}$ b) $j\omega$

c) $\dfrac{1}{(1+j\omega)}$ d) $\pi\delta(\omega)+\dfrac{1}{j\omega}$

Q.148 A linear network has the system function $H(s)=\dfrac{H(s+c)}{(s+a)(s+b)}$. The outputs of the network with zero initial conditions for two different inputs are tabled as

Input x(t)	Output y(t)
u(t)	$2+De^{-t}+Ee^{-3t}$
$e^{-2t}u(t)$	$Fe^{-t}+Ge^{-3t}$

Then the values of c and H are, respectively

a) 2 and 3 b) 3 and 2

c) 2 and 2 d) 1 and 3

Q.149 Which one of the following is the response y(t) of a causal LTI system described by $H(s)=\dfrac{(s+1)}{s^2+2s+2}$ for a given input $x(t) = e^{-t}\,u(t)$?

a) $y(t)=e^{-t}\sin t\,u(t)$

b) $y(t)=e^{-(t-1)}\sin(t-1)u(t-1)$

c) $y(t)=\sin(t-1)u(t-1)$

d) $y(t)=e^{-t}\cos t\,u(t)$

Q.150 A signal x(t)=6 cos 10πt is sampled at the rate of 14 Hz. To recover the original signal, the cut-off frequency f_c of the ideal low-pass filter should be

a) 5Hz < f_c < 9 Hz b) 9Hz

c) 10 Hz d) 14 Hz

Q.151 Which one of the following systems is a causal system? [y(t) is output and u(t) is a input step function].

a) y(t)=sin[u(t+3)]

b) y(t)=5u(t)+3u(t-1)

c) y(t)=5u(t)+3u(t+1)

d) y(t)=sin[u(t-3)+sin[u(t+3)]]

Q.152 The Fourier-series representation of a periodic current is $[2+6\sqrt{2}\cos\omega t+\sqrt{48}\,\sin 2\omega t]A$ .The effective value of the current is

a) $(2+6+\sqrt{24})A$ b) 8A

c) 6A d) 2A

Q.153 Match List I (Properties) with List II (Characteristics of the trigonometric form) in regard to Fourier series of periodic f(t) and select the correct answer using the codes given below the lists:

List I	List II
A).f(t)+f(-t) = 0	1.Even harmonics can exist
B).f(t)-f(-t) = 0	2.Odd harmonics can exist
C).f(t)+f(t-T/2) = 0	3. The dc and cosine terms can exist
D).f(t)-f(t-T/2) = 0	4.sine terms can exist
	5.Cosine terms of even harmonics can exist

Code:	**A**	**B**	**C**	**D**
a)	4	5	3	1
b)	3	4	1	2
c)	5	4	2	3
d)	4	3	2	1

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d	a	c	a	a	b	b	d	b	b	d	a	c	b	b
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
c	c	c	a	c	b	d	c	c	c	b	b	a	b	b
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
b	a	a	d	a	a	b	a	d	b	d	a	d	a	a
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
d	d	c	b	b	b	a	c	c	a	c	a	d	d	d
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
a	a	a	c	b	d	c	d	c	d	a	b	d	d	d
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
a	b	b	b	c	a	b	b	c	a	d	a	d	a	d
91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
b	a	b	c	c	a	b	a	a	a	c	d	d	d	a
106	107	108	109	110	111	112	113	114	115	116	117	118	119	
a	d	a	b	d	d	b	c	d	d	a	c	b	b	
120	121	122	123	124	125	126	127	128	129	130	131	132	133	134
d	d	d	b	b	a	b	b	b	c	c	d	c	a	d
135	136	137	138	139	140	141	142	143	144	145	146	147	148	149
a	a	b	b	a	d	b	a	b	c	b	a	d	a	a
150	151	152	153											
a	b	b	d											

EXPLANATIONS

Q.1 (d)

$x(n) \rightarrow M = 3$

$h(n) \rightarrow N = 5$

$y(n) \rightarrow M + N - 1 = 7$

DTFT of the output $y[n] \rightarrow Y(\omega)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y[e^{j\Omega}] = \sum_{k=-\infty}^{\infty} |h(k)|\, |x(k)|\, e^{-j\Omega n}$$

$$Y[0]_{max.} = \sum_{k=-\infty}^{\infty} L.B$$

$Y(0) \rightarrow 7\ (|h[n]| \le L \ \ X\ |x[n]| \le B) =$ 7LB

Q.2 (a)

$x(t) = e^{-3t}u(t) + e^{2t}u(-t)$

two-sided Laplace transform

$e^{-3t}u(t) \quad + \quad e^{2t}u(-t)$

$\downarrow \qquad \downarrow$

$\frac{1}{s+3}$, Re(s) > - 3 $\qquad -\frac{1}{s-2}$, Re(s) < 2

- 3 < Re (s) < 2

$\frac{1}{s+3} - \frac{1}{s-2}$

$\frac{-5}{s^2+s-6}$

Q.3 (c)

$$\text{(a) } X(0) = \sum_{n=-3}^{7} x(n)$$

$= -1 + 0 + 1 + 2 + 1 + 0 + 1 + 2 + 1$

$X(0) = 6$ False

$$\text{(b) } x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\, e^{j\Omega n}\, d\Omega$$

$$2\pi\, x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\, d\Omega$$

$= 2\pi \times 2 = 4\pi$ False

(c) $X(\omega) = X(-\omega)$

$|X(\omega)| \rightarrow$ even function

$\angle X(\omega) \rightarrow$ odd function

True

(d) False

Q.4 (a)

Multiplication of two linear system T_1 and T_2 result will be non linear

Q.5 (a)

If $x[n] = \begin{cases} \frac{2}{\pi}, & n = 0 \\ \frac{\sin 2n}{\pi n}, & n \ne 0 \end{cases}$

$x[n] = \begin{cases} \frac{2}{\pi}; & n = 0 \\ \frac{2}{\pi}\frac{\sin 2n}{\pi n}; & n \ne 0 \end{cases}$

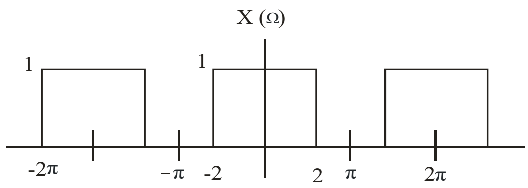
$x(n) \overset{DTFT}{\longleftrightarrow} X(e^{j\Omega})$

$\frac{\sin Wn}{\pi n} \overset{DTFT}{\longleftrightarrow} X(\Omega)$

$= \begin{cases} 1 & |\Omega| \le w \\ 0 & W \le |\Omega| \le \pi \end{cases}$

$\frac{\sin 2n}{\pi n} \leftrightarrow X(\Omega)$

$= \begin{cases} 1 & |\Omega| \le 2 \\ 0 & 2 \le |\Omega| \le \pi \end{cases}$



By parsevel's Theorems

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \int_{-W}^{W} 1(d\Omega)$$

$$= \frac{1}{2\pi}\int_{-2}^{2} d\Omega$$

$$E = \frac{1}{2\pi} \times 4 = \frac{2}{\pi}$$

Q.6 (b)

$$H_a(S) = \frac{k(s+2)\left(s^2 + \frac{\pi^2}{4}\right)}{s(s+2) + \pi^2}$$

equivalent-z $h_a(t)$ is sampled with 2Hz

$t = n\,T_s T_s = \frac{1}{2}$

$t = \frac{n}{2}$

$z = e^{sTs}$

$\log z = sT_s$

$z = e^{s/2}$

Putting $s = 0$

z = 1 unit circles

s = -2

$z = e^{-1} = \frac{1}{e}$ radius circle

Q.7 **(b)**

$$\frac{X(z)}{z^{19}} = \frac{z}{(z-\frac{1}{2})(z-2)(z=3)} = \frac{A}{2-\frac{1}{2}} + \frac{B}{z-2} + \frac{B}{z+3}$$

$$\frac{z^{-19}X(z)}{z} \Rightarrow$$

$$A = \frac{z}{(z-2)(z+3)}|z = \frac{1}{2} = \frac{\frac{1}{2}}{\frac{-3}{2}\times\frac{-7}{2}} = \frac{-2}{21}$$

$$B = \frac{z}{(z-\frac{1}{2})(z+3)}|z = 2 = \frac{2}{\frac{-3}{2}\times\frac{5}{2}} = \frac{4}{15}$$

$$C = \frac{z}{(z-\frac{1}{2})(z-2)}|z = -3 = \frac{-3}{\frac{-7}{2}\times -5} = \frac{-6}{35}$$

$$z^{-18}x(z) = \frac{-2}{21}.\frac{z}{z-\frac{1}{2}} + \frac{4}{15}.\frac{z}{z-2} - \frac{6z}{35z+3}$$

$$x(n-18) = \frac{-2}{21}\left(\frac{1}{2}\right)u(n) - \frac{4}{15}(2)^n 4(-n-1) +$$

$$\frac{6}{35}(-3)^n 4(-n-1)$$

by inverse Z transform of X(z) - converges for $|z| = 1$

$z = \frac{1}{2}$

z = 2

<u>z = -3</u>

RDC $\frac{1}{2} < |z| < 2$

$|z| \rightarrow 1$ converse

$x(n-18) = \frac{-2}{21}\,.\,1\,.\,1\,. = \frac{-2}{21}$

Q.8 **(d)**

$X(z)|_{z=1} = 1$, All option gives $X(z)|_{z=1} = 1$

Two poles, one $z = e^{j\pi/2}$ and second $e^{-j\pi/2}$ conjugates poles, two zero at original ROC is |z | >1. $X(z) = \frac{2z^2}{z^2+1}$

ROC of it dependent on |z|.

Q.9 **(b)**

one pole – $z = \frac{3}{4}$

one zero real zero of H(z), $|z| = \frac{3}{4}$

h(n) is real system then

zero are complex conjugate

a + jb, a - jb

$H(z) = \frac{(z-a+jb)(z-a-jb)}{z-\frac{3}{4}} = \frac{(z-a)^2+b^2}{z-\frac{3}{4}}$

H(z) > 1, N(z) > D(z) then rationalized to check stability & causality

$H(z) = \alpha z + \beta + \frac{\gamma}{z-\frac{3}{4}}$

$\frac{|z| > \frac{3}{4}}{z-\infty}$ ROC not includes ∞

System may be

-non causal + stable

Or anti causal + unstable

Q.10 **(b)**

$x(t) \leftrightarrow W$,

$y(t) = x^3(t) + x(t) + 1$

↓ ↓

3W W

$y(t) \leftrightarrow 3W$

$f_s = 2(f_m)max = 2 \times 3W = 6W$

f_s min = Nyquist sampling frequency

$(f_s)min = 6$ W

Q.11 **(d)**

x(t) = cos (10t) cos (100t)

y(t) = x(t) * h(t)

Y(ω) = X(ω) . H(ω)

$x(t) = \frac{1}{2}$ [cos 110 t – cos 90t]

↓ ↓

$|H(\omega)| = 1$, $\angle H(\omega) = ?$ (ω_1 ω_2)

$\angle H(\omega_1) = e^{(-j\,100 - 2j\,(110-100))} = e^{-j\,120}$

$\angle H(\omega_2) = e^{(-j\,100 - 2j\,(90-100))} = e^{-j\,80}$

$y(t) = \frac{1}{2}\cos[(110t - 120)] - \cos[(90t - 80)] = \cos\left(\frac{200t-200}{2}\right)\cos\left(\frac{20t-40}{2}\right) = \cos(100(t-1))\cos(10(t-2))$

Q.12 (a)

Inverse System. x[n] = w[n]

Q.13 (c)

a) $x_1(t) = 2e^{j\left(t+\frac{\pi}{4}\right)}u(t)$

Periodic Non Periodic

$\frac{2\pi}{1} = 2\pi 2e^{j\left(t+\frac{\pi}{4}\right)}u(t) \leftarrow$ Non Periodic

b) $x_2(n)$ – a periodic

non periodic

c) N = 4 ⟵ Periodic

d) $e^{-t}e^{-jt} \leftarrow$ Non Periodic

Q.14 (b)

$y(t) = \int_{-\infty}^{2t} x(\tau)d\tau \leftarrow$ Linear x(t) ⟶ u(t)

y(t) = 2t ⟶ Linear t > 0

delayed input

$y(t) = \int_{-\infty}^{2t} x(-p+\tau)d\tau$

$-p + \tau = c$

$y(t) = \int_{-\infty}^{2t-p} x(c)dc = \int_{-\infty}^{2t-p} x(c)dc$

delayed output

$y(t-p) = \int_{-\infty}^{2(t-p)} x(\tau)d\tau = \int_{-\infty}^{2t-2p} x(\tau)d\tau$

[delayed output ≠ delayed input] time varying

$y(t) = \int_{-\infty}^{2t} x(\tau)d\tau = \int_{-\infty}^{0} x(2t-\tau)d\tau$

$= \int_{-\infty}^{\infty} x(\tau - 2t)d\tau$

<u>Non causal</u> $|x(t)| < \infty$

$y(t) = \int_{-\infty}^{2t} x(\tau)d\tau \quad \int_{-\infty}^{0} |x(t)dt| < \infty$

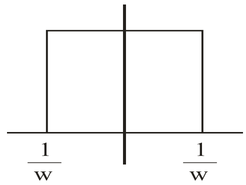
if x(t) . u(t)

than y(t). 2t u(t) t ⟶ ∞, y(t) ⟶ ∞,unbounded

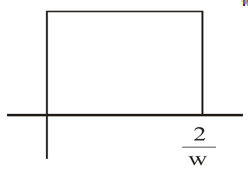
Q.15 (b)

a, b, c are band limited so those are not limited in time.

Rectangular puls ⟵Fourier transform⟶ Sinc(x) function time limited



For causally ⟶ it is shifted by - $\frac{1}{w}$

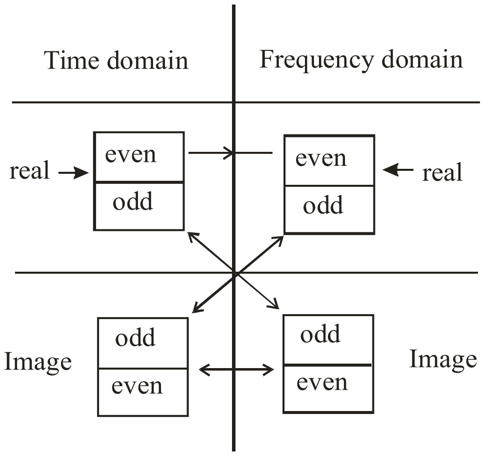


Time shifting adds nothing to magnitude spectrum component, added a term to phase spectrum component.

Q.16 (c)

$H(j\omega) = H_1(\omega) + j\,H_2(\omega)$

↓ odd function ↓ even function



$H_1(\omega)$ (odd function) + j $H_2(\omega)$ (even function), so that $H(\omega)$ is a purely imaginary function

Q.17 (c)

$F(t) = A[u(t) - u(t - c)]$

$= A\left[\frac{1}{s} - \frac{e^{-cs}}{s}\right]$

$= A\left[\frac{1-e^{-cs}}{s}\right]$

Q.18 (c)

$x(n) = \left(\frac{1}{2}\right)^2 u(n) = \left(\frac{1}{2}\right)^{-n} u(-n-1)$

↓

$|z| > \frac{1}{2}$ $\qquad |z| \angle \frac{1}{1/2} = 2$

ROC is $\left|\frac{1}{2}\right| \angle |z| \angle 2$

Q.19 (a)

F.T. $\begin{cases} y(t) = k\, x(t - t_d) \\ y(\omega) = k\, e^{-j\omega t_d} x(\omega) \end{cases}$

$H(\omega) = |H(\omega)| \angle |H(\omega)| = |H(\omega)| \angle \phi(\omega)$

Polar form → $re^{j\theta(\omega)}$

$\tau_g = -\frac{d\theta(\omega)}{d\omega}$, $\tau_p = -\frac{\theta(\omega)}{\omega}$

$|H(\omega)| = k$, $\angle \phi(\omega) = -j\omega t_d$

$\tau_g = \frac{-d(-\omega t_d)}{d\omega} = t_d$

$\tau_p = -\frac{-\omega t_d}{\omega} = t_d$

$\tau_g = \tau_p = t_d$ for a linear phase system

Q.20 (c)

m(t) is band limited to 20 kHz

Sampling frequency f_s

Low Pass Filter cut off frequency – f_L = 37 kHz

f_s minimum to reconstruct m(t) without distortion is

f_s minimum = $f_L + f_m$ = 37 + 20 = 57 kHz

Q.21 (b)

$y(t) = x(t) * k(t) = \int_{-\infty}^{\infty} x(\tau)\, k(t-\tau)\, d\tau$

$Y(\omega) = X(\omega) \cdot K(\omega)$

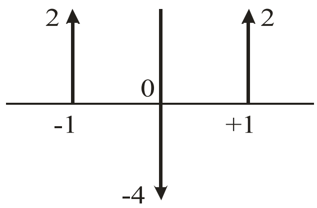
$y(0) = \int_{-\infty}^{\infty} x(\tau)\, k(t-\tau)\, d\tau$

$\frac{1}{2} \times 1 + \frac{1}{2} \times 1 + 2 \times \frac{1}{2} \times \frac{1}{2} \times 1$

$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)\, e^{j\omega t}\, d\omega$

$y(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)\, d\omega$

x``(t) =



$(j\omega)^2 X(\omega) = 2e^{+j\omega} - 4 + 2e^{-j\omega}$

$X(\omega) = \frac{4\cos\omega - 4}{-\omega^2} = \frac{4(1-\cos\omega)}{\omega^2} = \frac{4\sin^2 \omega/2}{\omega^2} = \frac{\sin^2 \omega/2}{\left(\frac{\omega}{2}\right)^2}$

$Y(\omega) = 2\left(\frac{\sin\omega}{\omega}\right)\left(\frac{\sin^{\omega}/2}{\omega^2}\right)^2 = 2 \sin c(\omega) \sin c^2 (\omega/2)$

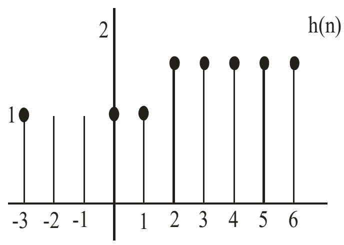
$y(0) = 2 \cdot 1 \cdot 1 = 2$

Q.22 (d)

For LTI System to be stable

$\sum_{n=-\infty}^{\infty} h(n) \angle \infty$ (finite) , impulse response summation is finite

$h(n) = u(n+3) + u(n-2) - 2u(n-7)$



System Stable

$\sum_{n=3}^{6} h(n) = \text{finite}$

and it is not causal because it depends on future value.

Q.23 (c)

Both the statement one Dirichlet's conditions

So both should be satisfied

Q.24 (c)

Fourier transform of unit step sequence

$$u(n) \xleftrightarrow{F.T} u(t) \xleftrightarrow{F.T} \frac{1}{j\omega} + \pi\delta(\omega)$$

$$u(n) \xleftrightarrow{D.T.F.T} \left[\frac{1}{1-z^{-1}} + \pi\delta(n)\right]_{z=e^{j\Omega}} Z = e^{j\Omega}$$

$$u(n) = \left[\frac{1}{1-e^{-j\Omega}} + \pi\delta(\Omega)\right]$$

Q.25 (c)

$\delta(t) \leftrightarrow 1$ Duality property

$1 \leftrightarrow 2\pi\,\delta(\omega)$

$e^{j\omega_0 t} \leftrightarrow 2\pi\,\delta(\omega - \omega_0)$

$e^{-j\omega_0 t} \leftrightarrow 2\pi\,\delta(\omega + \omega_0)$

$\delta(\omega - \omega_0) \leftrightarrow \frac{1}{2\pi} e^{j\omega_0 t}$

Q.26 (b)

As per Dirichlet's condition

Q.27 (b)

Step input is $t^2 e^{-t}$

$$e^{-t} \xleftrightarrow{\text{Linear Transform}} \frac{1}{s+1}$$

From linear transform properties -

$tx(t) \quad \frac{dx(s)}{ds}$

$$t\,e^{-t} \leftrightarrow -\frac{d}{ds}\left[\frac{1}{s+1}\right] = \frac{(0-1)}{(s+1)^2} = \frac{1}{(s+1)^2}$$

$$t^2\,e^{-t} \leftrightarrow -\frac{d}{ds}\left[\frac{1}{(s+1)^2}\right] = -\frac{2(s+1)^1}{(s+1)^4}$$

$$= \frac{+2}{(s+1)^3}$$

From step response to impulse response d/dt

use $\frac{d}{dt}$

$$\int \frac{d}{dt}[t^2 e^{-t}] = 2te^{-t} - t^2 e^{-t}$$

Both side Linear Transform

$$\frac{2}{(s+1)^2} - \frac{2}{(s+1)^3}$$

$$\frac{2(s+1)-2}{(s+1)^3} = \frac{2s}{(s+1)^3}$$

Q.28 (a)

Z – transform

$H(z) = 1 + z^{-1}$ for frequency response

$z = e^{j\Omega}$

$H(\Omega) = 1 + e^{-j\Omega}$

$H(\Omega) = e^{-j\Omega/2}[e^{j\Omega/2} + e^{-j\Omega/2}]$ $\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$

$H(\Omega) = e^{-j\Omega/2}\, 2\cos\left(\frac{\Omega}{2}\right)$

$H(\Omega) = 2\cos\left(\frac{\Omega}{2}\right) \angle \frac{-\Omega}{2}$

Q.29 (b)

Relation between Z – Transform & Linear Transform

$Z = e^{ST}$ $\qquad S = \frac{\ln z}{T}$

Q.30 (b)

Convolution of two sequence $x_1(n)$ & $x_2(n)$ is represented as

$x_1(n) * x_2(n) \overset{z}{\Leftrightarrow} x_1(z) \,.\, x_2(z)$

From properties. Convolution of two sequence multiplication in Z – domain

Q.31 (b)

Z – transform of - u(- n – 1) is

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

Where x(n) = - u(- n – 1) $0 < -n-1 < \infty$

$1 < -n < \infty$

$-\infty < n < -1$

$$X(z) = \sum_{n=-\infty}^{\infty} -u(-n-1)\, z^{-n}$$

$$X(z) = -\sum_{n=-\infty}^{-1} z^{-n}$$

- n replace with m

$$X(z) = -\sum_{m=1}^{\infty} z^{m} = -[z + z^2 + z^3 + \ldots\ldots$$

.]

$$X(z) = \frac{z}{z-1},\ |z| < 1$$

Q.32 (a)

$$X(z) = \frac{(1-e^{-T})z^{-1}}{(1-z^{-1})(1-e^{-T}-z^{-1})}$$

initial value x(0) is

$$z\underline{\text{Lim}}\ \infty X(z) = x(0) = z\underline{\text{Lim}}\ \infty \frac{(1-e^{-T})z^{-1}}{(1-z^{-1})(1-e^{-T}-z^{-1})}$$

$x(0) = 0$

Q.33 (a)

Unit step response y(n) + y(n – 1) = x(n)

Z – transform in both direction

$Y(z) + z^{-1} Y(z) = X(z)$

$\frac{Y(z)}{X(z)} = \frac{1}{1+z^{-1}} = \frac{z}{z+1}$

$Y(z) = \frac{z}{z+1} x(z)$

X(z) ↔ input z – transform

x(n) = unit step sequence

$X(z) = \frac{1}{1-z^{-1}} = \frac{1}{1-z}$

$Y(z) = \frac{z^2}{(z-1)(z+1)}$

Q.34 (d)

Basic definition of signal

Q.35 (a)

Check for linear system

y(t) – output of system $S = \int_{-\infty}^{t} x(\lambda)\, d\lambda$,

input x(t)

y(t) → Area under x(t) from -∞ to t So it is linear

Check for time invariant system

time invariant also because delay input and delayed output are equal

Q.36 (a)

X(n) = 5cos [0.2 πn]

Period $N = \frac{2\pi}{0.2\pi} . m$

$N = \frac{2\pi}{\omega}$ x m

N = 10 m,

m = 1(minimum)

N = 10 fundamental period

Q.37 (b)

if x(t) ↔ X(jω)

if h(t) = H(jω]

$y(t) = x(t) * h(t)$

$Y(j\omega) = X(j\omega).H(j\omega)$

Convolution project

Q.38 (a)

For distortion less, good quality transmission all frequency components should have the same transmission delay, t_d and same phase shift ϕ_s

Q.39 (d)

$F(s) = \frac{\omega}{s^2+\omega^2}$, steady state value of f(t)

From final value theorem is not application due to multiple pole at imaginary axis (system is marginally stable)

f(t) = sin ωt (periodic signal) value is swing between +1 and -1 [-1, 1]

Q.40 (b)

$H(s) = \frac{1}{s+1}$, $e^{-t} = h(t)$, x(t) = u(t)

$\frac{y(s)}{x(s)} = \frac{1}{s+1}$, $y(s) = \frac{1}{s+1} x(s)$, $x(s) = \frac{1}{s}$

$y(s) = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$

$y(t) = (1 - e^{-t})\, u(t)$

for steady state

$\lim_{t \to \infty} y(\infty) = \lim_{s \to 0} sy(s)$

$= \lim_{s \to 0} \frac{s\, 1}{s(s+1)} = 1$

$y(\infty) = (1 - e^{-\infty})\, u(t) = 1$

Q.41 (d)

Audio signal is band limited to 4 kHz .

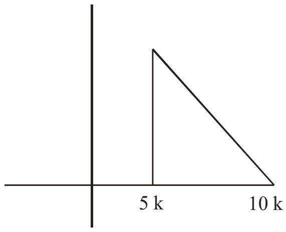
f_s = 5kHz

f_m, $nf_s \pm f_m$ ………………… n is a integer no. = 1, 2, 3, 4, ……………..

Q.42 (a)

A signal occupies a band 5 kHz to 10 kHz

It is a case of band pass sampling so that f_s minimum



$f_s = 2(f_H - f_L)$ ……. (1)

$f_s = \frac{2f_H}{[M]}$ ……………….. (2)

$[M] = \frac{f_H}{f_H - f_L}$

$f_s = 2(10 - 5) =$ 10 kHz. (1st Method),

when f_H & f_L are integral multiple of $(f_H - f_L)$

$f_s = \frac{2 \times 10}{a}$, $M = \frac{10}{10-5} = 2$

$f_s = \frac{20}{2} = 10$ kHz

Q.43 (d)

$\sin(\omega t+\alpha) \xrightarrow{\text{Linear Transform}}$

for $\sin \omega t \leftrightarrow \frac{\omega}{s^2+\omega^2}$ $\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$

for $\cos \omega t \leftrightarrow \frac{\omega}{s^2+\omega^2}$

$\sin(\omega t+\alpha) = \frac{e^{+j(\omega t+\alpha)}-e^{-j(\omega t+\alpha)}}{2j}$

$\Rightarrow$ Linear Transform

$\frac{1}{2j}\left[\frac{e^{j\alpha}}{(s-j\omega)} - \frac{e^{-j\alpha}}{(s+j\omega)}\right]$

$= \frac{1}{2j}\left[\frac{e^{j\alpha}(s+j\omega) - e^{-j\alpha}(s-j\omega)}{s^2+\omega^2}\right]$

$= \frac{1}{s^2+\omega^2}\left[\frac{s\,(e^{j\alpha}-e^{-j\alpha})}{2j}\right] + \frac{j\omega\,[e^{j\alpha}+e^{-j\alpha}]}{2j}$

$= \frac{1}{s^2+\omega^2}\cdot(s.\sin\alpha + \omega\cos\alpha)$ \

Q.44 (a)

$y(n) = \sum_{k=-\infty}^{n} x(k)$

it is a invertible system & with memory

Q.45 (a)

Find a periodic one

(a) Sin $(10\pi t)$ + sin $(20\pi t)$

$\downarrow$

$T_1 = \frac{1}{5} = \frac{2\pi}{10\pi}$ $T_2 = \frac{2\pi}{20\pi} = \frac{1}{10}$

$\frac{p}{q}$ = rational

$\frac{T_1}{T_2} = \frac{p}{q}$ = rational

T = LCM $(T_1 \& T_2)$

T = LCM $\left(\frac{1}{5}, \frac{1}{10}\right)$ = 1, fundamental period is 1 second.

(b) Sin (10t) + sin $(20\pi t)$

$\downarrow$

$T_1 = \frac{2\pi}{10} = \frac{\pi}{5}$ $T_2 = \frac{2\pi}{20\pi} = \frac{1}{10}$

$\frac{T_1}{T_2} = \frac{p}{q}$ = rational = 2π

Non-periodic

A periodic

(c) Sin $(10\pi t)$ + sin (20 t)

$\downarrow$

$T_1 = \frac{2\pi}{10\pi}$, $T_2 = \frac{2\pi}{20\pi} = \frac{T}{10} = \frac{1}{5}$

$\frac{T_1}{T_2} = \frac{p}{q} \neq$ (irrational) $\frac{5}{\pi}$ - A periodic

(d) Sin 10 t + sin 25 πt

$\downarrow$

$T_1 = \frac{2\pi}{10}$, $T_2 = \frac{2\pi}{25\pi}$

$T_1 = \frac{\pi}{5}$, $T_2 = \frac{2}{25}$

$\frac{T_1}{T_2} = \frac{p}{q} \neq$ (irrational)

$2\pi \times 5 = 10\pi$ - A periodic

Q.46 (d)

From definition of Fourier series parseval's Energy/ power theorem.

Q.47 (d)

Fourier Transform properties.

Q.48 (c)

From Laplace Transform & Fourier Transform comparison $\sigma = 0$

[Laplace Transform = Fourier Transform comparison]

$S = \sigma + j\omega$, $\sigma = 0$ (on the imaginary axis of the s – plane)

$S = j\omega$

Q.49 (b)

$f(t) \xleftrightarrow{\text{Fourier Transform}} F(j\omega)$

$F(j\omega) = \int_{-\infty}^{\infty} f(-t)e^{-j\omega t}dt$

$f(-t) \xleftrightarrow{\text{Fourier Transform}} ?$

$F'(j\omega) = \int_{-\infty}^{\infty} f(-t)e^{-j\omega t}dt$

$-t = -dt = dp$

$= -\int_{\infty}^{-\infty} f(p)e^{+j\omega p}dp$

$= \int_{-\infty}^{\infty} f(p)e^{-(-j\omega)p}dp = FG(-j\omega)$

Q.50 (b)

If f(t) is an even function then what is it Fourier transform

$F(j\omega) = \int_{-\infty}^{\infty} f(-t)\, e^{-j\omega t}\, dt$

$x_e = \frac{x(t)+x(-t)}{2}$

$F'(j\omega) = \int_{-\infty}^{\infty} f(-t)\, e^{-j\omega t}\, dt$

$$= \int_{-\infty}^{\infty} f(p)\, e^{-(-j\omega)p}\, dp$$

$= f(-j\omega)$

$$F'(j\omega) = 2\int_{0}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= 2\int_{0}^{\infty} f(t)\,(\cos \omega t - j \sin \omega t)dt$$

$$= 2\int_{0}^{\infty} f(t) \cos \omega t\, dt - 2j \int_{0}^{\infty} f(t) \sin \omega t\, dt$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\,(\cos \omega t - j \sin \omega t)dt$$

$$= \int_{-\infty}^{\infty} f(t) \cos \omega t\, dt - j \int_{-\infty}^{\infty} f(t) \sin \omega t\, dt$$

E x E E x o = o

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) \cos \omega t\, dt - 0$$

Q.51 (b)

$$\cos \omega_0 t \leftrightarrow \frac{s}{s^2 + \omega_0^2}$$

Q.52 (a)

Step response – $c(t) = 0.5(1 - e^{-2t})\, u(t)$

Impulse response = $\frac{d}{dt}$ (step response)

$= \frac{d}{dt}[0.5\, u(t) - 0.5\, e^{-2t}u(t)]$

$= 0.5\, \delta(t) - 0.5 \text{ x } - 2\, e^{-2t}u(t)$

$= 0.5\, \delta(t) + e^{-2t}u(t) - 0.5\, e^{-2t}\, \delta(t)$

$= e^{-2t}u(t)$

IInd Method Step response Linear Transform

$C(s) = \frac{0.5}{s} - \frac{0.5}{s+2} = 0.5\left[\frac{2}{s(s+2)}\right]$

$C(s) = \frac{Y(s)}{X(s)} = \frac{1}{s(s+2)}$

$Y(s) = \frac{1}{(s+2)}$ when input impulse

$y(t) = e^{-2t}u(t)$

Q.53 (c)

$x(n) = u(n)$ by formula (Z – Transform pair)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Q.54 (c)

Two signal band limited to 8 kHz, 4 kHz

As per sampling theorem, minimum sampling

Frequency is f_s minimum = 2 fm

f_s minimum = 2 x 8 = 16 kHz

As per question $f_s \rightarrow 12$ kHz

So, it does not obey sampling theorem

Q.55 (a)

a. $\sin \omega_0 t u(t - t_0) \leftrightarrow \frac{e^{-t_0 s}}{\sqrt{s^2 + \omega_0^2}} \sin \left(\omega_0 t_0 + \tan^{-1}\frac{\omega_0}{s}\right)$

b. $\sin \omega_0 (t - t_0)u(t-t_0) \leftrightarrow \frac{\omega_0}{s^2 + \omega_0^2}$

c. $\sin \omega_0 (t - t_0)u(t) \leftrightarrow \frac{1}{\sqrt{s^2 + \omega_0^2}} \sin \left(\omega_0 t_0 - \tan^{-1}\frac{\omega_0}{s}\right)$

d. $\sin\ \omega_0 t u(t) \leftrightarrow \left\{\frac{\omega_0}{s^2 + \omega_0^2}\right\} e^{-t_0 s}$

Q.56 (c)

Energy of power signal is infinite

Q.57 (a)

x(n) = [-1, , 1]

↑

x(-n) = [-1, , -1]

↑

y(n) = x(n) + x(-n) = 0, for all value of n

Q.58 (d)

Power formula -

$$T\underline{Lim} \infty \frac{1}{T} \int_{-T/2}^{T/2} |x^2(t)|\, dt$$

Q.59 (d)

Option a, b, c are linear system but in option d is non linear
$y(n) = x^2(n) \rightarrow$ non linear

Q.60 (d)
The output of a linear system for any input can be computed in both ways.
1) Summation of impulse response by convolution integral
2) Summation of step response by superposition integral

Q.61 (a)
if $x[-n] \xleftrightarrow{DTFT} X(e^{j\omega})$
From Discrete Time Fourier Transform table match following pair

a. $x[-n] \leftrightarrow X(e^{-j\omega})$

b. $nx[n] \leftrightarrow j\frac{d}{d\omega}X(e^{j\omega})$

c. $x^*[n] \leftrightarrow X^*(e^{-j\omega})$

d. $x[n-1] \leftrightarrow e^{-j\omega}X(e^{j\omega})$

Q.62 (a)
From Fourier transform properties $x(t) \leftrightarrow X(\omega)$

$|X(\omega)| \leftrightarrow$ even, $\angle X(\omega) \rightarrow$ odd

Q.63 (a)

$$G(s) = \frac{2}{s^{-s} - s - 2}$$

s Lim s G(s)=tLim ∞ g(t)

Final value theorem $\rightarrow$ which is not applicable because roots are in Right half – S plan.

Q.64 (c)

$\delta(t) \leftrightarrow 1$shift in time,

odd $e^{-to\, s}$ term

$\delta(t\text{-}1)=e^{-s}$to s – domain

Q.65 (b)
By scalling property

$e^{j\omega 0n}x(n) \leftrightarrow X(e^{-j\omega 0z})$

Q.66 (d)

Q.67 (c)

Q.68 (d)

Q.69 (c)
$x(n) \rightarrow -2 < n < 4$
$x(-n-2) \rightarrow -2 < -n-2 > 4$
$\rightarrow 0 < -n < 6$
$\rightarrow 0 > n > -6$
$n < -6, \ \& \ n > 0$
$\rightarrow x(-n-2)$ is guaranteed zero

Q.70 (d)

Q.71 (a)
$x(n) = [-1, -1, 0, 1, 1]$ (↑ under 1)
$-2 \le n \le 2$
$-2 < n + 3 \le 2$
$-5 < n < -1$

Q.72 (b)

Q.73 (d)

Q.74 (d)

a) $e^{-t}u(t) \leftrightarrow \frac{1}{1+j\omega}$

b) $x(t) = \begin{cases} 1, |t| \le 1 \\ 0, |t| > 1 \end{cases} \leftrightarrow \frac{2\sin\omega}{\omega}$

c) $\frac{dx(t)}{dt} \leftrightarrow j\omega X(j\omega)$

d) $\frac{2}{1+t^2} \leftrightarrow \frac{2}{1+\omega^2}$

Q.75 (d)
Property of Fourier Transform
$e^{j\omega_0 t}f(t) \leftrightarrow F(\omega - \omega_0)$

$x(t) \xrightarrow{\text{Fourier Transform}} \frac{2}{\omega} \sin \pi\omega$

$$e^{j5t}x(t) \leftrightarrow \frac{2}{\omega-5}\sin\ \pi(\omega-5)$$

Q.76 (a)

$u(t) \xleftrightarrow{\text{Fourier Transform}} \frac{1}{j\omega} + \pi\delta\ (\omega)$

$u(-t) \longleftrightarrow \frac{j}{\omega} + \pi\delta\ (\omega)$, even function

$\rightarrow \delta\ (\omega)$

$\frac{j}{t} + \pi\delta\ (t) \longleftrightarrow 2\pi\ u(\omega)$

$u(\omega) \leftrightarrow \frac{\delta(t)}{2} + \frac{j}{2\pi t}$

$u(f) \leftrightarrow \frac{\pi\delta(t)}{1} + \frac{j}{t}$

Q.77 (b)

$x(t) = -e^{2t}\ u(t) * t\ u(t)$

$u(t) \xleftrightarrow{\text{Laplace Transform}} \frac{1}{s}$

$tu(t) \xleftrightarrow{\text{Laplace Transform}} \frac{1}{s^2}$

$-e^{2t}\ u(t) = \frac{-1}{s-2}$

(convolution in time domain multiplication in frequency/ S – domain)

$x(t) \xleftrightarrow{\text{Laplace Transform}} \frac{-1}{s^2\ (s-2)}$

Q.78 (b)

$f(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{C}{(s+2)^3}$

from partial fraction method

A = 2, B = -2, C = -2, D = 6

$F(s) = \frac{2}{s+1} - \frac{2}{s+2} - \frac{2}{(s+2)^2} + \frac{6}{(s+2)^3}$

Q.79 (b)

$$X(s) = e^{-s}\left[\frac{-2}{s(s+2)}\right]$$

Initial final value of x(t)

$= \lim_{s \to \infty} s\ X(s)$

$= \lim_{s \to \infty} s.\ e^{-s}\left[\frac{-2}{s\ (s+2)}\right]$

= 0

Final value theorem

Final value of $x(t) = \lim_{s \to \infty} s\ X(s)$

$= \lim_{s \to \infty} s.\ e^{-s}\left[\frac{-2}{s\ (s+2)}\right]$

$= 1\left(\frac{-2}{2}\right)$

= -1

Q.80 (c)

$X(s) = \left(\frac{1-e^{-sT}}{s}\right)$

or $X(s) = \frac{1}{s} - \frac{1}{s}e^{-sT}ss$

$x(t) = u(t) - u(t - T)$

Q.81 (a)

Formula for inverse Z- transform is

$$\frac{1}{2\pi j}\oint X(z)z^{n-1}dz$$

Q.82 (b)

Absolutely summable

Q.83 (b)

Taking Z – transform of equation

$Y[z] - \frac{1}{2}\ y[z]\ .\ z^{-1} = X[z]$

and $Y[z] = \frac{X[z]}{[1-\frac{1}{2}\ z^{-1}]}$

Since $x[n] \xleftrightarrow{\text{Z-Transform}} X[z] = k$

Than

$Y[z] = \frac{k}{1-\frac{1}{2}\ z^{-1}}$

or $y[n] = k\ .\left(\frac{1}{2}\right)^n$

Q.84 (c)

A. Reconstruction ↔To convert the discrete time sequence back to a continuous time signal and then resample

B. Over sampling ↔ Sampling rate is chosen significantly greater than the Nyquist rate

C. Interpolation $\leftrightarrow$ Assign values between samples and signals
D. Decimation $\leftrightarrow$ A mixture of continuous and discrete time signals

Q.85 (a)

$A.\ 2t\frac{dy}{dt}+4y=2tx \leftrightarrow$

Linear, time-variable and dynamic

$B.\ y\frac{dy}{dt}+4y=2x \leftrightarrow$

Non linear, time invariant and dynamic

$C.\ 4\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=3\frac{dx}{dt} \leftrightarrow$

Linear, time invariant and dynamic

$D.\ \left(\frac{dy}{dt}\right)^2+2ty=4\frac{dx}{dt} \leftrightarrow$

Non-linear, time-variable and dynamic

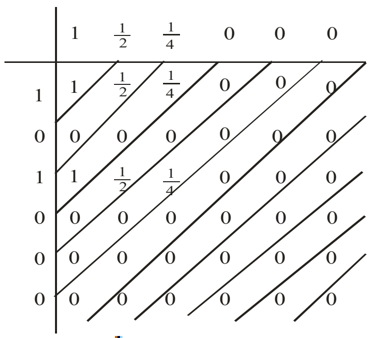
Q.86 (d)

2r(t – 3) – 2r(t – 4) -2u(t – 4)
= 2(t – 3) u(t – 3) – (2t – 8) u(t – 4) - 2u(t – 4)
= (2t – 6) u(t – 3) – (2t – 6) u(t – 4) - 2(t – 4)
= (2t – 6) u(t – 3) – (2t – 6) u(t – 4)
= (2t – 6) [u(t – 3) – u(t – 4)]

Q.87 (a)

Causal system can't be stable all the time when all poles are in left side of s – plan system will be stable but it is not necessary that ROC contain imaginary Axis.

Q.88 (d)



$\Rightarrow \left\{1, \frac{1}{2}, \frac{5}{4}, \frac{1}{2}, \frac{1}{4}, 0\right\}$

Q.89 (a)

x[n] = a^n u[n], h[n] = b^n u[n]
then y[n] = x[n] * h[n]
= a^n u[n] * b^n u[n]

$$= \sum_{k=-\infty}^{\infty} a^k u[k] b^{n-k} u[n-k]$$

Q.90 (d)

Since wave from is trapezoidal hence both rectangular pulse have different width and width of wave form = 8 units
So two rectangular waveforms of duration Six and two units respectively

Q.91 (b)

a. Periodic Function $\leftrightarrow$ Line discrete spectrum
b. Aperiodic Function $\leftrightarrow$ Continuous spectrum at all frequencies
c. Unit Impulse δ(t) $\leftrightarrow$ 1
d. sin ωt $\leftrightarrow$ δ(ω)

Q.92 (a)

f(t) = u(t) + u(t – 1) – 2u(t – 2)

$$F(s) = \frac{1}{s} + \frac{1}{s}.e^{-s} - \frac{2}{s}.e^{-2s}$$

Q.93 (b)

e^tu(t) = $\frac{1}{s-1}$, Re(s) > 1 only Laplace transform exist
Since not absolute integrable so no Fourier transform exist

Q.94 (c)

Q.95 (c)

$$y[n] = \sum_{k=0}^{\infty} x[n]$$

than $y[z] = \sum_{n=-\infty}^{\infty} y[n]\, z^{-n}$

$$= \sum_{n=-\infty}^{\infty}\sum_{k=0}^{\infty} n[k]\, z^{-n}$$

$= x[z] . \sum_{n=0}^{\infty} z^{-n}$ {since y[n] is causal}

$= x[z]\ [1 + z^{-1} + z^{-2} + \ldots\ldots..]$

$= x[z] = \frac{1}{1-z^{-1}}$

$= \frac{x[z]}{1-z^{-1}}$

Q.96 **(a)**

$\frac{x(z)}{z} = \frac{1}{(z-2)(z-3)}$

$\frac{x(z)}{z} = \frac{-1}{z-2} + \frac{1}{z-3}$

or $x(z) = \frac{-z}{z-2} + \frac{z}{z-3}$

Given ROC $\Rightarrow |z| < 2$ then

$x(n) = 2^n\, u(-n-1) - 3^n\, u(-n-1)$

$x(n) = (2^n - 3^n)\, u(-n-1)$

Q.97 **(b)**

A. Unit step function $\leftrightarrow \frac{z}{z-1}$

B. Unit impulse function $\leftrightarrow$ 1

C. sin ωt, t = 0, T, 2T, …….. $\leftrightarrow \frac{z \sin \omega T}{z^2 - 2z\cos\omega T + 1}$

D. cos ωt, t = 0, T, 2T …….. $\leftrightarrow \frac{z - \cos\omega T}{z^2 - 2z\cos\omega T + 1}$

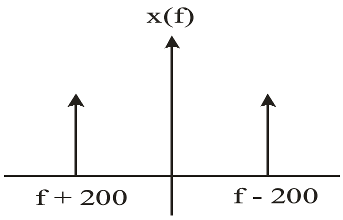
Q.98 **(a)**

$x(t) = 5\cos 400\pi t$. $f_s = 300$

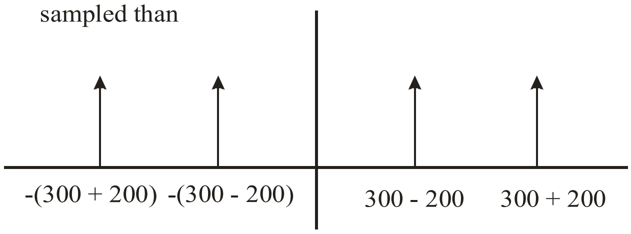
LPF cut off frequency = 150 Hz

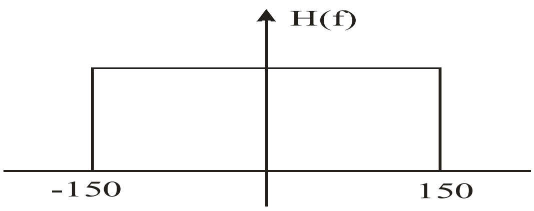
$x(t) = 5\cos 400\pi t$

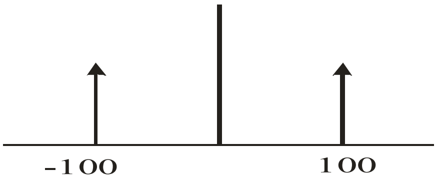
$x(f) = \frac{5}{2}[\delta(f-200) + \delta(f+200)]$



sampled than







Q.99 **(a)**

Q.100 **(a)**

$y[n] = \sum_{k=-\infty}^{n} x[k]$

$y[n-1] = \sum_{k=-\infty}^{n-1} x[k]$

$x[n] = y[n] - y[n-1]$

so inverse of given system is x[n] = y[n] - y[n-1]

Q.101 **(c)**

$\frac{27s+97}{s^2+33s} = F(s)$

Then $f(0^+) = \lim_{s\to\infty} s\, f(s)$

$= \lim_{s\to\infty} s\left[\frac{27s+97}{(s+33)s}\right]$

$f(0^+) = \lim_{s\to\infty} s\left[\frac{27+97/s}{(1+33/s)}\right] = 27$

Q.102 **(d)**

a. $y(n+2)+y(n+1) + y(n) = 2x(n+1) + x(n)$ ↔Linear, time-invariant dynamic

b. $n^2 y^2(n) + y(n) = x^2(n)$ ↔ Non-linear,time-variable memory less

c. $y(n+1) + ny(n) = 4nx(n)$ ↔ Linear, time variable dynamic

d. $y(n+1)\, y(n) = 4x(n)$ ↔ Non linear, time-invariant, dynamic

Q.103 **(d)**

$h(n) = a^n\, u(n+2)\ |a| < 1$

Q.104 **(d)**

Rectangular pulse convolute with another rectangular pulse result will be Rectangular pulse of duration 2T

Q.105 **(a)**

By the half – wave symmetry property in Fourier series extension

Q.106 (a)

$$x(t)=\begin{cases}A, & 0<t<T/2\\ -A, & T/2<t<T\end{cases}$$

$$a_n=\frac{1}{T}\int_T x(t)\cos n\omega_0 t\,dt$$

$$=\frac{1}{T}\int_0^{T/2} A\cos n\omega_0 t\,dt+\frac{1}{T}\int_{T/2}^{T}(-A)\cos n\omega_0 t\,dt$$

$$=\frac{1}{T}\left[\frac{A\sin n\,\omega_0 t}{n\omega_0}\right]_0^{T/2}-\frac{1}{T}\left[\frac{A\sin n\,\omega_0 t}{n\omega_0}\right]_{T/2}^{T}$$

$$=\frac{A}{n\omega_0 T}\left[\sin\left(n\,\omega_0\frac{T}{2}\right)-\left\{\sin n\,\omega_0 T_0-\sin n\,\omega_0\frac{T}{2}\right\}\right]$$

$$=\frac{A}{n2\pi}[\sin n\pi-\sin 2n\pi+\sin n\pi]$$

$$=\frac{A}{2n\pi}[2\sin n\pi-0]$$

$$=\frac{A\sin n\pi}{n\pi}$$

$$=0$$

Q.107 (d)

a. Continuous ↔Non-periodic
b. Discrete ↔ Periodic
c. Periodic ↔ Discrete
d. Non-periodic ↔Continuous

Q.108 (a)

e^{-at} u(t) * e^{-bt} u(t)

$$=\frac{e^{-at}-e^{-bt}}{b-a}u(t)$$

So $e^{k_1 t}$ u(t) * $e^{-k_2 t}$ u(t)

$$=\frac{e^{k_1 t}-e^{-k_2 t}}{k_2-(-k_1)}u(t)$$

$$=\frac{e^{k_1 t}-e^{-k_2 t}}{k_2+k_1}u(t)$$

Q.109 (b)

Put $x(n)=\delta(n)$
Then $y(n)=h(n)$
$h(n-2)=\delta(n)-\delta(n-4)$
or $h(n)=\delta(n+2)-\delta(n-2)$
Where n is replace by n - 2
y(n-2)=x(n)-x(n-4)

Q.110 (d)

It is a Laplace transform property

Q.111 (d)

$x(t)=t\,e^{-at}u(t)$
Let $g(t)=e^{-at}u(t)$
Then $g(j\omega)$ = Fourier Transform $[g(t)]=\frac{1}{a+j\omega}$
Then $x(t)=t\,g(t)$
or $x(\omega)=+j\frac{d}{d\omega}[g(j\omega)]$
$=+j\frac{d}{d\omega}\left(\frac{1}{a+j\omega}\right)$
or $x(j\omega)=\frac{1}{(a+j\omega)^2}$

Q.112 (b)

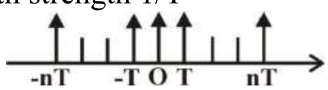
$$\frac{e^{-as}}{s}\xleftrightarrow{\text{Inverse Laplace Transform}}u(t-a)$$

Q.113 (c)

Rectangular pulse of different width are convolved together gives Trapezoidal wave

Q.114 (d)

a. Impulse train ↔Impulse train with strength 1/T



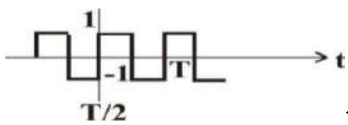
b. Full wave rectified sine function ↔Only even harmonics are present

c. $\sin\frac{2\pi t}{6}\cdot\cos\frac{4\pi t}{6}$ ↔

$$\alpha_3=\frac{1}{2j};\alpha_{-3}=-\frac{1}{2j};$$

$$\alpha_1=-\frac{1}{2j};\alpha_{-1}=\frac{1}{2j}$$

d.



↔Only odd harmonics are present

Q.115 (d)

All other options (a, b, c) are causal but in option (d) its impulse response depends on future value of applied input.

Q.116 (a)

Q.117 (c)

a. Free and Forced response ↔Non-homogenous differential equation
b. Z-transforms ↔ Discrete time system
c. Probability ↔ Random processes
d.Fourier series ↔Dirichlet conditions

Q.118 (b)

$A \xleftrightarrow{\text{Fourier Transform}} 2\pi A\delta(\omega)$

or $A \xleftrightarrow{\text{Fourier Transform}} A\delta(f)$

Put A = 1

$1 \xleftrightarrow{\text{Fourier Transform}} \delta(f)$

Q.119 (b)

$$\frac{d^2y}{dt^2} = x(t-2)u(t-2) + \frac{d^2x}{dt^2}$$

Taking laplace transform

s^2 y(s) = e^{-2s} x(s) + s^2 x (s)

or $\frac{Y(s)}{X(s)} = \frac{s^2 + e^{-2s}}{s^2} = 1 + \frac{e^{-2s}}{s^2}$

Q.120 (d)

f(t) = u(t – 1) + u(t – 2) – 2u(t – 3)

{x(t –a) ↔ X(s) e^{-a2}}

F(s) = $\frac{1}{s}e^{-s} + \frac{1}{s}e^{-2s} - \frac{2}{s}e^{-3s}$

F(s) = $\frac{1}{s}[e^{-s} + e^{-2s} - 2e^{-3s}]$

Q.121 (d)

A. $\frac{10}{s(s+10)}$ = F(s)

⇒ by partial fraction

$$\begin{cases} \frac{1}{s} \xleftrightarrow{\text{InverseLaplaceTransform}} u(t) \\ \frac{1}{s+a} \xleftrightarrow{\text{InverseLaplaceTransform}} e^{-at}u(t) \end{cases}$$

F(s) = $\frac{1}{s} - \frac{1}{s+10}$

Taking Inverse Laplace Transform

f(t) = u(t) – e^{-10t} u(t)

f(t) = (1– e^{-10t}) u(t)

B. $\frac{10}{s(s^2+10)}$ = F(s)

By partial fraction

F(s) = $\frac{1}{s} - \frac{1}{s+10}$

Taking Inverse Laplace Transform we get

f(t) = u(t) – cos $\sqrt{10}$ t u(t)

= (1 – cos $\sqrt{10}$ t) u(t)

C. $\frac{(s+10)}{(s+10)^2+100}$ = f(s)

Then Inverse Laplace Transform of F(s) = f(t)

= e^{-10t} cos 10t u(t)

D. F(s) = 10

Then Inverse Laplace Transform [F(s)] = f(t) = 10 s(t)

Q.122 (d)

sin ωt $\xleftrightarrow{\text{Laplace Transform}} \frac{\omega}{s^2+\omega^2}$

Q.123 (b)

$$L[f(t)] = F(s) = \int_0^{\infty} f(t)e^{-st}dt$$

hence

1. L[f(t – a) u(t – a)] = F(s) . e^{-as} True
2. L[f(t)] = $\frac{-dF(s)}{ds}$ True
3. L[(t – a) f(t)] = L[t f(t) – a f(t)] = $\left[\frac{-dF(s)}{ds} - aF(s)\right]$
4. L $\left[\frac{df(t)}{dt}\right]$ = sF(s) – f(0^+) True

Q.124 (b)

a. Real and even symmetric ↔ Real and even symmetric
b. Real and odd symmetric ↔ Imaginary odd symmetric
c. Imaginary and even symmetric ↔ Imaginary and even symmetric
d. Imaginary and odd symmetric ↔ Real odd symmetric

Q.125 (a)

a. Fourier series ↔Discrete, aperiodic
b.Fourier transform ↔ Continuous, aperiodic
c. Discrete time Fourier ↔ Continuous, periodic
d.Discrete Fourier transform ↔ Discrete, periodic

Q.126 (b)

a. Delta function ↔ Constant function
b. Gate function ↔ Sampling function
c. Normalized Gaussian function ↔ Gaussian function
d. Sinusoidal function ↔ Delta function

Q.127 (b)

Options (a, c, d) have difference between its delayed input & its delayed output
For y(n)=x(n)-x(n-1),
delayed input = delayed output
So it is time invariant

Q.128 (b)

a. y(n) = x(n) ↔ Linear, casual
b. $y(n) = x(n^2)$ ↔ Linear, non-casual
c. $y(n) = x^2(-n)$ ↔ Nonlinear, non-casual
d. $y(n) = x^2(n)$ ↔ Nonlinear, casual

Q.129 (c)

For $n \geq 0 \rightarrow a^n$ & $n < 0 \rightarrow b^n$

$$\sum_{-\infty}^{\infty} |h(n)| < \infty$$

$$= \sum_{-\infty}^{-1} b^n + \sum_{0}^{\infty} a^n$$

| a | < 1 & | b | > 1 are condition for system to be BIBO stable

Q.130 (c)

$$x(t) = \sum_{k-\infty}^{\infty} \delta(t - kT) \text{ then}$$

Taking Fourier transform

$$X[f] = \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta(t - {}^{k}/_{T})$$

Q.131 (d)

Q.132 (c)

a. exp(-αt) u(t), α> 0 ↔ $\left(f - \frac{a}{t_0}\right)$
b. exp(-α|t|), α> 0 ↔ $\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$
c. t exp(-αt) u(t), α> 0 ↔ $\frac{1}{(\alpha + j2\pi f)^2}$
d. exp (j 2παt / t_0) ↔ $\frac{1}{\alpha + j2\,\pi f}$

Q.133 (a)

Q.134 (d)

$H(f) = |H(f)|\, e^{-t\theta(f)} \equiv 2e^{-j4\pi f}$
Then y(t) = x(t) * H(t)
$y(t) = X(t) * 2e^{-4\pi jf}$
y(t) = 2X(t - 4π)

Q.135 (a)

If input = u(t) ⟶ output = δ(t)
If input = nu(n) ⟶ output = u(n)

Q.136 (a)

a. $[(1/2)^n + (1/3)^n]\, u(n) \leftrightarrow$ |z| > ½

b. $(1/2)^n u(n) - (1/3)^n u(-n-1) \leftrightarrow |z| < 1/3$

c. $(1/2)^n u(-n-1) + (1/3)^n u(n) \leftrightarrow (1/3) < |z| < (1/2)$

d. $[(1/2)^n + (1/3)^n]\, u(-n-1) \leftrightarrow |z| < 1/3$ and $|z| > 1/2$

Q.137 (b)

Q.138 (b)

Q.139 (a)

$a^n u(-n-1) \leftrightarrow \frac{z}{z-a}\ |Z| > |a|$

$a^n u(-n-1) \leftrightarrow \frac{z}{z-a}\ |Z| < |a|$

Q.140 (d)

Q.141 (b)

It is a trigonometric periodic power signal

Q.142 (a)

$x_1(t) = 2\sin \pi t + \cos 4\pi t$

$T_1 \rightarrow \frac{2\pi}{\pi}$, $T_2 \rightarrow \frac{2\pi}{4\pi} = \frac{1}{2}$

$\frac{T_1}{T_2} = \frac{2}{\frac{1}{2}} = \frac{4}{1}$

$x_2(t) = \sin 5\pi t + 3\sin 13\pi$

$= \frac{2\pi}{5\pi}, \frac{2\pi}{13\pi}$

$T_1 \rightarrow \frac{2}{5}$, $T_2 \rightarrow \frac{2}{13}$

$\frac{T_1}{T_2} = \frac{13}{5}$

T = LCM (T_1 & T_2)

T = 2

Q.143 (b)

By pulling all options value in expression

a) u(t)

$u(t) + \int_0^\infty u(\tau)\, x(t-\tau)\, d\tau = \delta(t) + x(t)$,

not equal

b) $\delta(t)$

$\delta(t) + \int_0^\infty \delta(\tau)\, x(t-\tau)\, d\tau = \delta(t) + x(t)$

Q.144 (c)

$H(t) = h_1(t) * h_2(t)$

= cascading of two systems

= convolution of two system

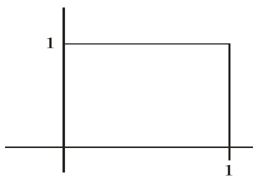
$= \delta(t-0.5) * \delta(t-0.5) = \delta(t-1)$

Q.145 (b)

h(n) = as per figure

$2\delta(n) - \delta(n-2)$

x(t) =



Square pulse of unit amplitude

$y(t) = x(n) * h(n)$

$x(n) = u(n) - u(n-1)$

$y(n) = [u(n) - u(n-1)] * [2\delta(n) - \delta(n-2)]$

$= 2u(n) - u(n-2) - 2u(n-1) + u(n-3)$

$= 2[u(n) - u(n-1)] - [u(n-2) - u(n-3)]$

$= \{0 - 1\} \rightarrow$ 2unit $\{2 - 3\} \rightarrow$ 1 unit

Q.146 (a)

$x(t) = e^{-b|t|} \leftrightarrow \frac{2b}{b^2+\omega^2}$

$e^{-b|t|} \leftrightarrow e^{-bt}\, u(t) + e^{bt}\, u(-t)$

$\downarrow$

$\frac{1}{b+j\omega} + \frac{1}{b-j\omega} = \frac{2b}{b^2+\omega^2}$

Q.147 (d)

Fourier Transform

$u(t) \leftrightarrow \pi\, \delta(\omega) + \frac{1}{j\omega}$

Q.148 (a)

$H(s) = \frac{(s+c)}{(s+a)(s+b)}$

$Y(s) = H(s) \,.\, X(s)$

For first input $x(t) = u(t)$

X(s) Laplace of input signal is $\frac{1}{s}$

$Y(s) = \frac{H(s+c)}{s(s+a)(s+b)}$

$= \frac{A}{s} + \frac{B}{s+a} + \frac{C}{s+b}$

$A = \frac{Hc}{ab} = 2 \quad s = 0$

a = 1

b = 3

ab = 3

For second input $x(t) = e^{-2t}u(t)$

X(s) Laplace of input signal is $\frac{1}{s+2}$

$Y(s) = \frac{F}{s+a} + \frac{G}{s+b}$

Then c = 2, H = 3

Q.149 (a)

For a given input $x(t) = e^{-t}\, u(t)$

Laplace transform of x(t) is

$X(s) = \frac{1}{s+1}$

$Y(s) = H(s) \,.\, X(s)$

$Y(s) = \frac{1}{s+1} \times \frac{(s+1)}{s^2+2s+2}$

$Y(s) = \frac{1}{s^2+2s+2}$

$Y(s) = \frac{1}{(s+1)^2+1}$

$(t) = e^{-t} \sin tu(t)$

Q.150 (a)

$f_s = 14$ Hz

$f_m = 5$ Hz

f_s minimum = f_{Nquist}

$= 2 \times f_m = 2 \times 5 = 10$ Hz

Low pass filter f_c = ?

$f_c = f_s – f_m = 14 – 5 = 9$ Hz

f_c minimum = f_{Nquist} - f_m

= 10 – 5 = 5 Hz

5Hz < f_c< 9 Hz

Q.151 (b)

Options (a, c, d) are non causal because output depend on future value of input

Option b) y(t)=5u(t)+3u(t-1) is causal system

Q.152 (b)

I_{RMS} = effective value of the current is =

$\sqrt{2^2 + 36 + 24}$

$= \sqrt{64}$ = 8 A

Q.153 (d)

a)f(t)+f(-t)=0 ↔

Sine terms can exist

b) f(t)-f(-t) = 0 ↔

The dc and cosine terms can exist

c) f(t)+f(t-T/2) = 0 ↔

Odd harmonics can exist

d) f(t)-f(t-T/2) = 0 ↔

Even harmonics can exist